Numerical modeling of cavitating venturi – a flow control element of propulsion system

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Introduction

1. What is cavitating venturi?
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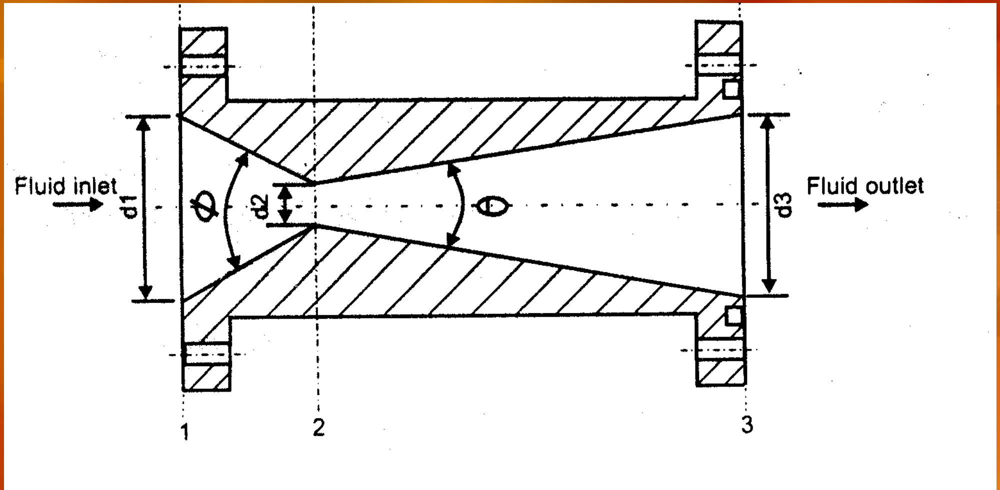
3. Past work
    - Experimental
    - Analytical
4. Present Contribution

What is cavitating venturi ?

A venturi operating with a throat pressure equal to the vapor pressure of the fluid corresponding to the temperature is called a ‘cavitating venturi’



Use of cavitating venturi

- Propellant flow and mixture ratio in the combustion chamber is controlled by cavitating venturi
- It maintains constant propellant flowrate for fixed inlet conditions (pressure and temperature) and wide range of outlet pressure
- During ignition, it maintains constant flowrate while pressure is building from ambient condition
- At steady-state condition, it maintains constant flowrate while pressure fluctuates due combustion oscillation

Past Work (Experimental)

Randall (1951) – Journal of American Society

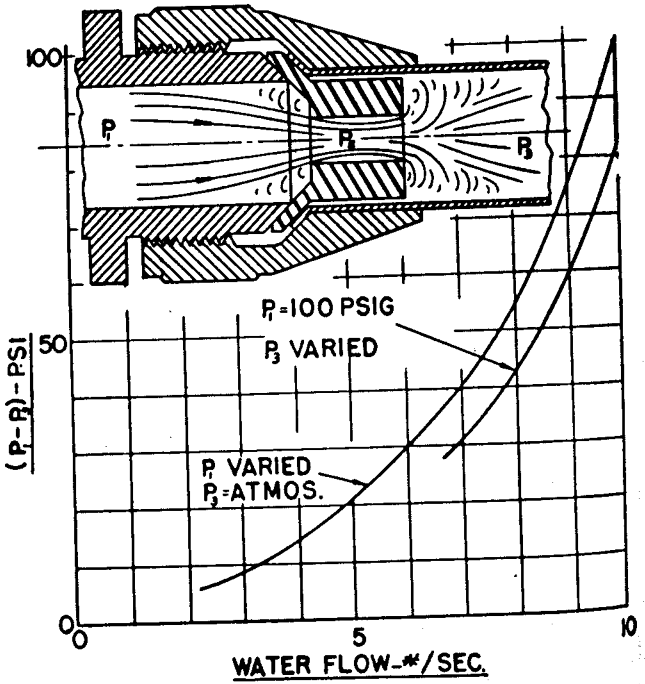


FIG. 1. THICK PLATE ORIFICE AND FLOW CHARACTERISTICS

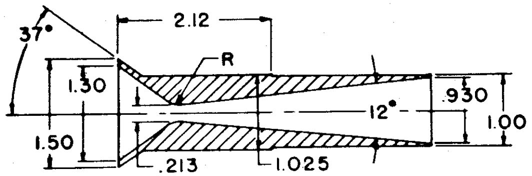


FIG. 4 TYPICAL CAVITATING VENTURI ORIFICE

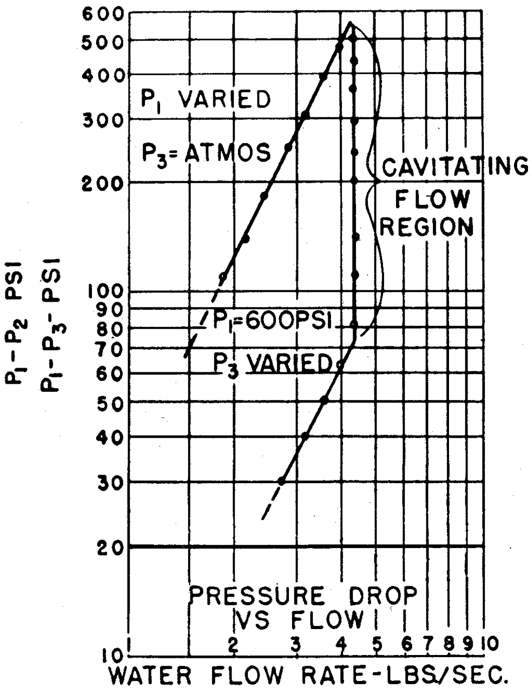


FIG. 5 TYPICAL CAVITATING VENTURI CALIBRATION

Past Work (Analytical)

