Some Impacts Of Mesh Density On The Accuracy Of Finite Difference Thermal Model Results

Perry Ramsey
ITT Industries
Aerospace/Communications Division
Fort Wayne, IN

Abstract

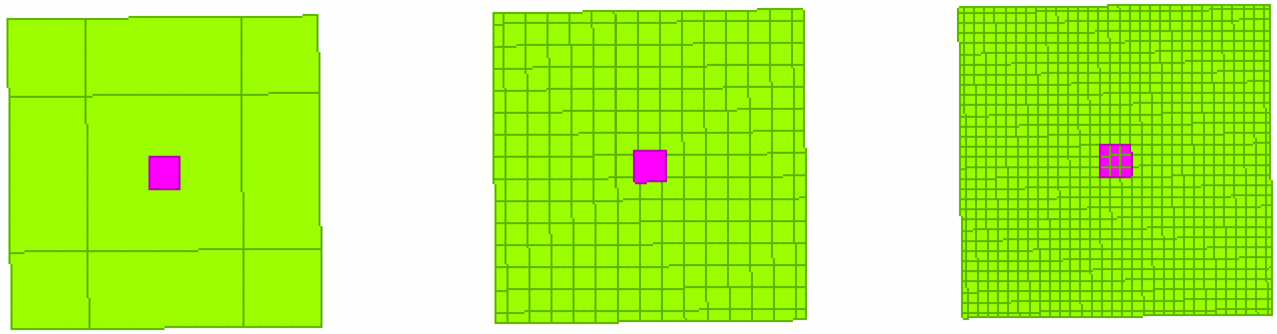
- Some modeling experiments were made concerning the accuracy of temperatures between surfaces of different size as calculated using finite difference methods.
- Application of a point source to a node in a rectangular mesh results in varying temperatures depending on the size of the mesh. This is a computational artifact that results from the inherent properties of a regular mesh.
- If the size of source is held constant, the calculated temperature still depends on the underlying mesh dimension. The temperature rise is not monotonic, but was found to rise to a maximum when the mesh size matched the source size, then asymptotically decreased to stable value.
- A method for compensating for the mismatch is developed and compares well with results from ultra-fine meshes. This takes the form of an additional resistance that needs to be added between the source and the mesh when the mesh is more than 2.4 times as large as the source.

How Does the Mesh Density Affect Results in Practical Terms

- The standard process is to break things into pieces
  - This linearizes the problem despite irregular boundary conditions
- Intuitively, finer grids give better answers
- But finer grids come at a price
  - Transient model solution time for a 2-D problem varies $(1/L)^5$
  - Plus model generation time, storage, post processing, etc.
- How small is small enough?

A Concrete Example is a Chip on a Circuit Board

- There's a heat source in the middle of a meshed sink
  - (For simplicity, we'll only consider a square mesh)
- Do any of these nodalizations give accurate results?
- If not, is there anything that can be done to fix it?

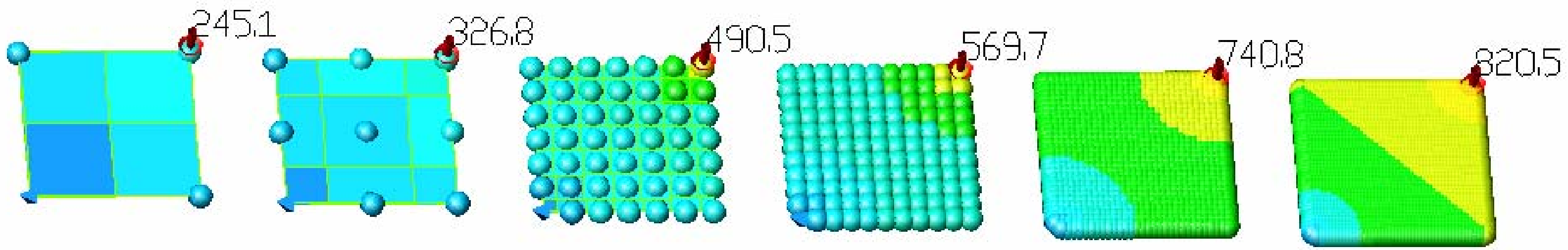


Really a Universal Issue

- Not just a chip on a board
  - Localized solar heating
  - Electrical heater
  - Electronics box on a spacecraft panel
- It’s also a factor on the sink end
  - Boards are often mounted on tiny standoffs, which are the same problem in reverse

