CFD-Thermal Interactions Short Course, TFAWS 2003

Integrated Fluid-Thermal Analysis from a Thermal-Structures Perspective

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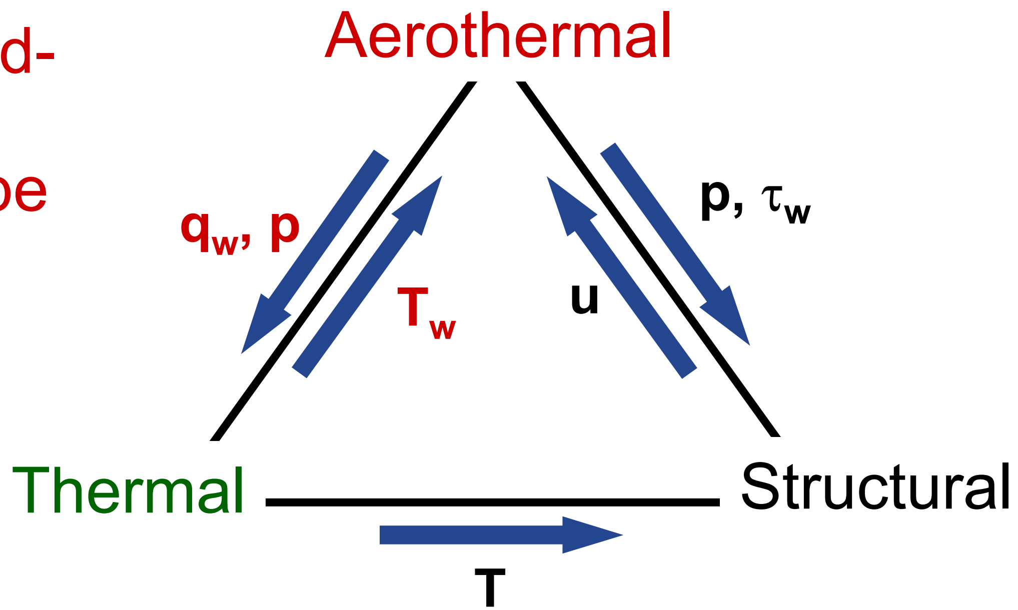
Integrated Fluid-Thermal Analysis for High Speed Flight Vehicles

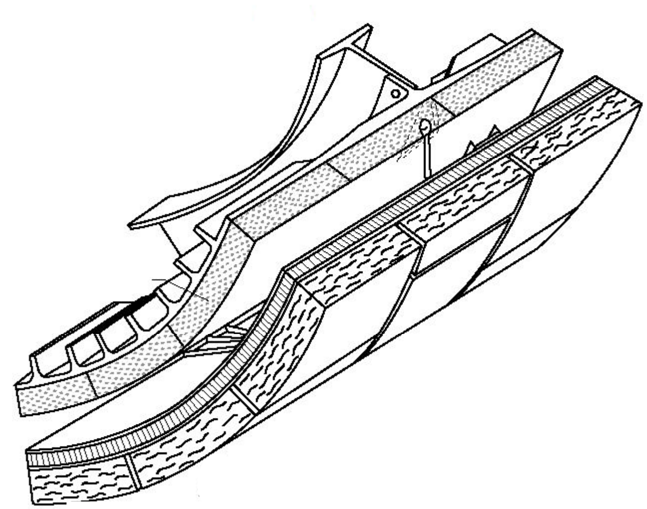
- Coupling at the fluid-thermal interface depends on the type of structure: insulated or non-insulated (hot)



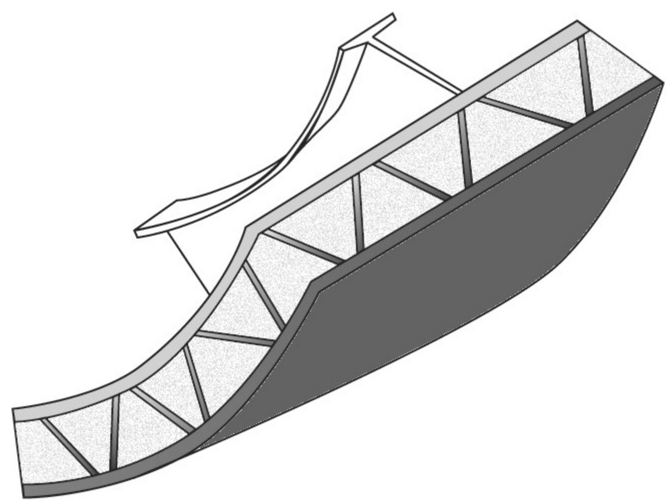
- "Next Generation" Thermal Analysis Methods for Hot Built-up Structures

Thermal-Structural Airframe Concepts for Reusable Launch Vehicles

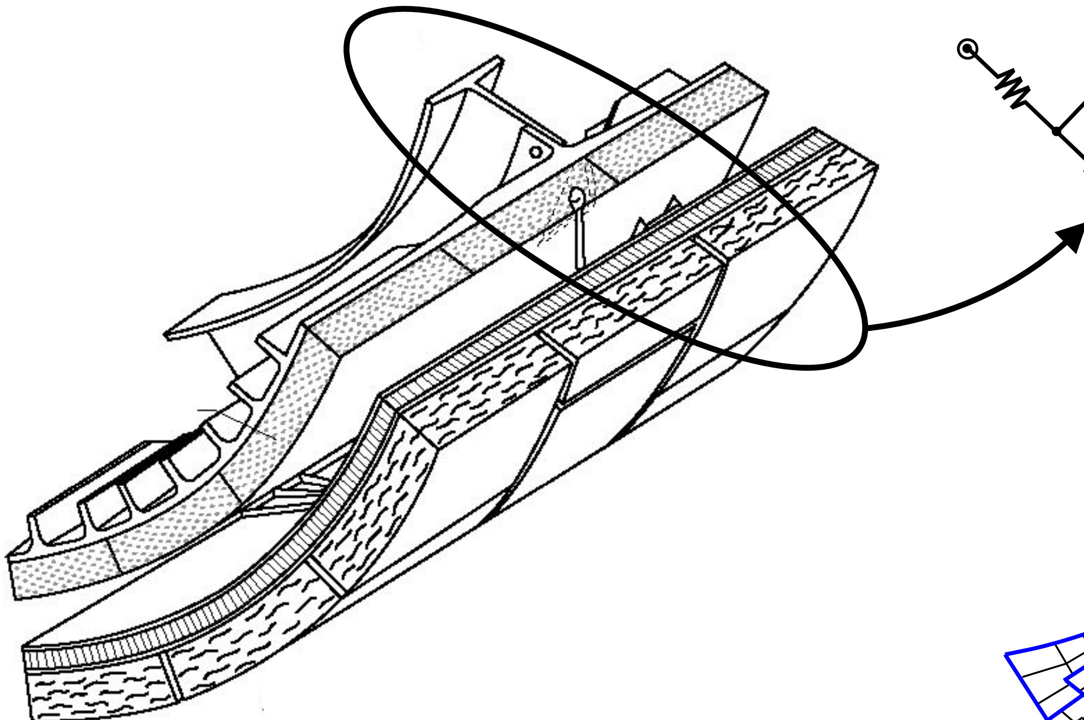
Cryotank with aeroshell and insulating thermal protection system

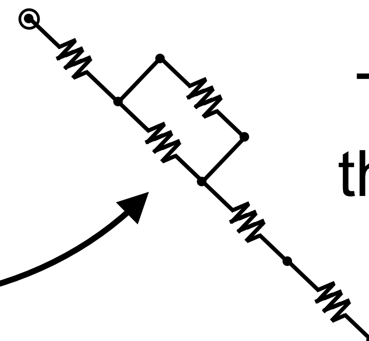


Integrated hot structure where thermal protection also carries load

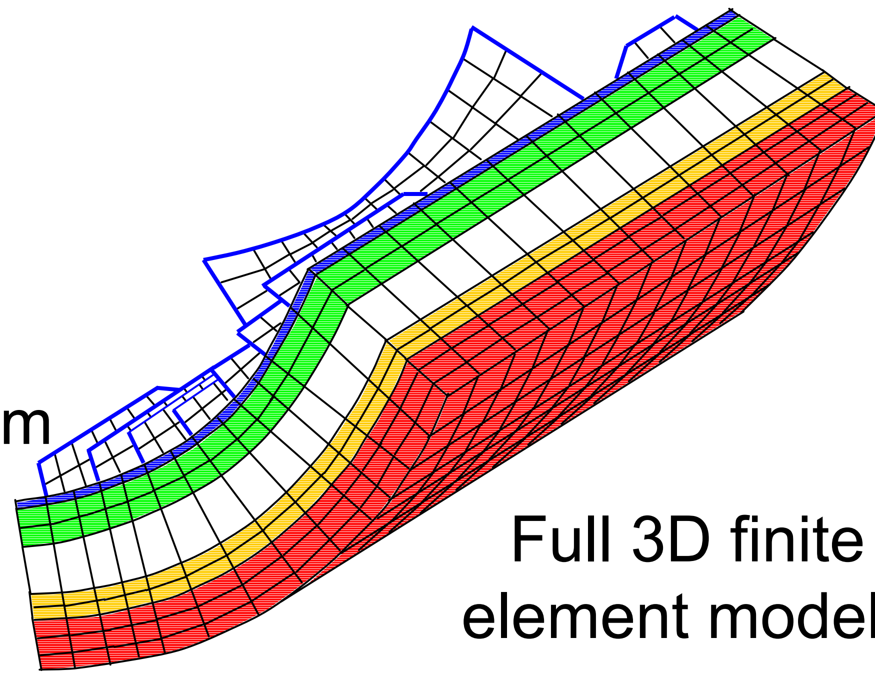


Airframe Thermal Analysis State-of-the-Art





Through-the-thickness plug models



Full 3D finite element models

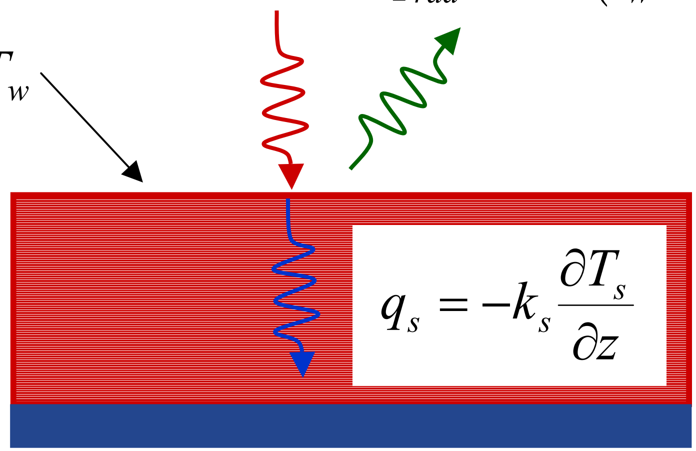
Transient nonlinear problem
- Conduction
- Radiation exchange
- Convection