- Flowrate through cavitating venturi is calculated from the following equation:

$$\dot{m} = C_d A_t \sqrt{2 \rho g_c \left(p - p_{sat} \right)}$$

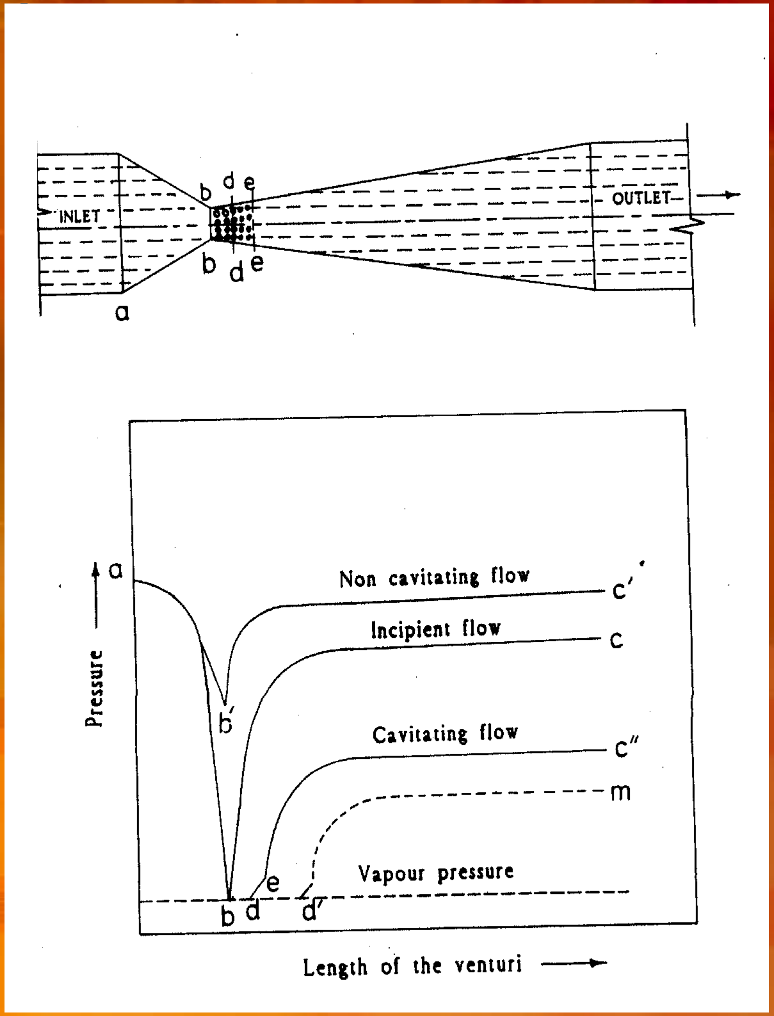
- There has been no published reference of an effort to model cavitating venturi by CFD or network analysis methods
- Modeling of phase change and two-phase flow are required to compute flow through cavitating venturi

Present Contribution

- Finite volume model (FVM) of cavitating venturi using Generalized Fluid System Simulation Program (GFSSP) (http://eodd.msfc.nasa.gov/GFSSP/)
- FVM solves for mass, momentum and energy conservation equations in venturi
- Numerical predictions of cavitations at the throat
- Predictions of choked flow of liquid when it cavitates

2. Flow Characteristics

- Pressure decreases in the converging section and increases in the diverging section
- With decrease of downstream pressure, pressure at throat reaches vapor pressure (Incipient cavitations)
- With further reduction of downstream pressure, two phase condition extends (Cavitating flow)
- Vapor bubble collapses further downstream and flow becomes single phase



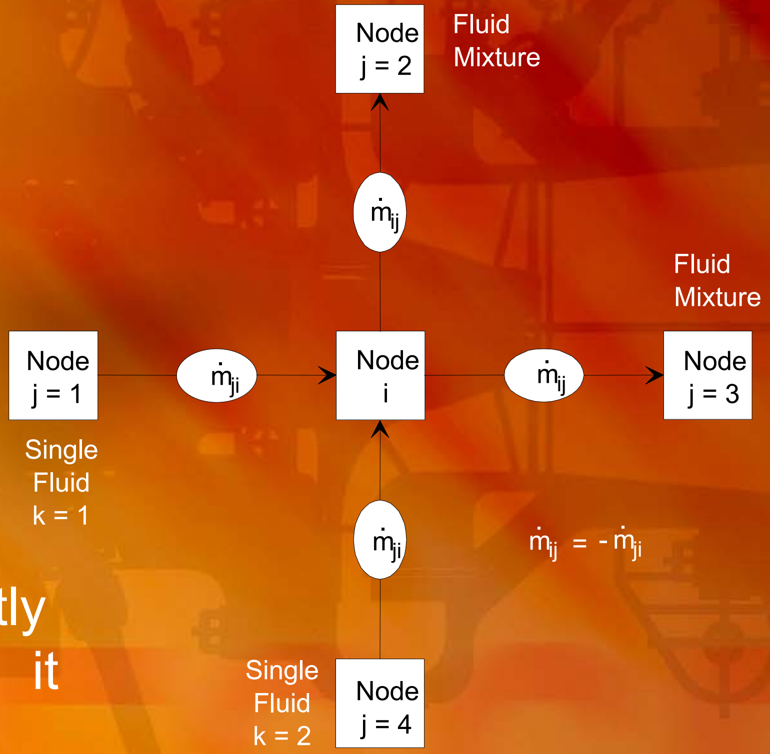
3. Numerical Model

- Governing Equations in Finite Volume Scheme
- Solution of Governing Equations
- Generalized Fluid System Simulation Program (GFSSP)

