Numerical modeling of cavitating venturi – a flow control element of propulsion system

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Introduction

1. What is cavitating venturi?
2. Use of cavitating venturi
3. Past work
   • Experimental
   • Analytical
4. Present Contribution
What is cavitating venturi?

A venturi operating with a throat pressure equal to the vapor pressure of the fluid corresponding to the temperature is called a ‘cavitating venturi’
Use of cavitating venturi

- Propellant flow and mixture ratio in the combustion chamber is controlled by cavitating venturi
- It maintains constant propellant flowrate for fixed inlet conditions (pressure and temperature) and wide range of outlet pressure
- During ignition, it maintains constant flowrate while pressure is building from ambient condition
- At steady-state condition, it maintains constant flowrate while pressure fluctuates due combustion oscillation
Past Work
(Experimental)

Randall (1951) – Journal of American Society
Past Work (Analytical)

• Flowrate through cavitating venturi is calculated from the following equation:

\[ m = C_d A_t \sqrt{2 \rho g_c (p - p_{sat})} \]

• There has been no published reference of an effort to model cavitating venturi by CFD or network analysis methods.

• Modeling of phase change and two-phase flow are required to compute flow through cavitating venturi.
Present Contribution

- Finite volume model (FVM) of cavitating venturi using Generalized Fluid System Simulation Program (GFSSP) (http://eodd.msfc.nasa.gov/GFSSP/)
- FVM solves for mass, momentum and energy conservation equations in venturi
- Numerical predictions of cavitations at the throat
- Predictions of choked flow of liquid when it cavitates
2. Flow Characteristics

- Pressure decreases in the converging section and increases in the diverging section.
- With decrease of downstream pressure, pressure at throat reaches vapor pressure (Incipient cavitations).
- With further reduction of downstream pressure, two phase condition extends (Cavitating flow).
- Vapor bubble collapses further downstream and flow becomes single phase.
3. Numerical Model

- Governing Equations in Finite Volume Scheme
- Solution of Governing Equations
- Generalized Fluid System Simulation Program (GFSSP)
GOVERNING EQUATIONS

MASS CONSERVATION EQUATION

\[
\frac{m_{t+\Delta t} - m_t}{\Delta t} = \sum_{j=1}^{n} m_{ij}
\]

Node \( j = 1 \), \( m_{ji} \)

Node \( j = 3 \), \( m_{ij} \)

Node \( j = 2 \), \( m_{ij} \)

Node \( j = 4 \), \( m_{ij} \)

Node \( i \), \( \dot{m}_i \)

Note: Pressure does not appear explicitly in Mass Conservation Equation although it is earmarked for calculating pressures
GOVERNING EQUATIONS

MOMENTUM CONSERVATION EQUATION

- Represents Newton’s Second Law of Motion

\[ \text{Mass} \times \text{Acceleration} = \text{Forces} \]

- Unsteady
- Longitudinal Inertia
- Transverse Inertia
- Pressure
- Gravity
- Friction
- Centrifugal
- Shear Stress
- Moving Boundary
- Normal Stress
- External Force
MOMENTUM CONSERVATION EQUATION

Mass x Acceleration Terms in GFSSP

Unsteady

\[
\left( m_\text{ij} u \right)_\tau - \left( m_\text{ij} u \right)_{\tau + \Delta \tau} \\
g \Delta \tau
\]

Longitudinal Inertia

\[
MAX \{ m_\text{ij}, 0 \} \left( u_\text{ij} - u_\text{u} \right) - MAX \{ m_\text{ij}, 0 \} \left( u_\text{ij} - u_\text{u} \right)
\]

Transverse Inertia

\[
+ MAX \{ m_\text{trans}, 0 \} \left( u_\text{ij} - u_\text{p} \right) - MAX \{ m_\text{trans}, 0 \} \left( u_\text{ij} - u_\text{p} \right)
\]
GOVERNING EQUATIONS

ENERGY CONSERVATION EQUATION

• Based on Upwind Scheme

Enthalpy Equation

Rate of Increase of Internal Energy =

Enthalpy Inflow - Enthalpy Outflow + Heat Source

\[
m\left(h - \frac{p}{\rho J}\right)_{i+\Delta\tau}^{\tau+\Delta\tau} - m\left(h - \frac{p}{\rho J}\right)_{i} = \sum_{j=1}^{j=n} \left\{ \text{MAX} \left[ -\dot{m}_j, 0 \right] h_j - \text{MAX} \left[ \dot{m}_j, 0 \right] h_i \right\} + Q_i
\]
GOVERNING EQUATIONS