CFD-Thermal Interactions for Insulated Structures are (Approximately) Decoupled

$$q_{aero} = -k_f \frac{\partial T_f}{\partial z}$$

$$q_{rad} = \varepsilon\sigma\left(T_W^4 - T_\infty^4\right)$$

$$T_f = T_s = T_w$$



insulation

$k_s << 1$

substructure

$k_s >> 1$

Heat conducted into the structure is small

$$q_s \approx 0$$

$$\Downarrow$$

$$q_{aero} \approx q_{rad}$$

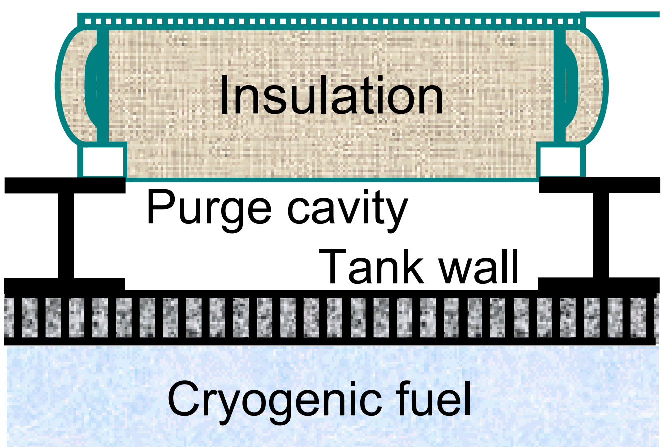
Iterate the fluid energy equation at the wall boundary until

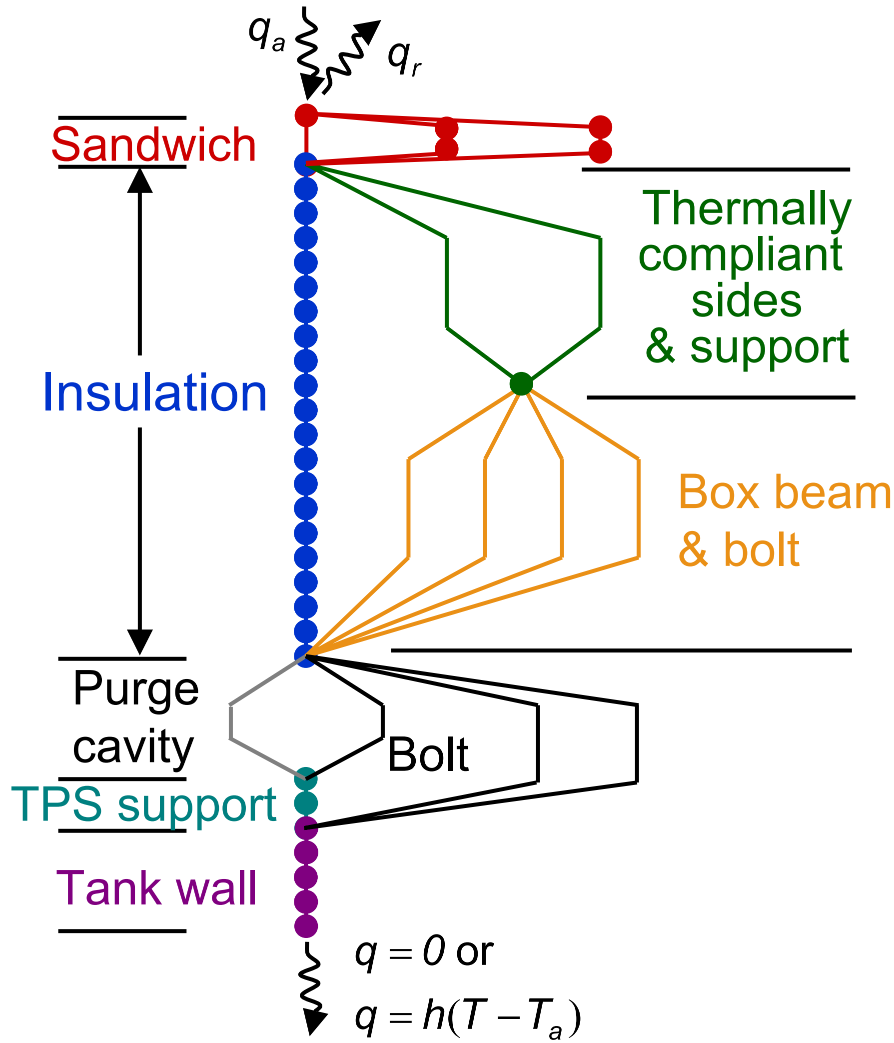
$$q_W = k_f \left.\frac{\partial T_f}{\partial z}\right|_W = \varepsilon\sigma\left(T_W^4 - T_\infty^4\right)$$

Through-the-Thickness Plug Models are Adequate for Insulated Structures since In-plane Temperature Gradients are Small

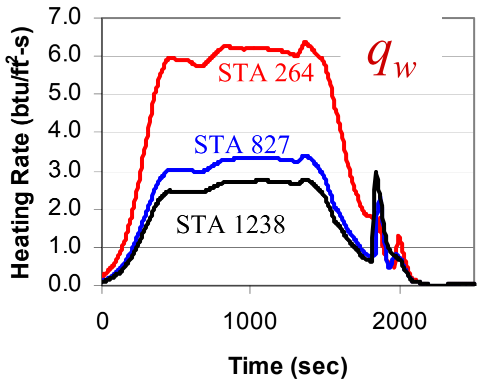
Through-the-Thickness Plug Model of Complex Metallic TPS Concept