GOVERNING EQUATIONS

MASS CONSERVATION EQUATION

$$\frac{m_{\tau+\Delta\tau}-m_{\tau}}{\Delta\tau}=\sum_{j=1}^{j=n}\dot{m}_{ij}$$



Node
j = 2

Fluid
Mixture

$\dot{m}_{ij}$

Fluid
Mixture

Node
j = 1

$\dot{m}_{ji}$

Node
i

$\dot{m}_{ij}$

Node
j = 3

Single
Fluid
k = 1

$\dot{m}_{ji}$

$\dot{m}_{ij} = -\dot{m}_{ji}$

Single
Fluid
k = 2

Node
j = 4

Note : Pressure does not appear explicitly in Mass Conservation Equation although it is earmarked for calculating pressures

GOVERNING EQUATIONS

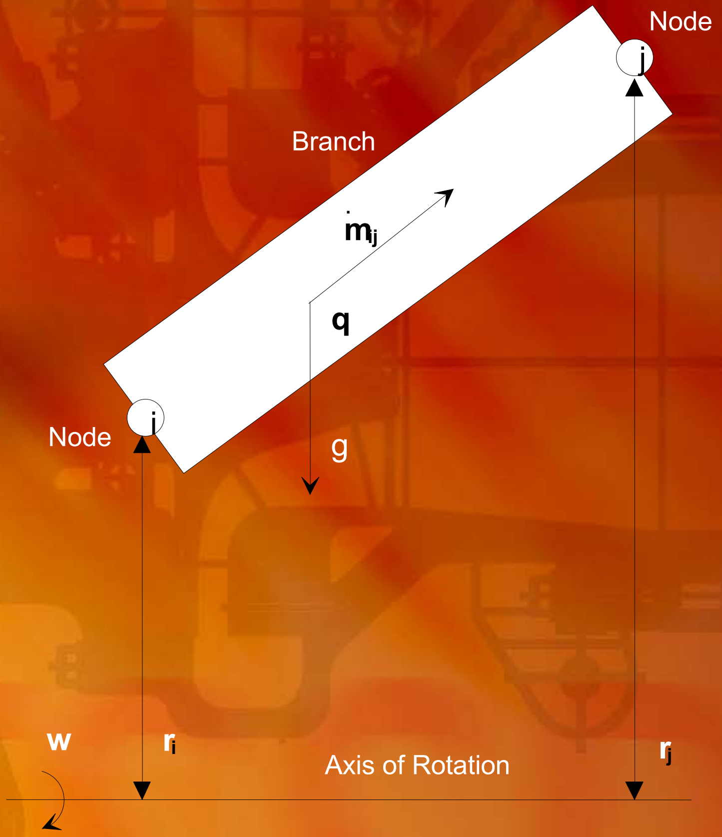
MOMENTUM CONSERVATION EQUATION

- Represents Newton’s Second Law of Motion

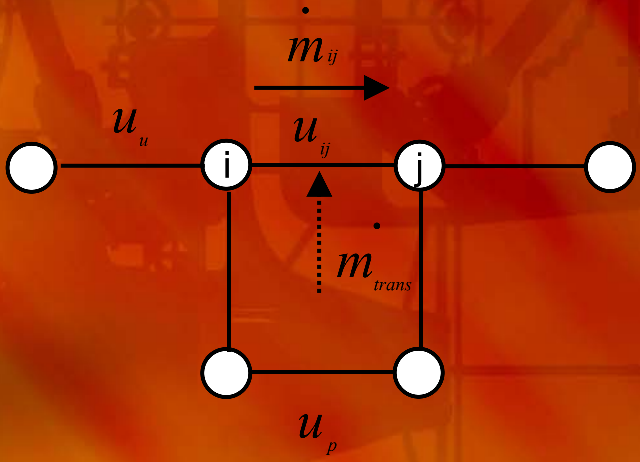
Mass x Acceleration = Forces

- Unsteady
- Longitudinal Inertia
- Transverse Inertia

- Pressure
- Gravity
- Friction
- Centrifugal
- Shear Stress
- Moving Boundary
- Normal Stress
- External Force



MOMENTUM CONSERVATION EQUATION



Mass x Acceleration Terms in GFSSP

Unsteady

$$\frac{(mu_{ij})_{\tau+\Delta\tau} - (mu_{ij})_{\tau}}{g_c \Delta\tau}$$

Longitudinal Inertia

$$MAX\left|\dot{m}_{ij},0\right|\left(u_{ij} - u_{u}\right) - MAX\left|-\dot{m}_{ij},0\right|\left(u_{ij} - u_{u}\right)$$