Start with the "Zeroth Order" Problem

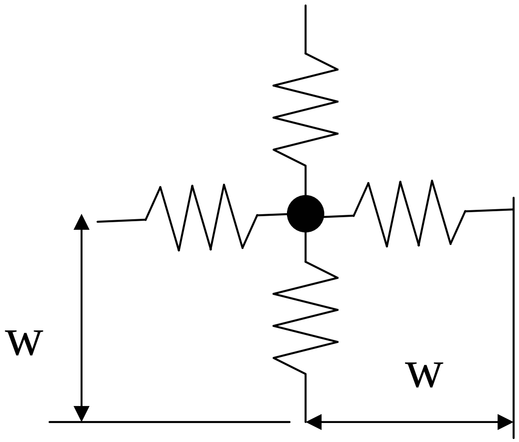
- Put a point source in one corner
- Make the opposite corner a point boundary (sink)
- Try some different resolutions



- Temperature of source is rising without limit despite no change in physical problem

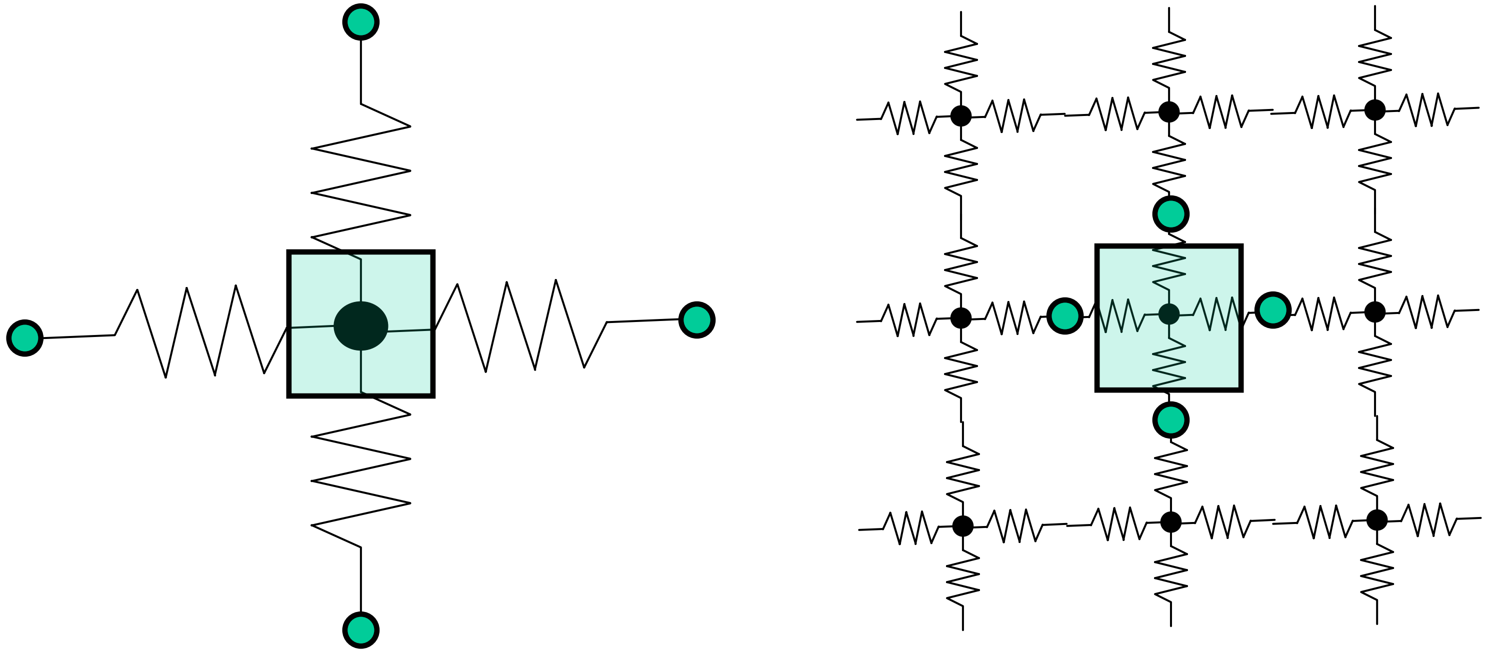
Mesh Looks the Same at All Scales

- A point source is an analytical fiction
  - Local heat flux becomes large as grid scale becomes small
- The resistance from a node to its neighbor is 1/R=G=4kwt/w=4kt
- This is independent of the grid size w
- So ΔT in the first step from the source is the same no matter how the size of the grid!