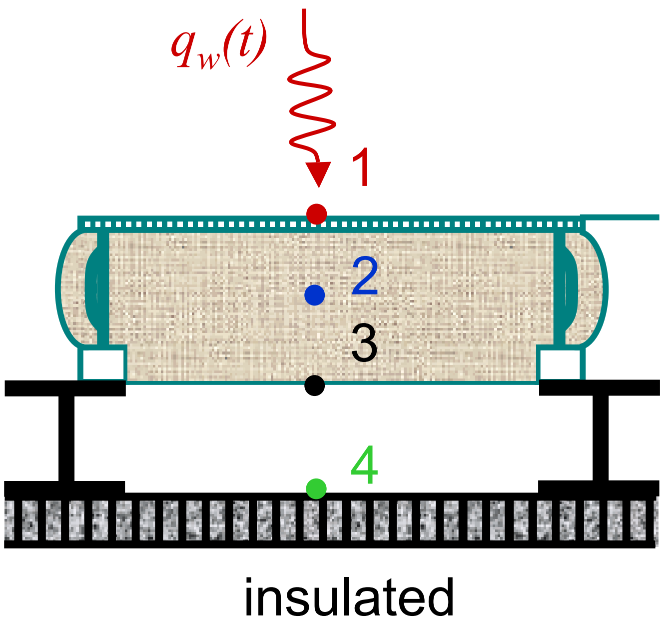
Armor TPS panel

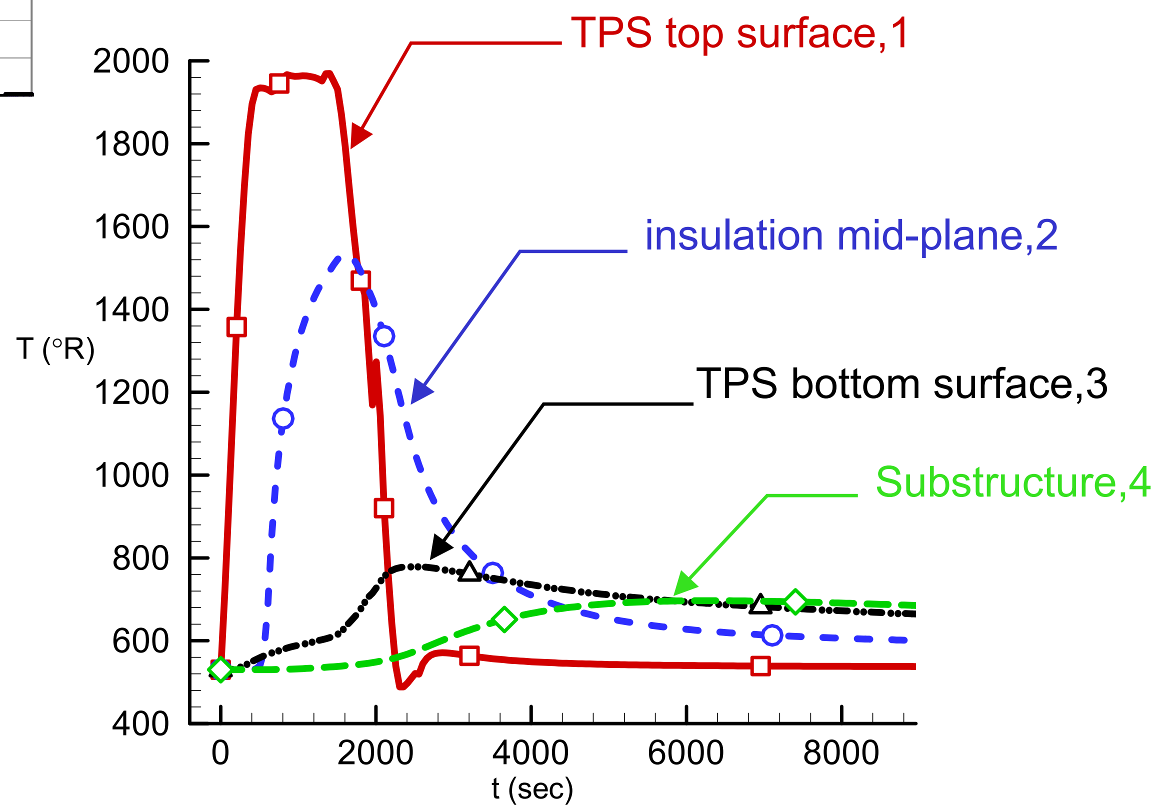




Thermal Response Predicted with TPS-it







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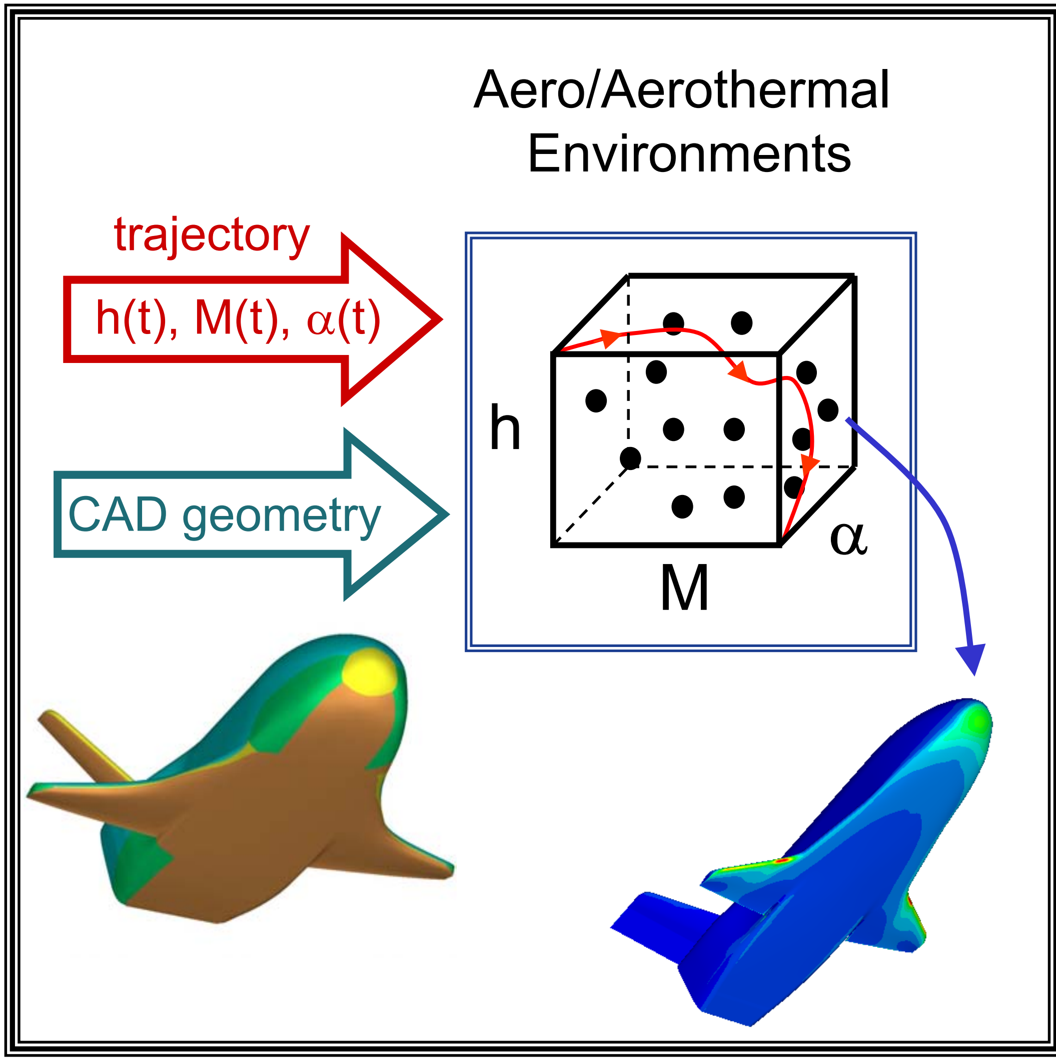
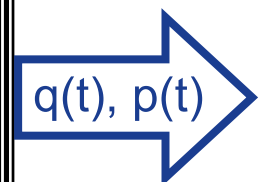
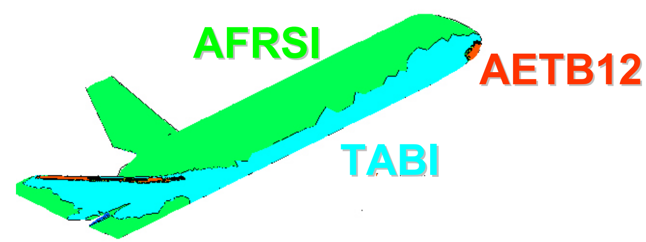
Decoupled CFD-Thermal Interactions Simplify the Design Process





TPS Sizing and Material Selection



are performed independently

CFD-Thermal Interactions for Hot (non-insulated) Structures are Coupled

Structure absorbs thermal energy

- Heating strongly depends on wall temperature

$$q_{aero} = -k_f \frac{\partial T_f}{\partial z} = q_{aero}(T_W)$$

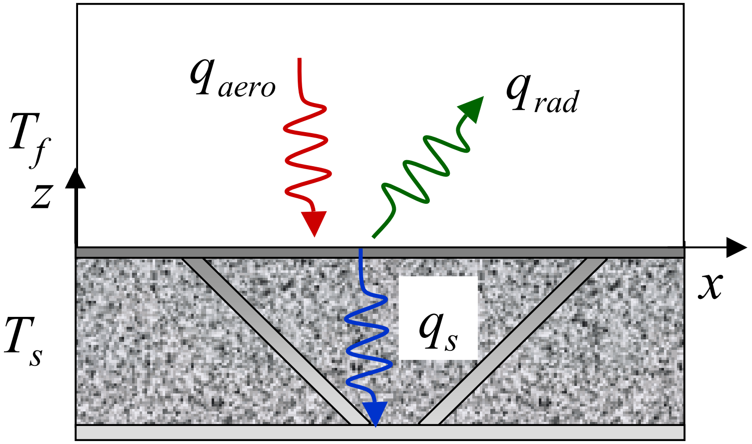
- Wall temperature strongly depends on thermal energy absorbed by structure $\quad T_W = T_W(q_s)$

At the fluid-solid interface

$$\left.\begin{aligned} -k_f \frac{\partial T_f}{\partial z} &= k_s \frac{\partial T_s}{\partial z} = q_W \\ T_f &= T_s = T_W \end{aligned}\right\} \longrightarrow$$

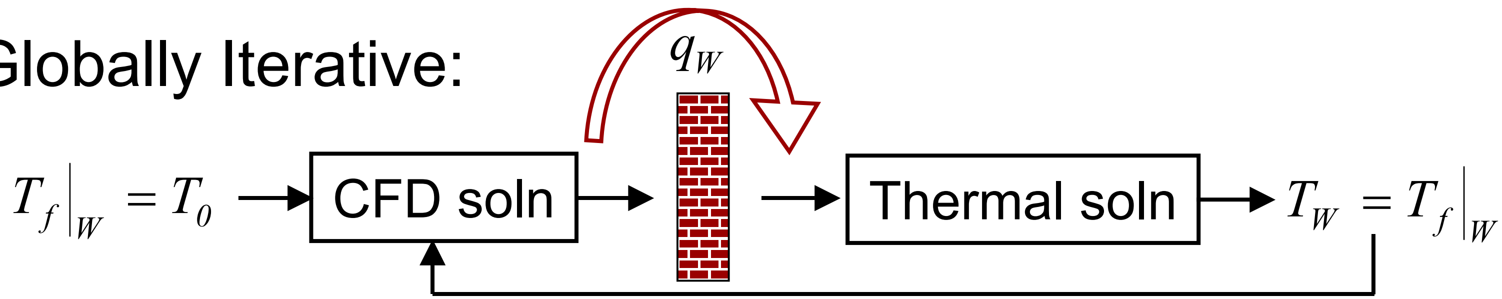
Fluid: $k_f,\ T_f$

Solid: $k_s,\ T_s$

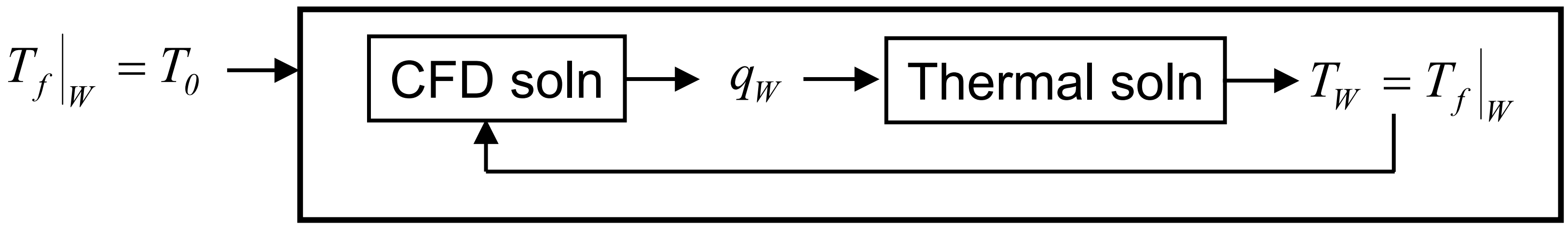


CFD-Thermal Coupling Approaches

Globally Iterative:



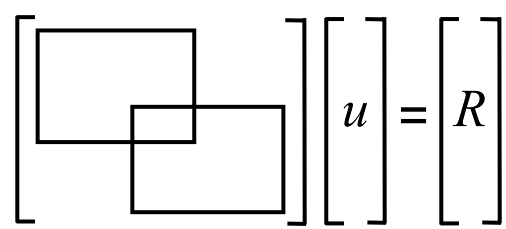
Locally iterative:



Fully Coupled:

- Solid is a fluid with $u=v=w=0$

- Cast thermal problem in conservation form, use same CFD algorithm, coupled energy equation at interface

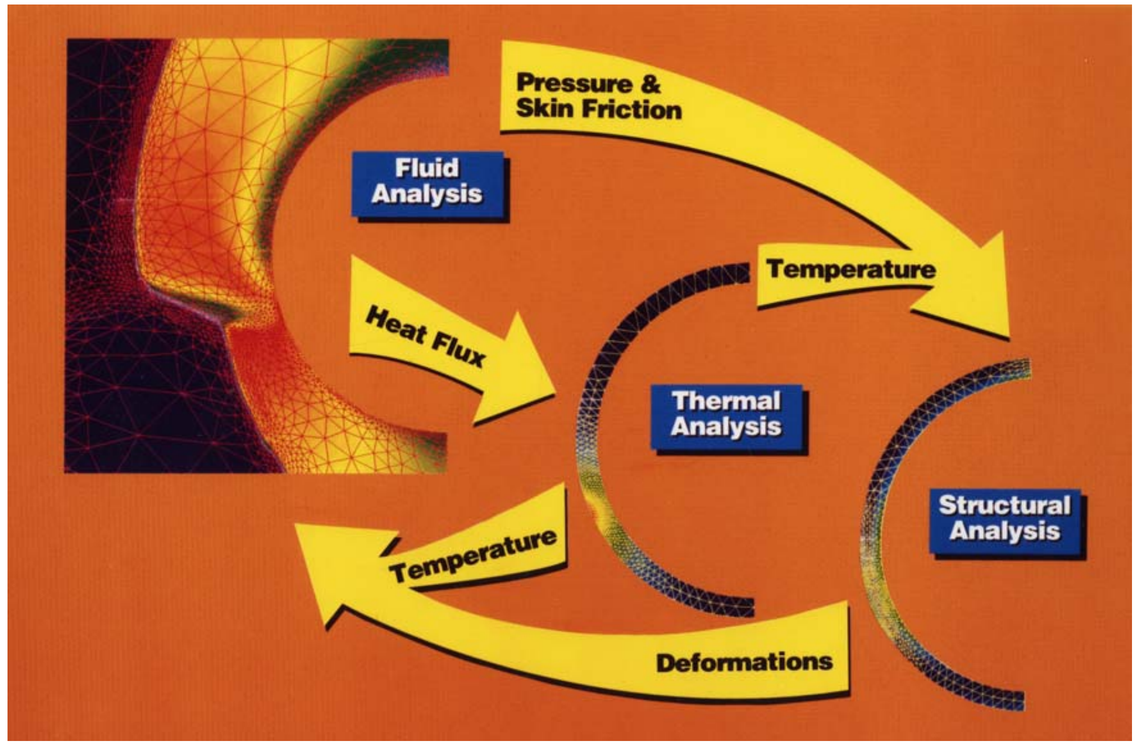


Integrated Fluid-Thermal-Structural Analysis using Unstructured Meshes

Fully coupled analysis using Taylor-Galerkin finite element formulation

- artificial viscosity for high speed flow
- flux-based formulation for heat transfer