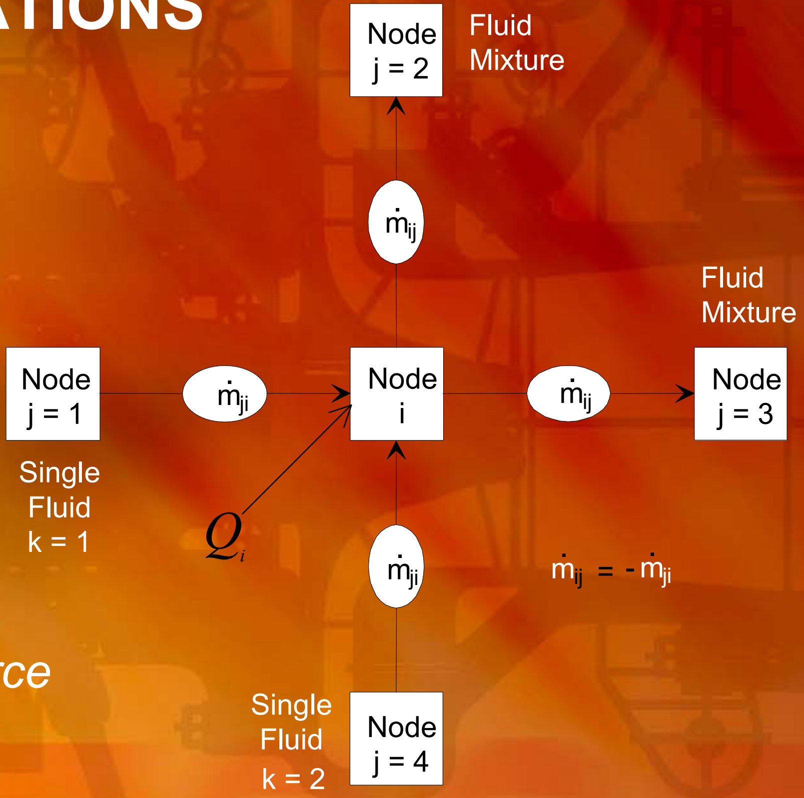
Transverse Inertia

$$+MAX\left|\dot{m}_{trans},0\right|\left(u_{ij} - u_{p}\right) - MAX\left|-\dot{m}_{trans},0\right|\left(u_{ij} - u_{p}\right)$$

GOVERNING EQUATIONS

ENERGY CONSERVATION EQUATION

- Based on Upwind Scheme



Enthalpy Equation

Rate of Increase of Internal Energy =

Enthalpy Inflow - Enthalpy Outflow + Heat Source

$$\frac{m\left(h-\frac{p}{\rho J}\right)_{\tau+\Delta\tau} - m\left(h-\frac{p}{\rho J}\right)_{\tau}}{\Delta\tau} = \sum_{j=1}^{j=n}\left\{MAX\left[-\dot{m}_{ij},0\right]h_j - MAX\left[\dot{m}_{ij},0\right]h_i\right\}+Q_i$$

GOVERNING EQUATIONS

EQUATION OF STATE

For unsteady flow, resident mass in a control volume is calculated from the equation of state for a real fluid

$$m = \frac{pV}{RTz}$$

Z is the compressibility factor determined from higher order equation of state

GOVERNING EQUATIONS

EQUATION OF STATE

- GFSSP uses two separate Thermodynamic Property Packages GASP/WASP and GASPAK
- GASP/WASP uses modified Benedict, Webb & Rubin (BWR) Equation of State
- GASPAK uses “standard reference” equation from
  - National Institute of Standards and Technology (NIST)
  - International Union of Pure & Applied Chemistry (IUPAC)
  - National Standard Reference Data Service of the USSR

SOLUTION PROCEDURE

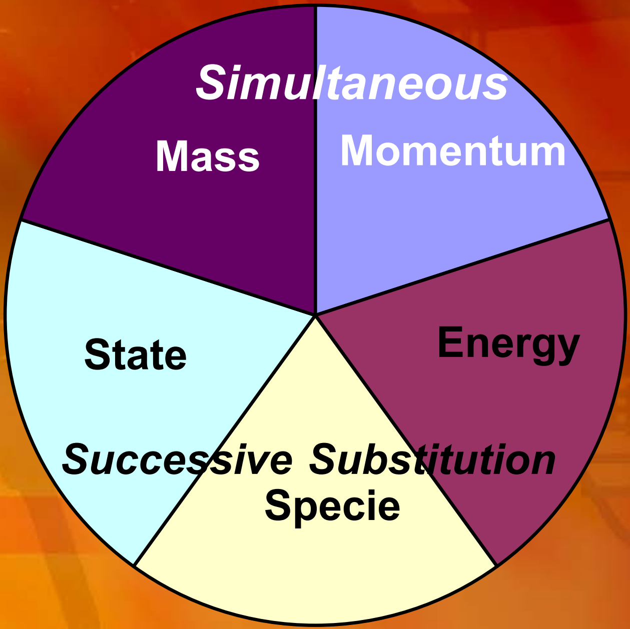
- Non linear Algebraic Equations are solved by
  - Successive Substitution
  - Newton-Raphson
- GFSSP uses a Hybrid Method
  - SASS (Simultaneous Adjustment with Successive Substitution)
  - This method is a combination of Successive Substitution and Newton-Raphson