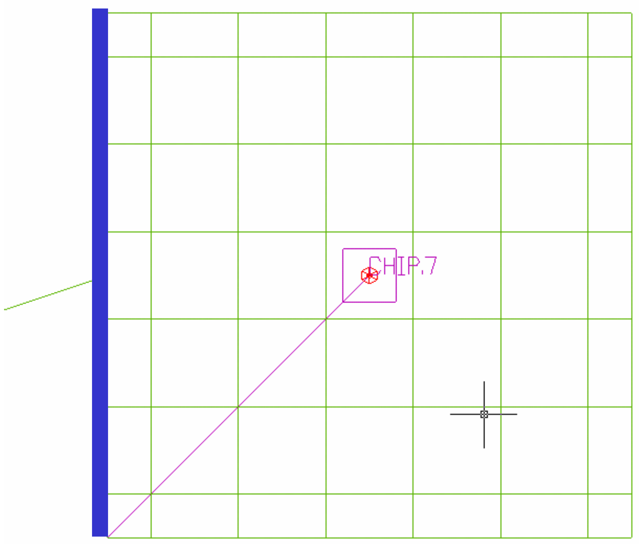
This Implies that the Scale of the Mesh Has to Match the Scale of the Source

- A finer mesh will always give higher temperatures for a point source
  - ΔT to green dots will be the same



Try a Different Model to Concentrate on One Problem

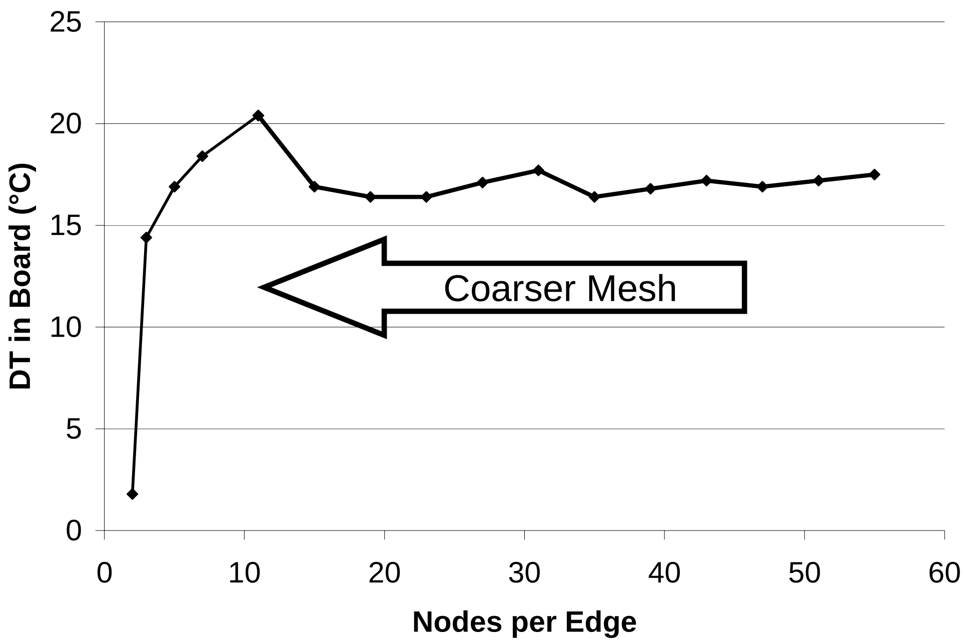
- Connect edge to sink (ΔT independent of scale)
- Make the source a rectangle in the middle
  - Constant size
- Specifics
  - Chip is 20 x 20 mm
  - Q = 1 W
  - θ_{cb} = 2.5 °C/W
  - 200 mm x 200 mm board
  - 8 oz copper
  - Board edge resistance 0.5 °C/W
  - Sink is at 0 °C