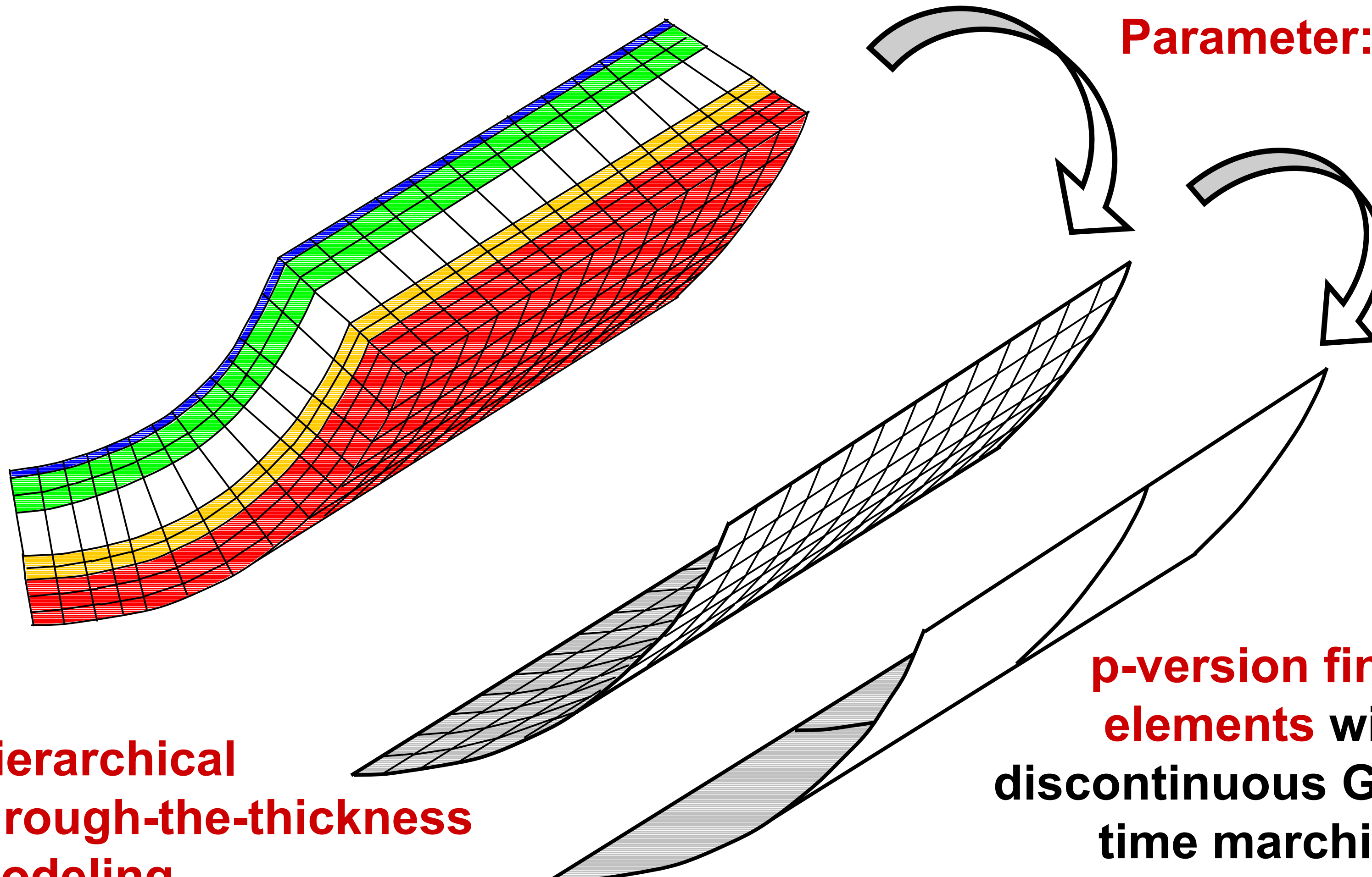
Structural analysis of built-up structures rarely use meshes of the “continuum” (3D elasticity equations).



Dechaumphai et. al. LaRC, Circa 1990

Built-up structures are “modeled” with plates, shells, beams, and 3D elements. ⟹ Need thermal analogues

“Next Generation” Thermal Analysis Methods for Hot Built-up Structures



Parameter: p_z

Hierarchical through-the-thickness modeling

p-version finite elements with discontinuous Galerkin time marching
Parameters: h, p_{ip}, Δt, p_t

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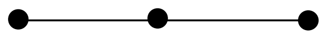
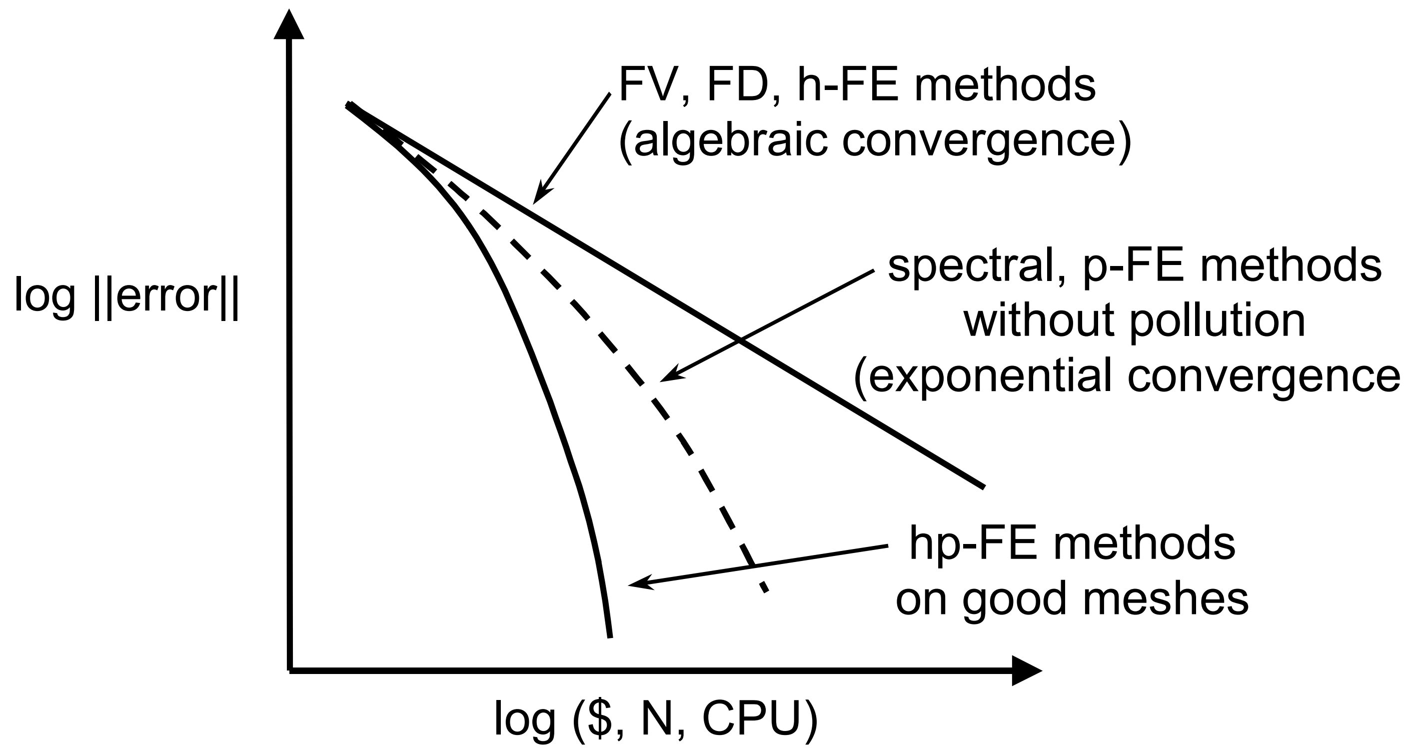
Why p-Version Finite Elements?