GFSSP Solution Scheme

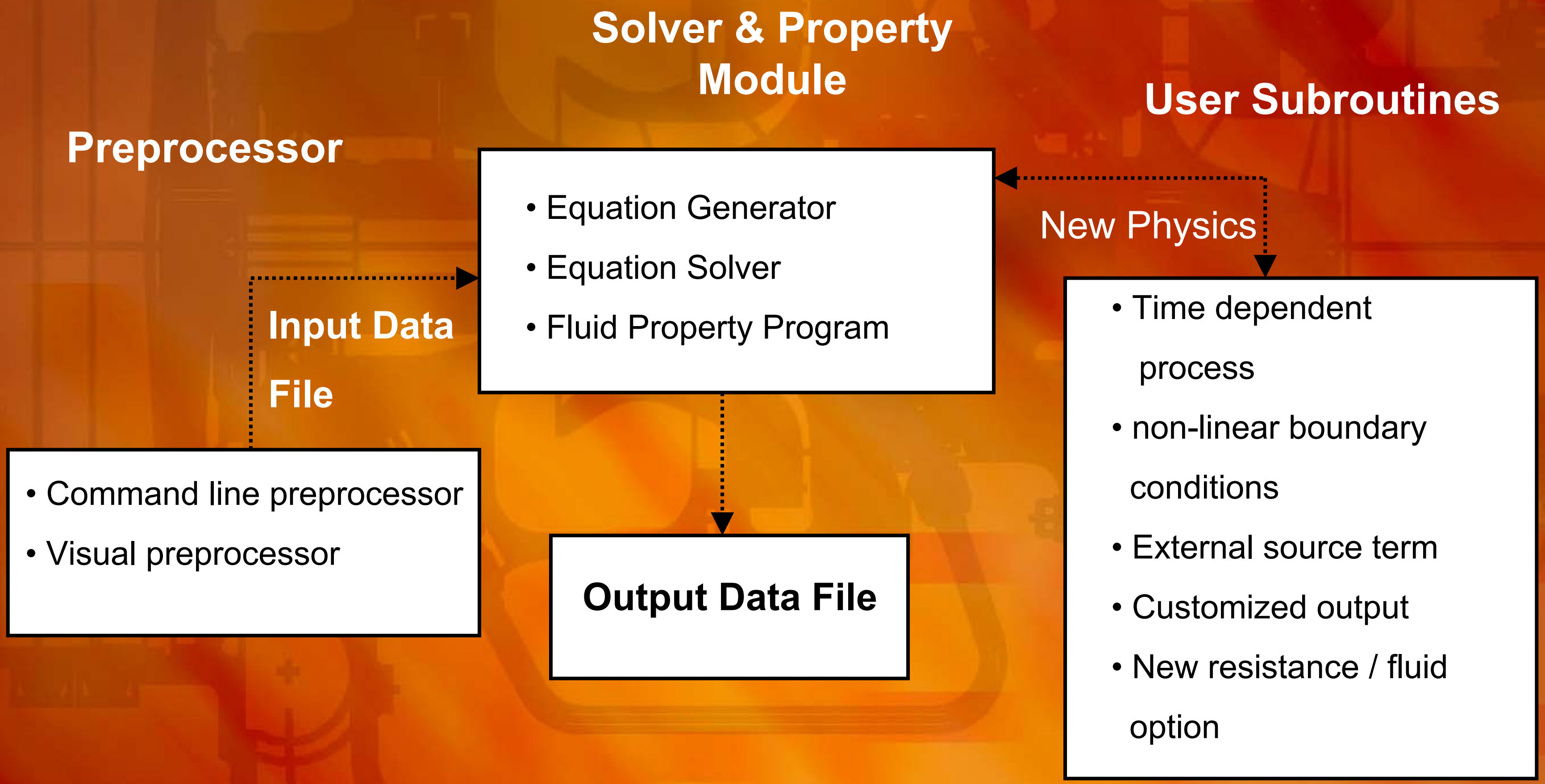
SASS : **S**imultaneous **A**djustment with **S**uccessive **S**ubstitution

Approach : Solve simultaneously when equations are strongly coupled and non-linear

Advantage : Superior convergence characteristics with affordable computer memory



GFSSP PROCESS FLOW DIAGRAM



Solver & Property Module

- Equation Generator
- Equation Solver
- Fluid Property Program

User Subroutines

New Physics

- Time dependent process
- non-linear boundary conditions
- External source term
- Customized output
- New resistance / fluid option

Preprocessor

Input Data File

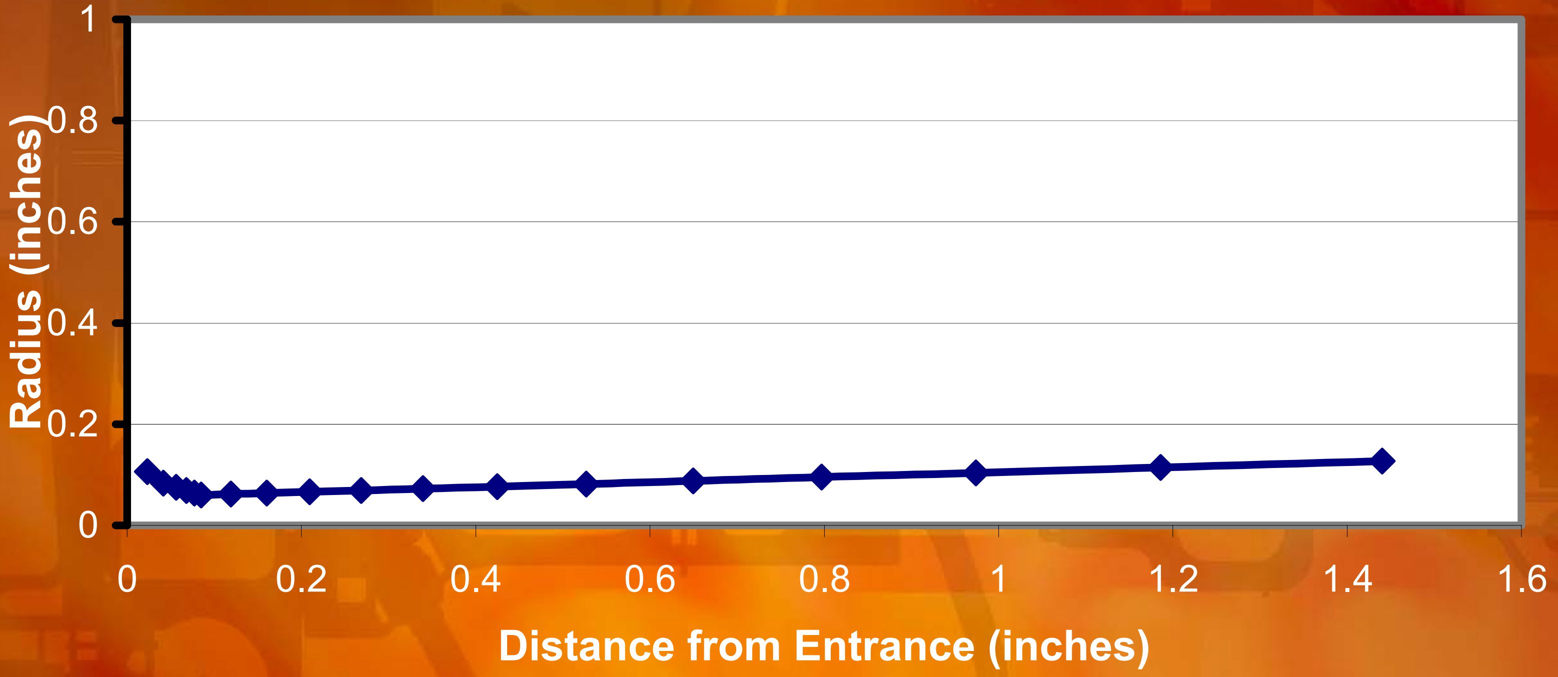
- Command line preprocessor
- Visual preprocessor

