NEW METHODS TO SOLVE FOR RADIATION FROM FINITE ELEMENT MODELERS

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ABSTRACT

Finite element methods often result in radiation enclosure models consisting of many thousands of small triangular and or quadrilateral flat surfaces (facets). While methods exist for handling these large radiation problems, they are typically limited to diffuse gray body considerations. Furthermore, such methods approximate curved surfaces with a large number of locally flat facets. Traditional radiation solvers support most or all of the desired capabilities, but can be difficult to use in conjunction with finite element methods.

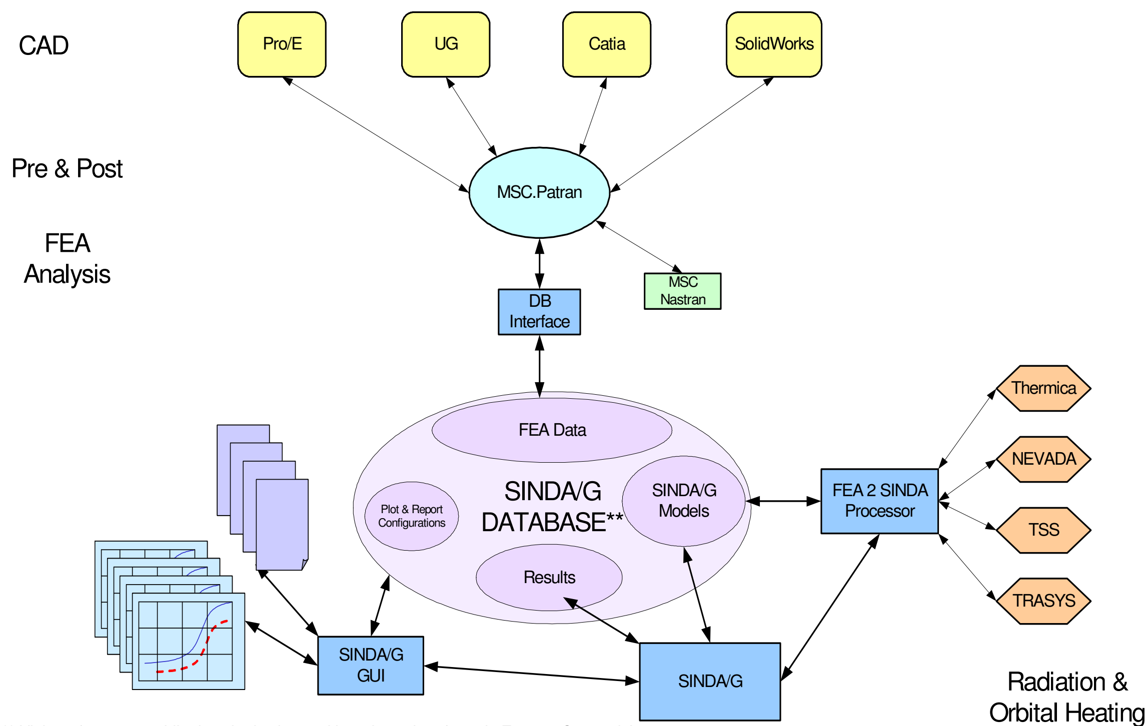
This paper introduces and describes thermal radiation capabilities of SINDA/G for PATRAN (SG4PATRAN), developed under NASA's SBIR Program. Features in this software address these modeling and accuracy issues by integrating traditional radiation solvers such as THERMICA, TSS, NEVADA, and TRASYS with the PATRAN modeling environment, ultimately using SINDA/G to compute the temperatures. Two new radiation surface modeling methods are provided in addition to the standard facet approach. The facet approximation can be eliminated by using Primitive surfaces, and large radiating facet model size can be effectively reduced by using Radiation Super Elements. Examples are presented illustrating computational speed and accuracy trade-offs for the various methods.

INTRODUCTION

SG4PATRAN is another finite element (FE) to SINDA/G network model integration by NAI. The thermal modeling environment is integrated seamlessly within PATRAN to form a product that goes from importing models to creating new models to solving those models. In SG4PATRAN, SINDAG model construction occurs through 2 programs. PAT2SG extracts the model from the PATRAN file and writes it to the HDF5 file. SDB2SG, the second program, completes translation by applying various finite-element based algorithms to the model resulting finally in a SINDAG input file.

INTEGRATION

Applying enclosure radiation to the model signals SDB2SG to invoke an external radiation solver (THERMICA, NEVADA, TSS, or TRASYS). SDB2SG creates the input and launches the radiation solver to obtain the radiation solution. The radiation results, being based on element and surface entities, are not in SINDAG-suitable form, which requires radiation to be expressed in terms of nodal interconnectivity. Thus, SDB2SG reads these results, performs calculations needed to map results to the nodes, and incorporates this in the SINDAG input file. The following figure shows the data flow for enclosure radiation and modeling in PATRAN.