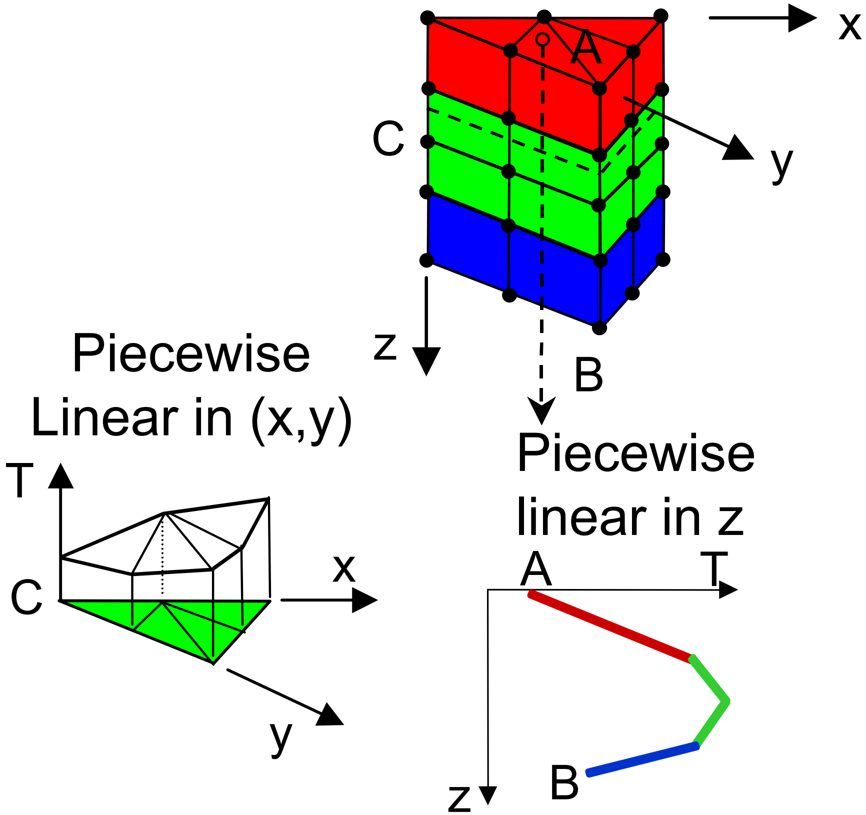
$$T(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_p x^p$$



Higher accuracy for fixed number of unknowns
Fewer elements for fixed accuracy
Element shapes that reflect actual geometry

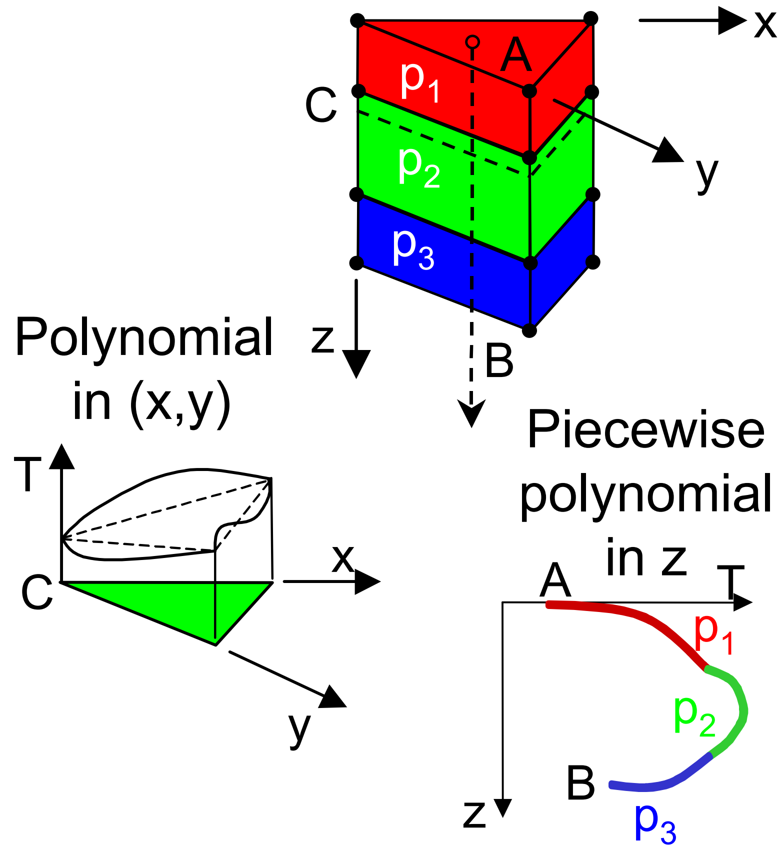
Finite Element Options for Multi-layered Plates

Conventional elements (p=1)



Improve accuracy by adding more elements

p-Version elements



Improve accuracy by increasing polynomial degree

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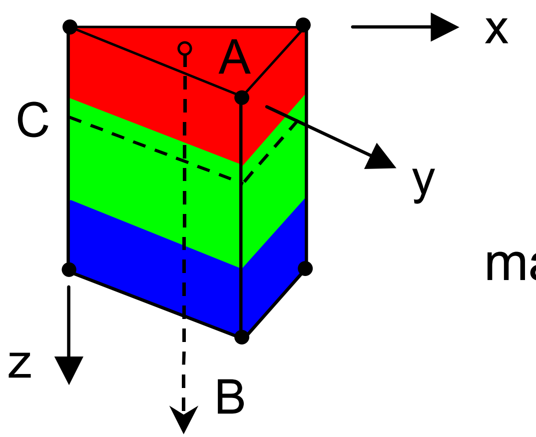
Homogenized Through-the-Thickness Modeling of Conduction in Multi-layered Plates

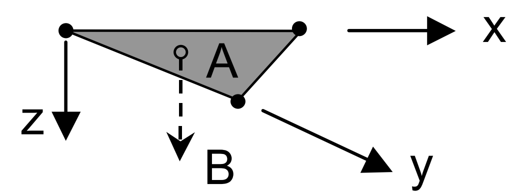
p-element



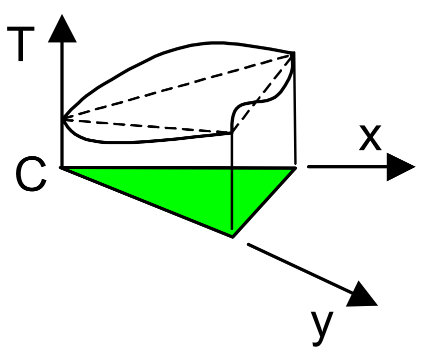
Hierarchical model

- Geometrically collapsed
- Thermal "higher-order plate theory"
- Structurally compatible

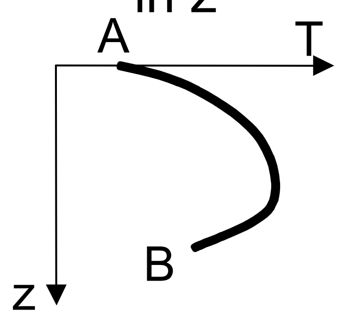
mathematically equivalent



Polynomial in (x,y)

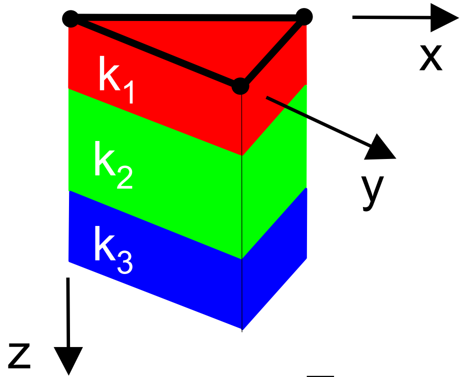


Single polynomial in z



+ Fewer degrees of freedom than multiple layers of p-elements
+ Good for single-layer
– Bad for multiple layers
  - Lacks convergence with increasing model order
  - Jump in the flux across material interfaces

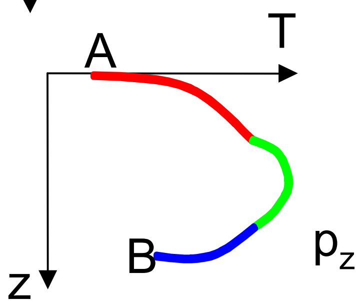
Optimal Through-the-Thickness Modeling of Conduction in Multi-layered Plates

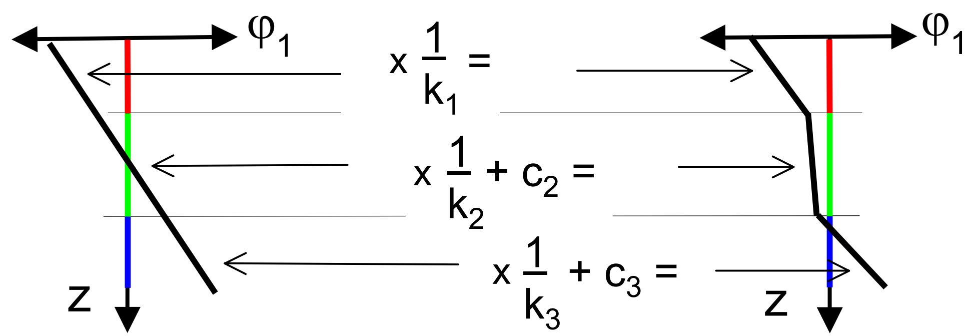


Basis functions are single polynomials defined piecewise by scaling the homogenized basis functions by the thermal conductivity of each layer

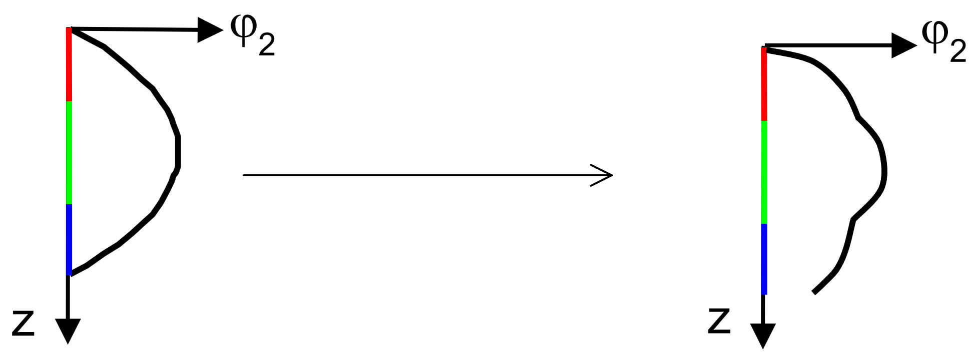
Homogenized basis functions

Optimal basis functions





Same number of DOF's as homogenized hierarchical model



Converges with model order and plate thickness

Ref: Volgelius & Babuska

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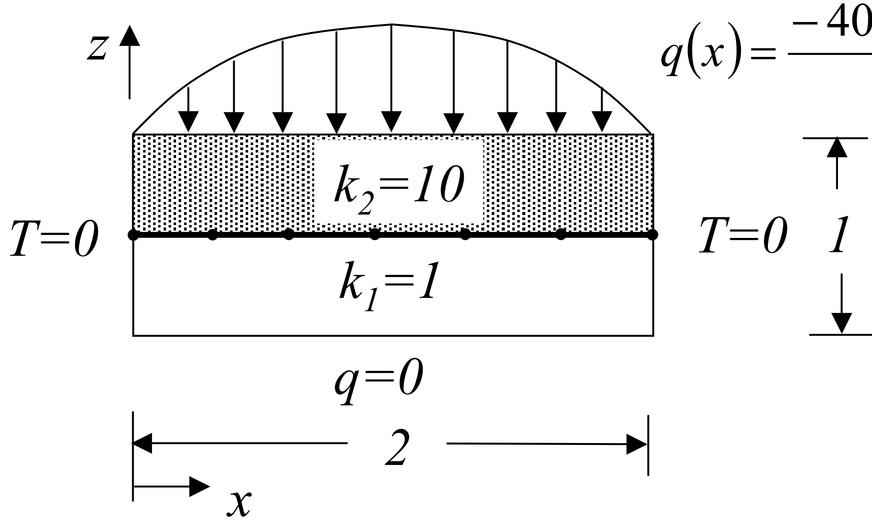
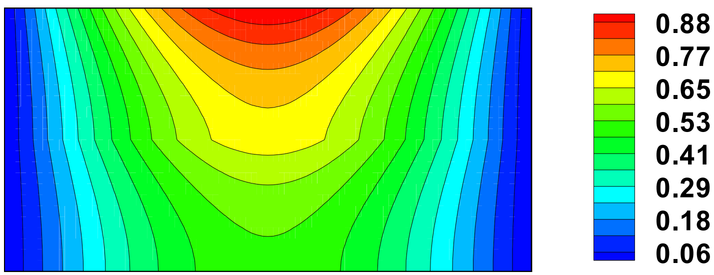
Steady-State Conduction in a Two-Layer Plate