EQUATION OF STATE

For unsteady flow, resident mass in a control volume is calculated from the equation of state for a real fluid

\[ m = \frac{pV}{RTz} \]

\( Z \) is the compressibility factor determined from higher order equation of state
GOVERNING EQUATIONS

EQUATION OF STATE

• GFSSP uses two separate Thermodynamic Property Packages GASP/WASP and GASPAK
  - GASP/WASP uses modified Benedict, Webb & Rubin (BWR) Equation of State
  - GASPAK uses “standard reference” equation from
    • National Institute of Standards and Technology (NIST)
    • International Union of Pure & Applied Chemistry (IUPAC)
    • National Standard Reference Data Service of the USSR
SOLUTION PROCEDURE

• Non linear Algebraic Equations are solved by
  – Successive Substitution
  – Newton-Raphson

• GFSSP uses a Hybrid Method
  – SASS (Simultaneous Adjustment with Successive Substitution)
  – This method is a combination of Successive Substitution and Newton-Raphson
GFSSP Solution Scheme

**SASS** : Simultaneous Adjustment with Successive Substitution

**Approach** : Solve simultaneously when equations are strongly coupled and non-linear

**Advantage** : Superior convergence characteristics with affordable computer memory
GFSSP PROCESS FLOW DIAGRAM

Preprocessor

Input Data File

- Command line preprocessor
- Visual preprocessor

Solver & Property Module

- Equation Generator
- Equation Solver
- Fluid Property Program

Output Data File

User Subroutines

New Physics

- Time dependent process
- non-linear boundary conditions
- External source term
- Customized output
- New resistance / fluid option
4. Results

1. Finite Volume Discretization of Venturi
2. Pressure Distribution
3. Density Distribution
4. Compressibility Factor
5. “Choked” Flowrate
6. Comparison with Bernoulli model
Nozzle Geometry

Distance from Entrance (inches)

Radius (inches)
Pressure Distribution in Cavitating Venturi

- Pinlet = 413 psia
- Pinlet = 601 psia
- Pinlet = 977 psia
- Pinlet = 1381 psia
Density Distribution in a Cavitating Venturi

Density (lb/ft²) vs Distance from Entrance (inches)

Pin=413 psia
Pin=601 psia
Pin=977 psia
Pin=1381 psia
Compressibility factor distribution in a Cavitating Venturi

![Graph showing compressibility factor distribution with different pressures (Pin=413 psi, Pin=601 psi, Pin=977 psi, Pin=1381 psi).](image)
Effect of Inlet Pressure on Flowrate

Flowrate (lbm/s) vs. Exit Pressure (psia)

Pin=413 psi
Pin=601 psi
Pin=977 psi
Pin=1381 psi
Comparison of Predicted Choked Flowrate with Bernoulli Model (Fluid : Hydrogen)

<table>
<thead>
<tr>
<th>$P_{\text{INLET}}$ (PSIA)</th>
<th>$T_{\text{INLET}}$ (R)</th>
<th>$P_{\text{SAT}}$ (PSIA)</th>
<th>$\rho_{\text{INLET}}$ (LBM/FT$^3$)</th>
<th>$A_{\text{THROAT}}$ (IN$^2$)</th>
<th>$M_{\text{DOT}}$ (GFSSP) (LB/S)</th>
<th>$C_D$</th>
<th>$M_{\text{DOT}}$ (BERNOULLI) (LB/S)</th>
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</thead>
<tbody>
<tr>
<td>413</td>
<td>46.3</td>
<td>55.28</td>
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</table>

Discrepancies in flowrate are due to constant density assumption in Bernoulli model.
Conclusions

• Cavitating flow in venturi can be predicted by solving conservation equations of mass, momentum and energy conservation equations in conjunction with thermodynamic equation of state
• Bernoulli model overpredicts the flowrate due to constant density assumption
• Rapid drop in compressibility indicates that sound velocity drops significantly at throat which may be the reason for occurrence of choked flow at low velocity
References:


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