Change the Resolution and Look for an Asymptote

- Try varying resolutions

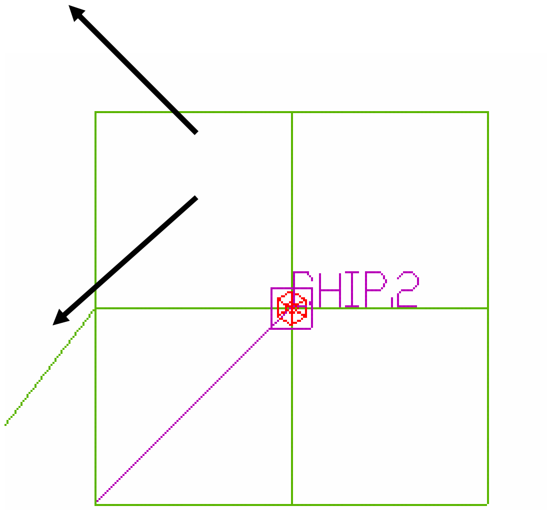
Number of Nodes Each Edge	Distance Between Nodes (mm)	Chip Temp (°C)	Rise in Board (°C)
2	200.0	4.8	1.8
3	100.0	17.4	14.4
5	50.0	19.9	16.9
7	33.3	21.4	18.4
11	20.0	23.4	20.4
15	14.3	19.9	16.9
19	11.1	19.4	16.4
23	9.1	19.4	16.4
27	7.7	20.1	17.1
31	6.7	20.7	17.7
35	5.9	19.4	16.4
39	5.3	19.8	16.8
43	4.8	20.2	17.2
47	4.3	19.9	16.9
51	4.0	20.2	17.2
55	3.7	20.5	17.5



- Amazingly, it's not monotonic

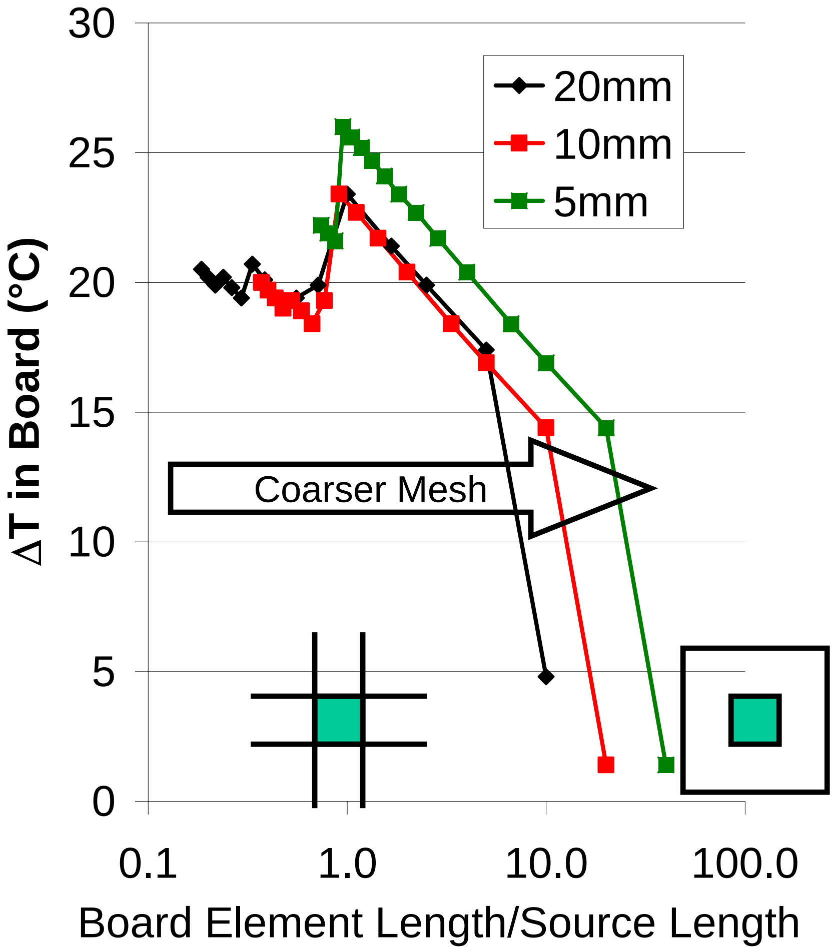
The 2x2 Case is Catastrophic

- There is a dead short to the sink
  - The heat is being dumped directly to the edge and not experiencing any resistance from the board