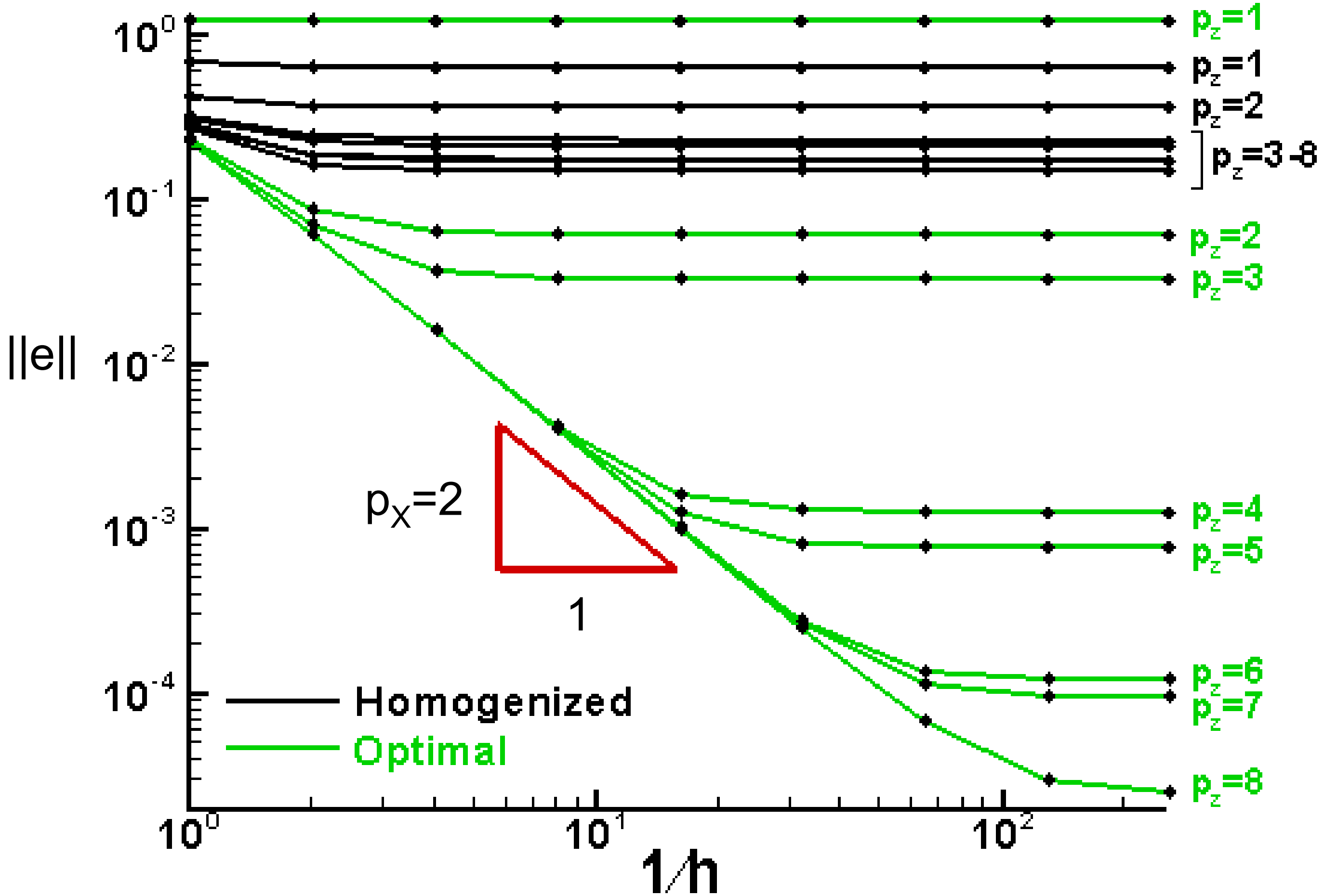
$$q(x) = \frac{-40x(L-x)}{L^2}$$

Exact Solution

$$T = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{L}\right) \begin{cases} A_n \sinh\left(\frac{n\pi z}{L}\right) + B_n \cosh\left(\frac{n\pi z}{L}\right) \\ C_n \sinh\left(\frac{n\pi z}{L}\right) + D_n \cosh\left(\frac{n\pi z}{L}\right) \end{cases}$$



Convergence of Through-the-Thickness Hierarchical Models of Conduction in Two-Layer Plate



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A Posteriori Error Estimation

- Error in finite element solution $e = u - \hat{u}$
- Global error equation

$$-\nabla \cdot (\kappa \nabla u) = Q$$

$$+\nabla \cdot (\kappa \nabla \hat{u}) - \nabla \cdot (\kappa \nabla u) = Q + \nabla \cdot (\kappa \nabla \hat{u})$$

$$-\nabla \cdot (\kappa \nabla e) = Q + \nabla \cdot (\kappa \nabla \hat{u})$$

- Can solve this global problem using the same FE approach, but this would be as computationally expensive as obtaining the solution
- Instead, solve a local problem on each element

$$-\int_K \nabla \cdot (\kappa \nabla e) v \, d\Omega = \int_K Q v \, d\Omega + \int_K \nabla \cdot (\kappa \nabla \hat{u}) v \, d\Omega$$

Local Problem for Element Error

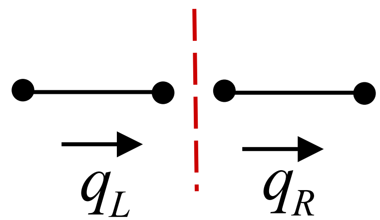
- Weak formulation of element error problem

$$\int_K (\kappa\nabla e)\cdot\nabla v\, d\Omega = \int_K Q v\, d\Omega - \int_K (\kappa\nabla\hat{u})\cdot\nabla v\, d\Omega + \underbrace{\int_{\partial K}(\kappa\nabla u)\cdot\vec{n}\, v\, ds}_{q \text{ is unknown}}$$

- Approximate boundary flux $\hat{q} = \bar{q} + \tilde{q}$

$\bar{q} =$ Average flux

$\tilde{q} =$ Correction to equilibrate $\hat{q}$



Find $\tilde{q}_K$ = polynomial of degree p_z such that

$$\int_{\partial K}\tilde{q}_K\,\theta_n\, ds = \int_K (\kappa\nabla\hat{u})\cdot\nabla\theta_n\, d\Omega - \int_K Q\theta_n\, d\Omega - \int_{\partial K}\bar{q}_K\,\theta_n\, ds$$

$\theta_n =$ nodal basis functions $, n = 1, \ldots, p_z$

$$[M]\{\tilde{q}\} = \{R\}$$

Ref: Ainsworth & Oden

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On Each Element, Solve the Local Problem for the Estimated Error

- Approximate element error

Find $\hat{e} = \sum_{i=0}^{p_x+1} \sum_{j=0}^{p_z+1} \varphi_i(x)\, \psi_j(z)\, \varepsilon_{ij}$ such that

$$\int_K \left(\kappa \nabla \hat{e}\right) \cdot \nabla v\, d\Omega = \int_K Q v\, d\Omega - \int_K \left(\kappa \nabla \hat{u}\right) \cdot \nabla v\, d\Omega + \int_{\partial K} \hat{q}\, v\, ds$$

for all admissible v

$$[K]\{e\} = \{F\}$$

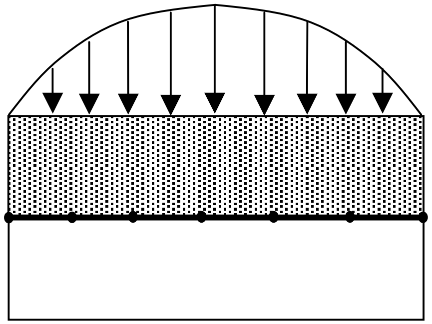
- Element error indicator

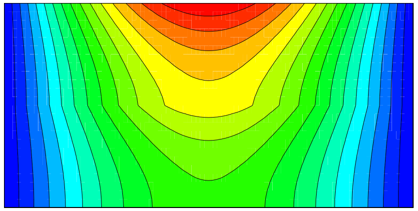
$$\| \hat{e} \|_K = \sqrt{\int_K \left(\kappa \nabla \hat{e}\right) \cdot \nabla \hat{e}}$$