Output Data File

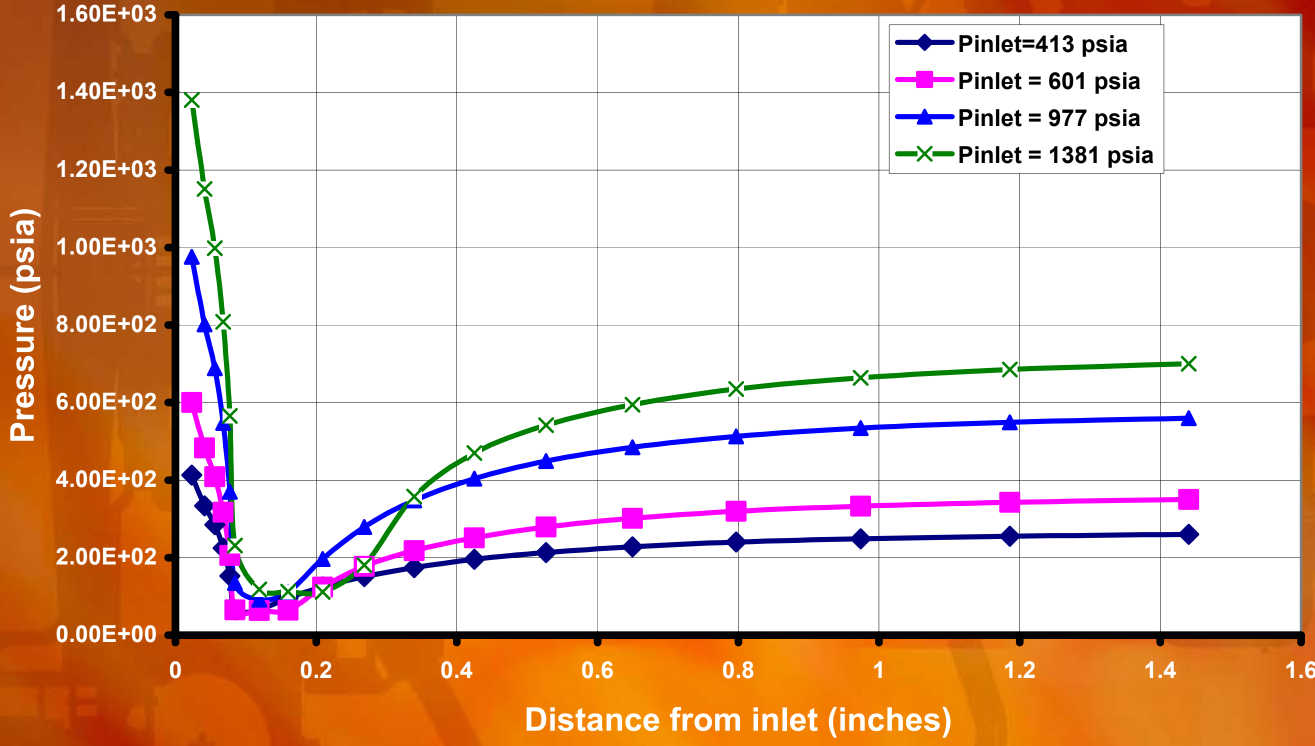
4. Results

1. Finite Volume Discretization of Venturi
2. Pressure Distribution
3. Density Distribution
4. Compressibility Factor
5. “Choked” Flowrate
6. Comparison with Bernoulli model