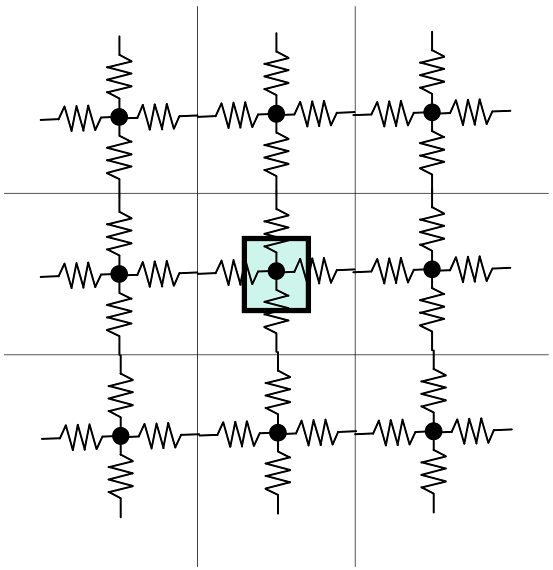
Look From Perspective of Board Element vs. Chip Element Size

- Maximum ΔT observed where chip = board element length
- Matching the length results in over-prediction
- Just exceeding the mesh length results in false spreading and under-prediction

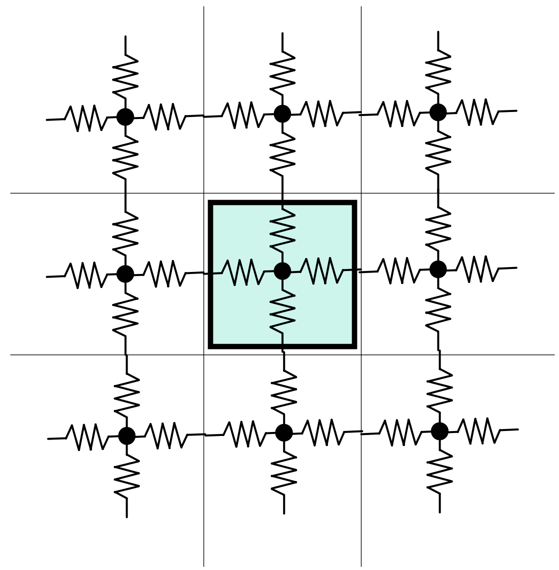


Relationship Between Source and Grid Size is Critical

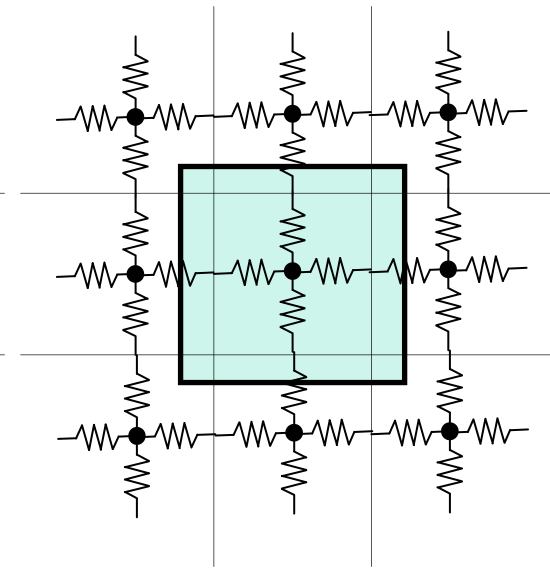
- This exemplifies that the meshing has to adequately represent the size of heat sources



Underestimates – misses spreading resistance



Overestimates – forces all heat through center

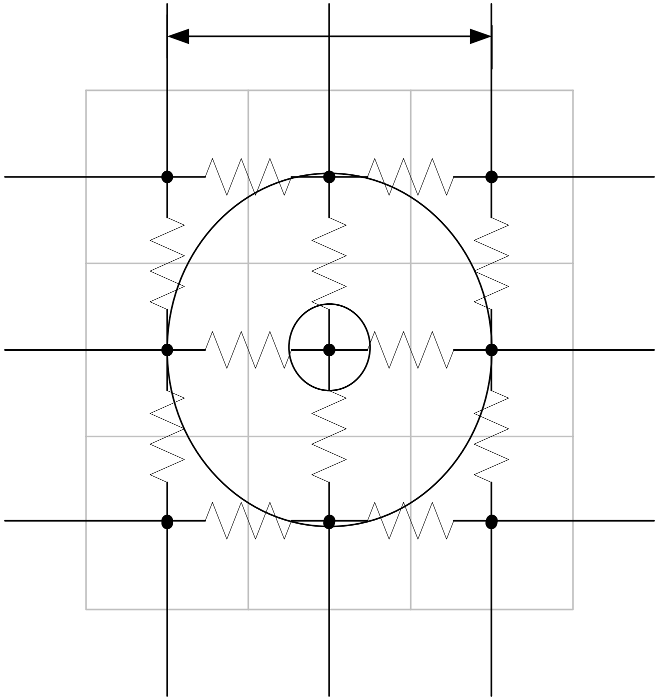


Underestimates – Falsely spreads to surrounding nodes

Try a Simple Theoretical Approach to Simplify The Problem

- Reality is that there is spreading from a finite source
- Recall the formula for resistance through the wall of a pipe