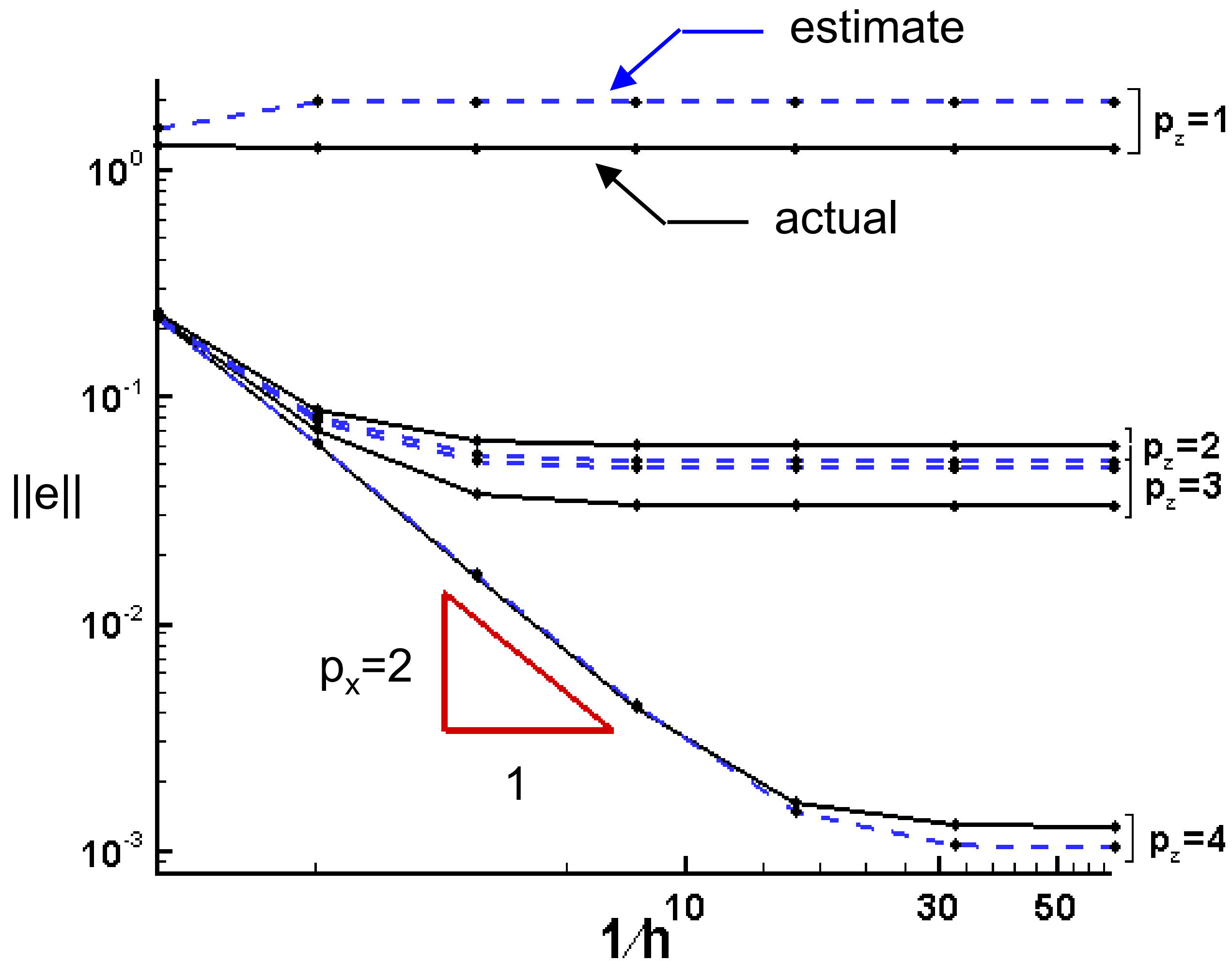
- Global error estimate

$$\| \hat{e} \|_\Omega = \sqrt{\sum_K \| \hat{e} \|_K^2}$$

Performance of the Error Estimate on the Two-Layer Example

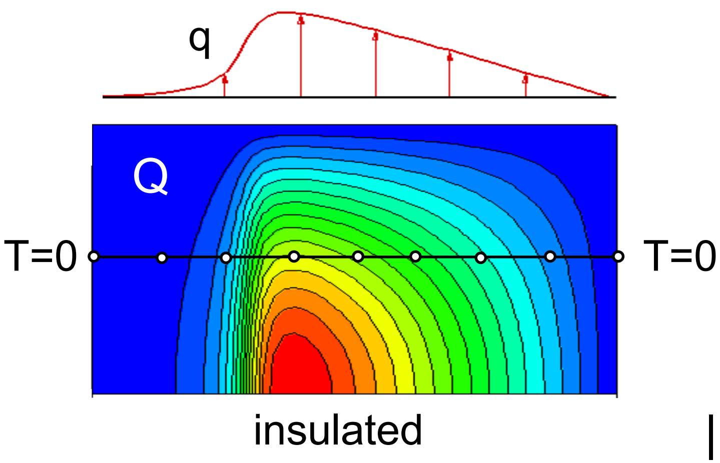


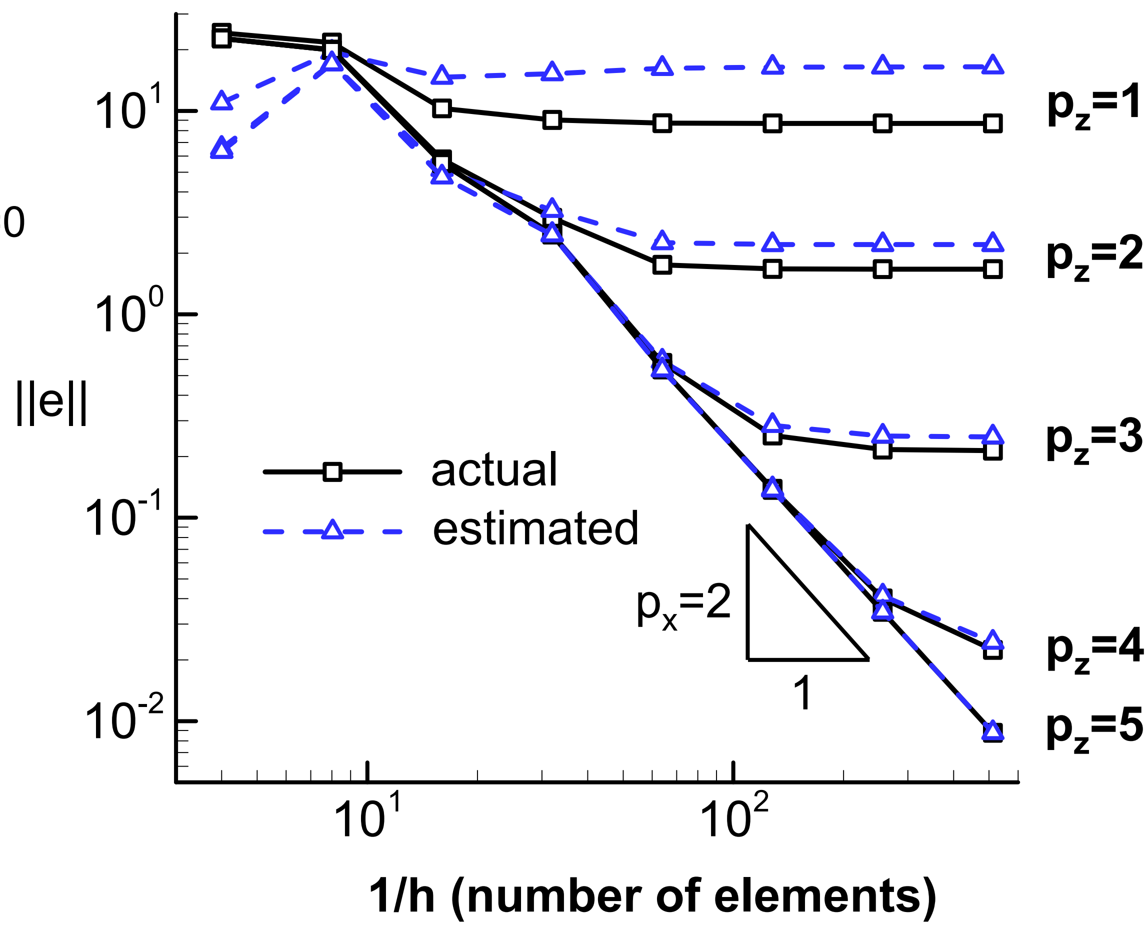




Performance of Error Estimate
Steady Conduction with Internal Heat Generation

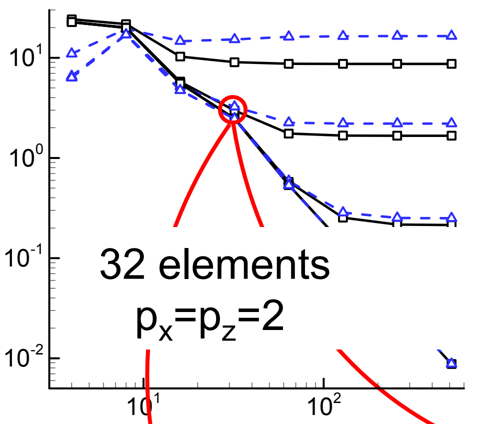
Rough Exact Solution

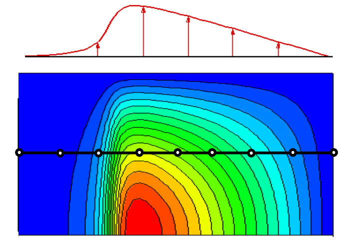




Performance of Element Error Indicator

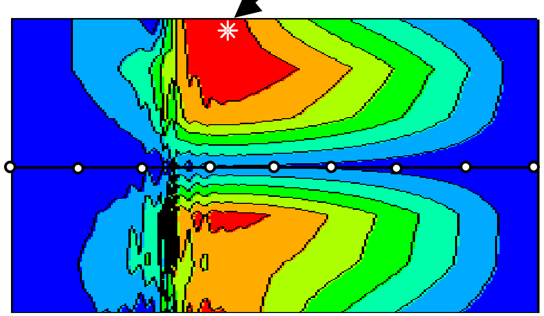
Steady Conduction with Internal Heat Generation