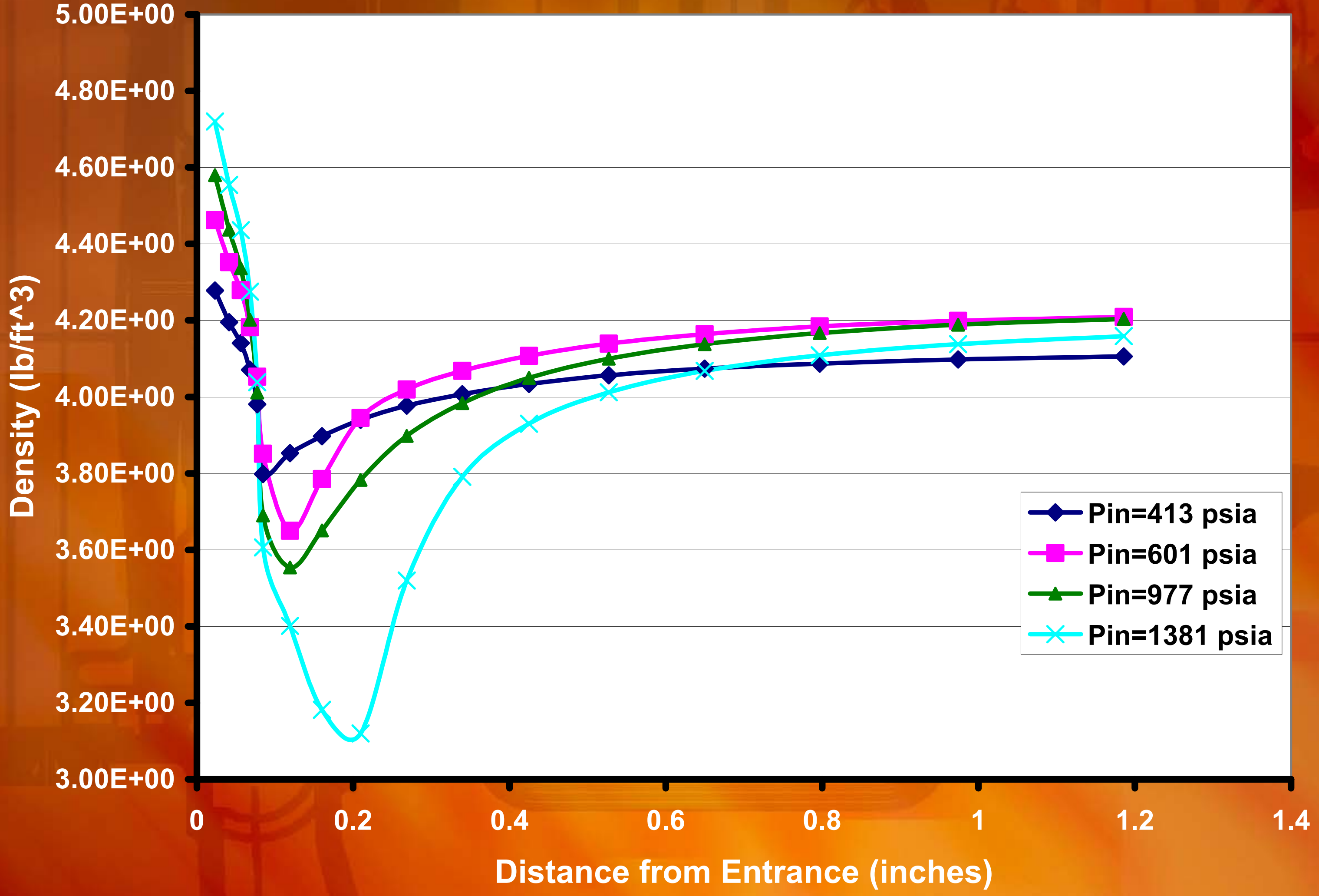
Nozzle Geometry



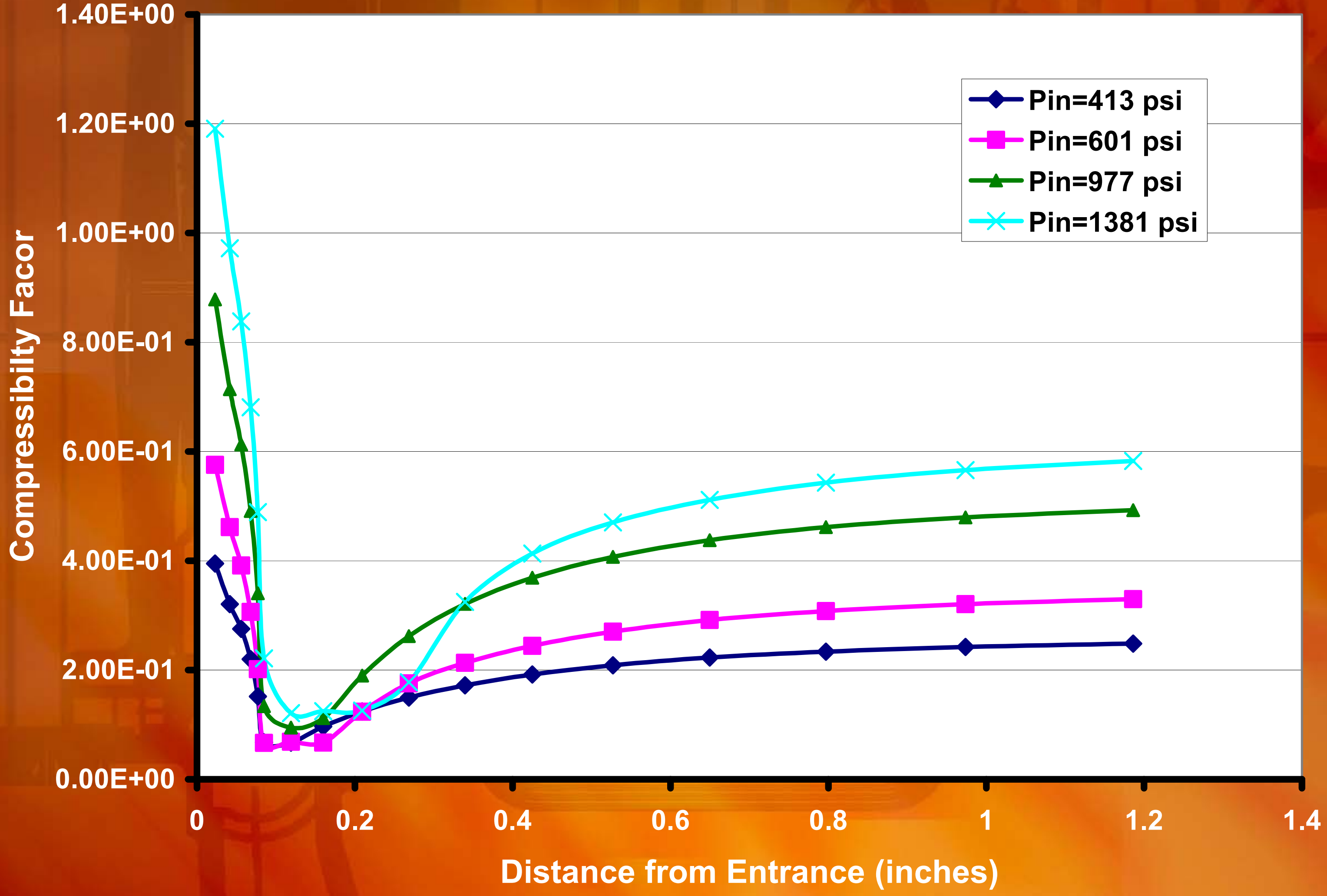
Pressure Distribution in Cavitating Venturi