$$R = \frac{\ln(r_2 / r_1)}{2\pi k t}$$



- Compare this to the resistance created by the mesh

ITT Industries

Compare to What a Mesh Does

- r1 is the one-half the size of the source ($L_1/2$)
- r2 is the element dimension (L_2)
- Spreading resistance is

$$R_{spread} = \frac{\ln(2L_2 / L_1)}{2\pi kt}$$

- The mesh gives resistance (through 4 resistors) is

$$R_{mesh} = \frac{L_2}{4ktL_2} = \frac{1}{4kt}$$

Difference Shows When Additional Resistance is Required for Safety

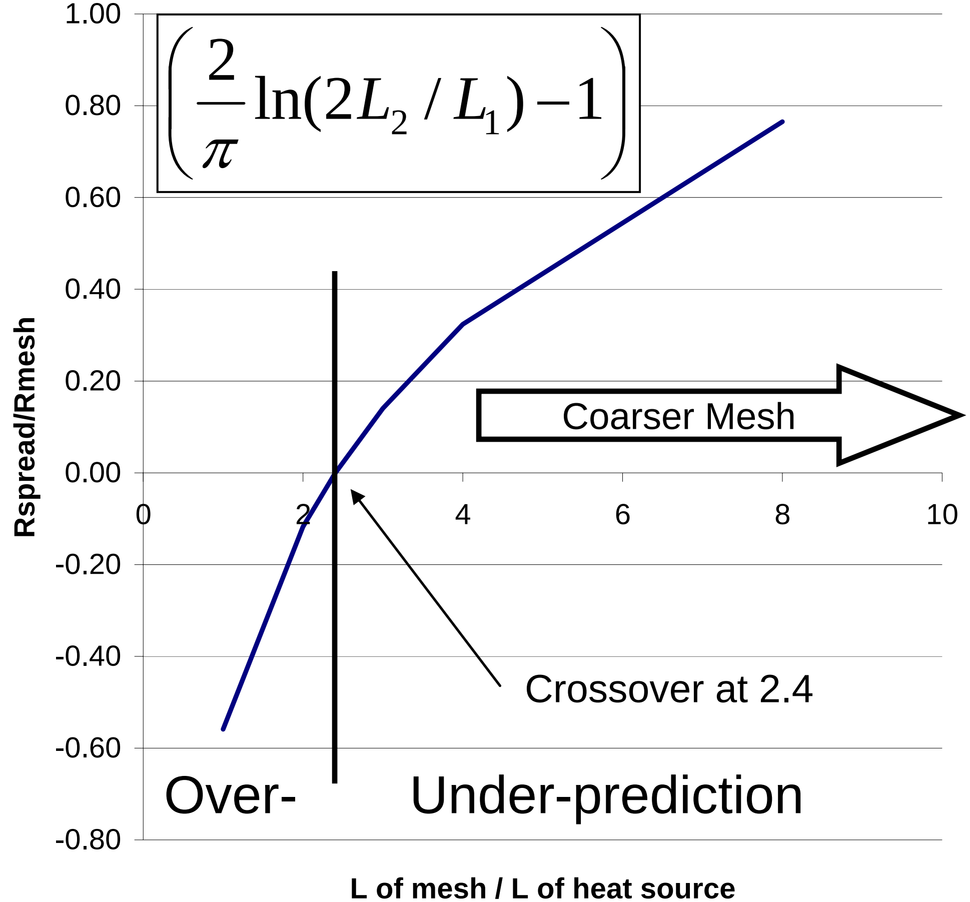
- The difference is

$$R_{spread} - R_{mesh} = \frac{1}{4kt}\left(\frac{2}{\pi}\ln(2L_2 / L_1) - 1\right)$$