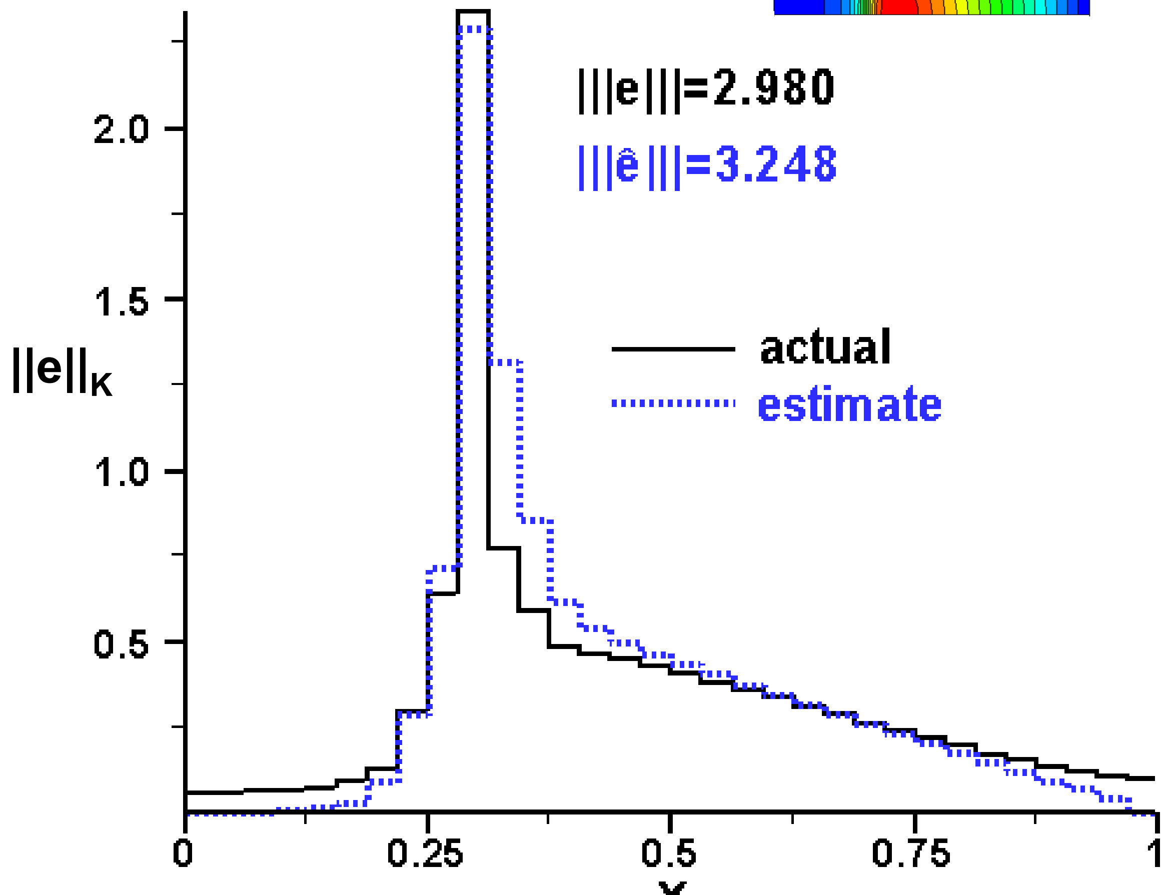


actual error

Max. error



Max. gradient



Estimating Contributions of Hierarchical Modeling and Finite Element Error

- Hierarchical modeling error: $e_{HM} = u - u_{HM}$
- Finite element error: $e_{FE} = u_{HM} - \hat{u}$
- Total error: $e = u - \hat{u} = e_{HM} + e_{FE}$

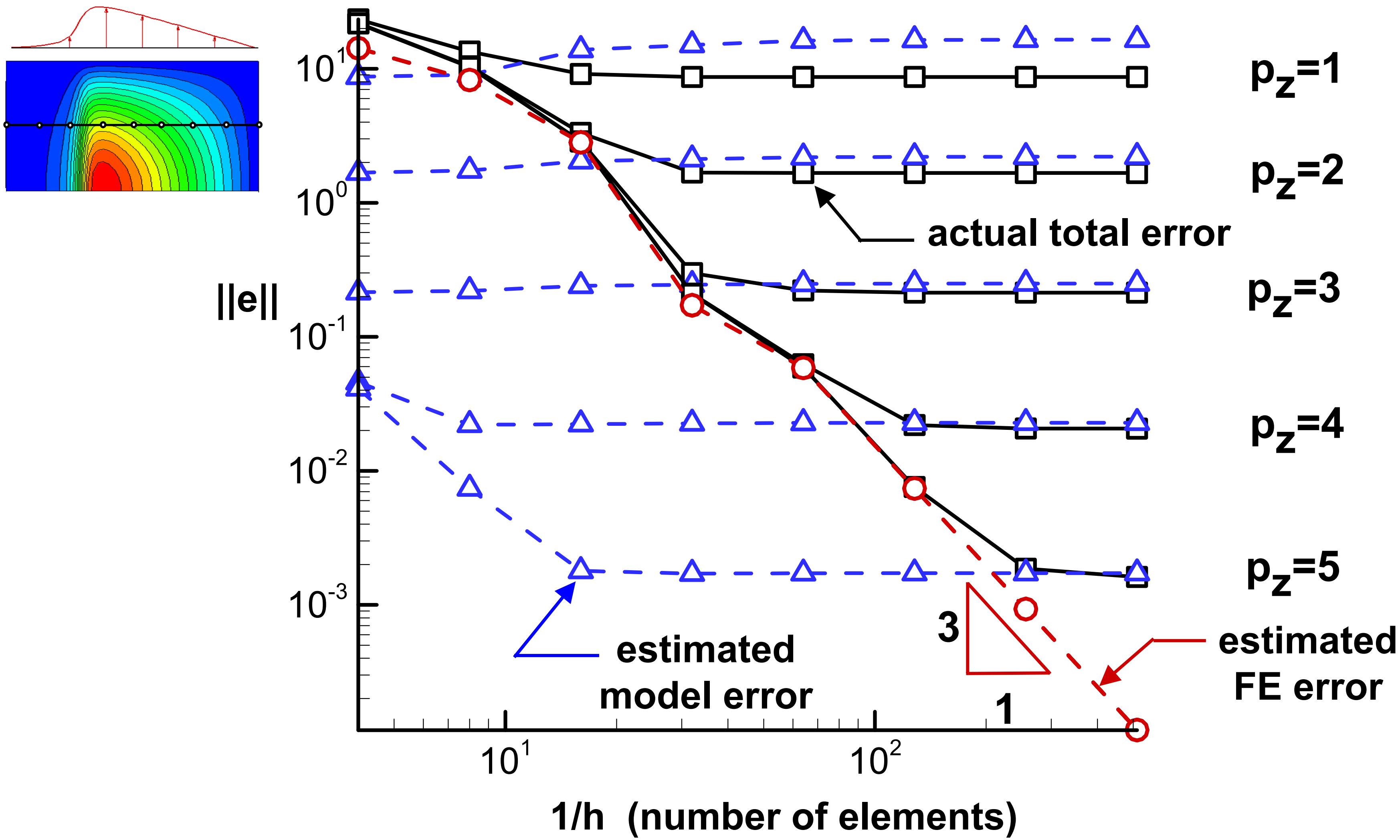
$$\| e \|^2 = \| e_{HM} \|^2 + \| e_{FE} \|^2$$

- Solve local problem twice

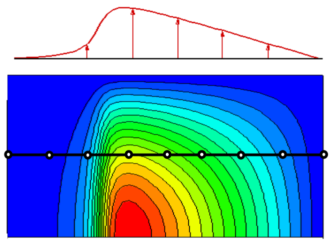
$$\hat{e}_{HM} = \sum_{i=0}^{p_x} \sum_{j=0}^{p_z+1} \varphi_i(x)\, \psi_j(z)\, c_{ij}$$

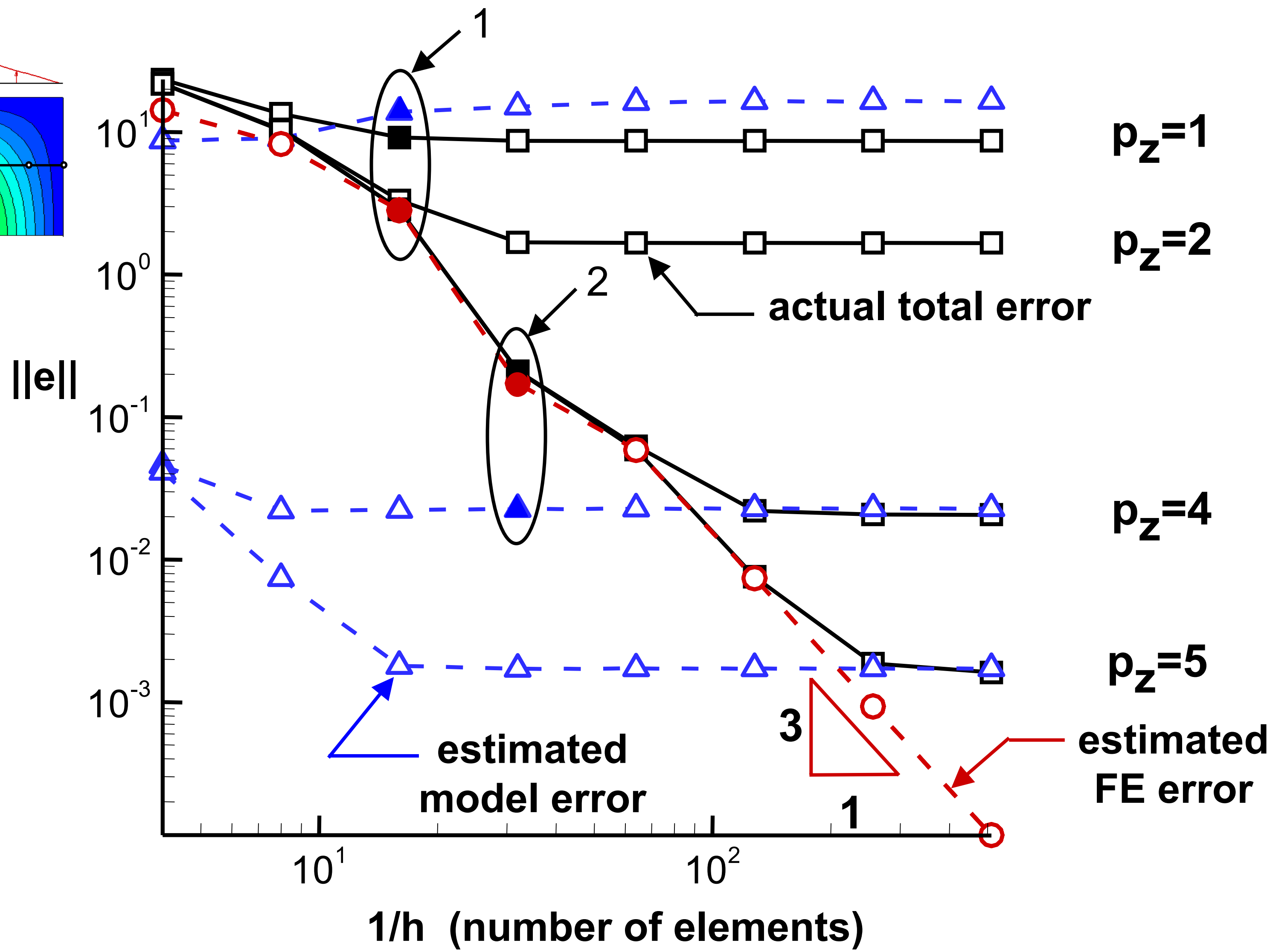
$$\hat{e}_{FE} = \sum_{i=0}^{p_x+1} \sum_{j=0}^{p_z} \varphi_i(x)\, \psi_j(z)\, d_{ij}$$

Performance of Estimated Error Contributions



Error Estimates are Sufficiently Accurate to Drive an Adaptive Strategy





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Concluding Remarks

- Examples shown here were constructed to have exact solutions to study behavior of the solution method and error estimates.

- Similar approach has been used with same success for transient 3D linear conduction using 2D elements and steady-state 3D nonlinear conduction.

- Can homogenization using p-version finite elements be used to accurately represent the thermal effects of all the internal structure on the surface?