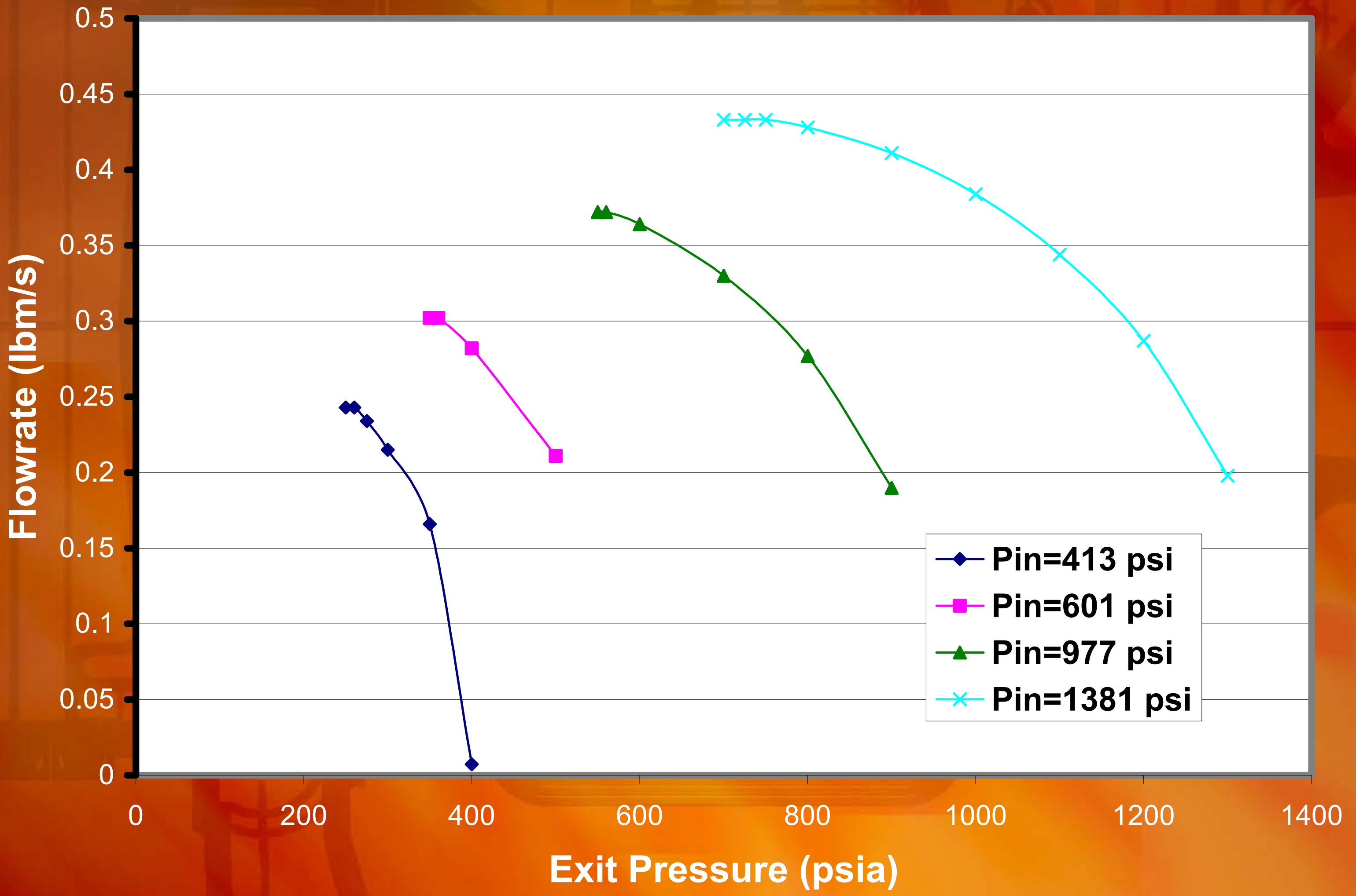
Density Distribution in a Cavitating Venturi



Compressibility factor distribution in a Cavitating Venturi



Effect of Inlet Pressure on Flowrate



Comparison of Predicted Choked Flowrate with Bernoulli Model (Fluid : Hydrogen)

P_{INLET} (PSIA)	T_{INLET} (R)	P_{SAT} (PSIA)	ρ_{INLET} (LBM/FT^3)	A_{THROAT} (IN^2)	M_{DOT} (GFSSP) (LB/S)	C_D	M_{DOT} (BERNOULLI) (LB/S)
413	46.3	55.28	4.265	0.0113	0.243	0.9	0.266
601	46.3	55.28	4.378	0.0113	0.302	0.9	0.332
977	46.3	55.28	4.56	0.0113	0.372	0.9	0.441
1381	46.3	55.28	4.715	0.0113	0.433	0.9	0.537

Discrepancies in flowrate are due to constant density assumption in Bernoulli model

Conclusions

- Cavitating flow in venturi can be predicted by solving conservation equations of mass, momentum and energy conservation equations in conjunction with thermodynamic equation of state
- Bernoulli model overpredicts the flowrate due to constant density assumption
- Rapid drop in compressibility indicates that sound velocity drops significantly at throat which may be the reason for occurrence of choked flow at low velocity

References & Acknowledgements

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