- This is the spreading resistance missed by FD model
- Only applicable up to where source smaller than mesh
- Where positive, the finite difference mesh is underestimating impedance to surroundings
  - Hence the source temperature is underestimated
- If it's negative, you're over-predicting final temperature
  - Usually, that's the safe side, but be sure it is for your case

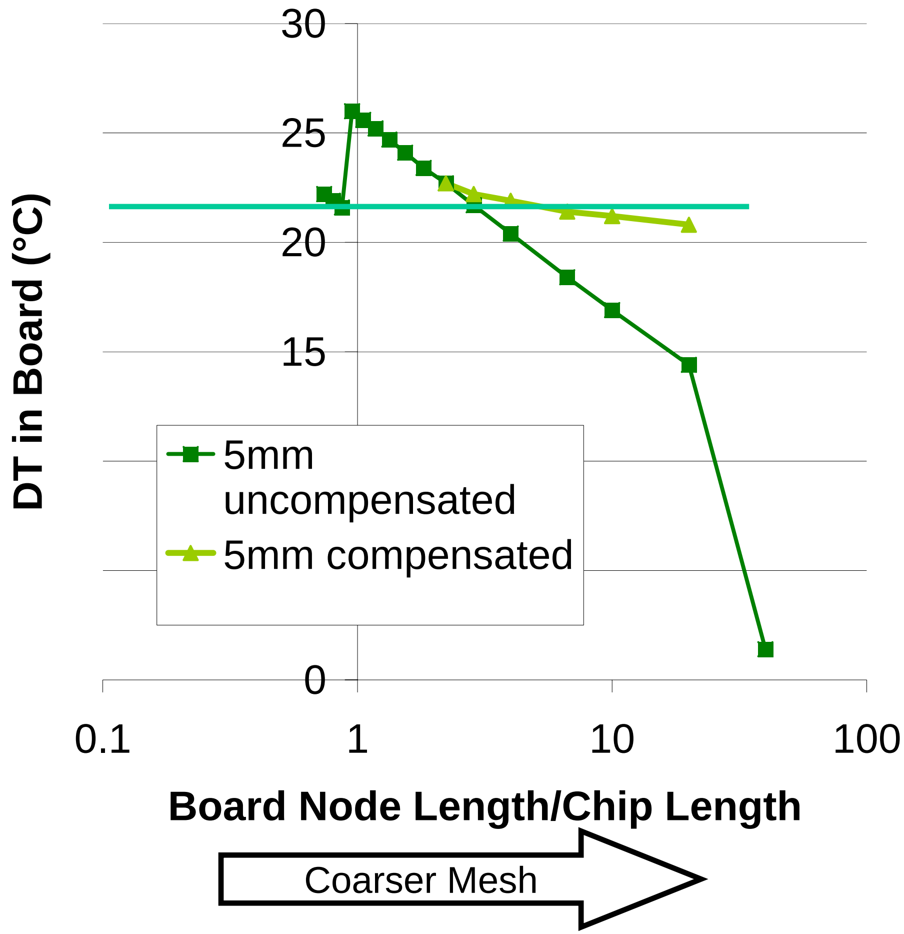
For Mesh Larger Than 2.4 Times Source, FD Model Overestimates

- Crossover is at 2.4
  - If mesh is finer than this, (down to size of mesh) results are on the high side
  - If coarser, add resistance equal to factor * 1/(4kt)



In Practice, Good Results By Adding Compensation

- In practice, the compensation is in the right area
- Eliminates a significant non-conservative error



ITT Industries

A Method Exists to Compensate for Mismatch Between Source and Mesh

- When heat source is significantly smaller than mesh, FD model will underestimate source temperature
- This is NOT an error in any analytical code, but an inherent property of finite difference meshes
- A compensation formula has been developed based on the classical circular conduction formula
- Additional compensating resistance is needed when underlying mesh is larger than 2.4 times source (or sink)

Professional Thermal Analyst. Feel free to attempt this maneuver anyway.

Modeling Guidelines

- ➢ Keep the underlying mesh within 2x the dimension of your smallest heat source
  - Otherwise you may be creating a non-conservative model
- ➢ If heat source is smaller than above guideline, use the compensation formula to add resistance in series with the attachment resistance
- ➢ Prefer models that work good to those that look good. It's better to move the modeled part from its real location or change its size than to fall victim to spreading resistance traps
- ➢ Don't let small heat sources hang over multiple node boundaries
  - Creates false spreading
- ➢ Especially if one of those nodes is attached to your ultimate sink
  - Creates a dead short