** High performance public domain database with end user interface via Fortran, C++ and Java

The ability to exchange radiation solvers strengthens this architecture, not only allowing cross-checking during development, but more importantly, providing a measure of portability between users of different radiation codes. This exchangablity would allow one user to run a PATRAN model using NEVADA and then deliver the same model to another user who could run it using THERMICA.

The radiation model produced by SDB2SG generally does not include every geometric entity in the PATRAN model unless enclosure radiation is applied to every entity. Applying enclosure radiation to an entity is the only way to represent that entity in the radiation model.

RADIATION FEATURES

With SG4PATRAN, three ways exist to apply enclosure radiation to the geometry.

- FR (Facet Radiation)
- SER (Super Element Radiation)
- PR (Primitive Radiation)

Facet Radiation: FR is the most basic, creating a radiation surface for each element face with an enclosure radiation load. An element face may be the front or back of a plate (but not an edge), any face of a solid element, or any edge (but not a face) of an axisymmetric element. Usually, FR is the most accurate option, but for large, fine-mesh FE models, the number of surfaces makes solution impractical for most radiation codes.

Super Element Radiation: SER extends FR by allowing numerous facets to be treated collectively as a single surface entity in the radiation solver. SER should only be applied elements that form a single connected surface. Though possible to model disconnected regions as a single super element, such practices should be avoided. Also, SER can only be applied to elements that all reference the same property. Judiciously used, SER can vastly reduce radiation processing time while sacrificing relatively little accuracy.

Primitive Radiation: PR is new to finite element modeling systems such as PATRAN, and involves using new primitive surface entities added to PATRAN by MSC specifically for this project. Applying enclosure radiation to a primitive surface causes the surface to be modeled as a mathematically exact shape (cylinder, cone, parabola, etc.). When applying primitive enclosure radiation loads, the AxB mesh, a property of the load and not of the surface, can be specified. The AxB mesh subdivides the surface along parametric coordinates into smaller daughter surfaces, refining the mesh for the radiation model. This AxB mesh need not be aligned, coincident, or congruent with the underlying finite element mesh. However, SDB2SG must map AxB mesh based radiation results back to the finite element mesh to finish creating the SINDAG model, so mesh congruence is relevant to accuracy.

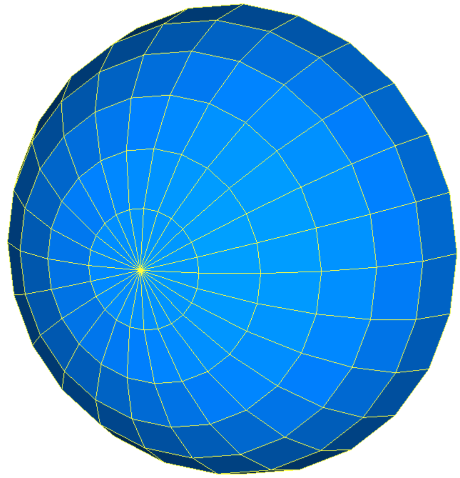
Because of this, radiation-to-conduction mesh congruence defines the two general classes of PR scenario.

Congruent meshes – every finite element face associated with the primitive surface falls neatly inside the daughter surface boundaries of the AxB mesh

Non-congruent meshes – one or more finite element faces violates the edge boundaries defined by the AxB mesh lines

The following examples describe all possible congruent mesh scenarios. All other cases are non-congruent.

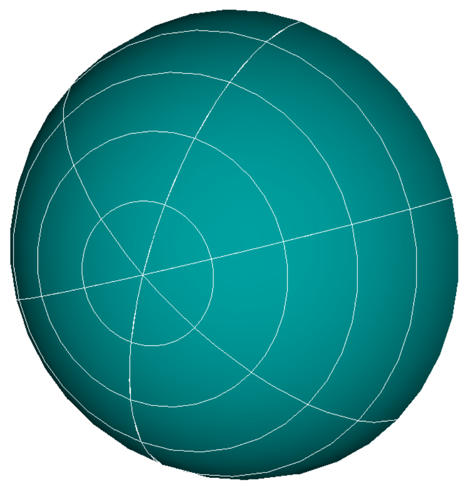
AxB = 1x1 – every element face always falls within the one and only primitive; there is only 1 daughter surface



Coincident mesh – finite element meshing done according to an identical AxB parametric scheme – one finite element per primitive surface – *this is the most accurate available option*.

Contained mesh – finite element meshing done according to CxD mesh where C and D are integer multiples of A and B, respectively; also possible but unlikely: unstructured mesh within each daughter primitive – as long as no element faces cross AxB mesh lines.

It is also worth noting – all congruent mesh scenarios, with no fractional element face associations, are similar to the SER option, though accounting for true versus facet-approximated surface area can result in small, systematic differences in the results.



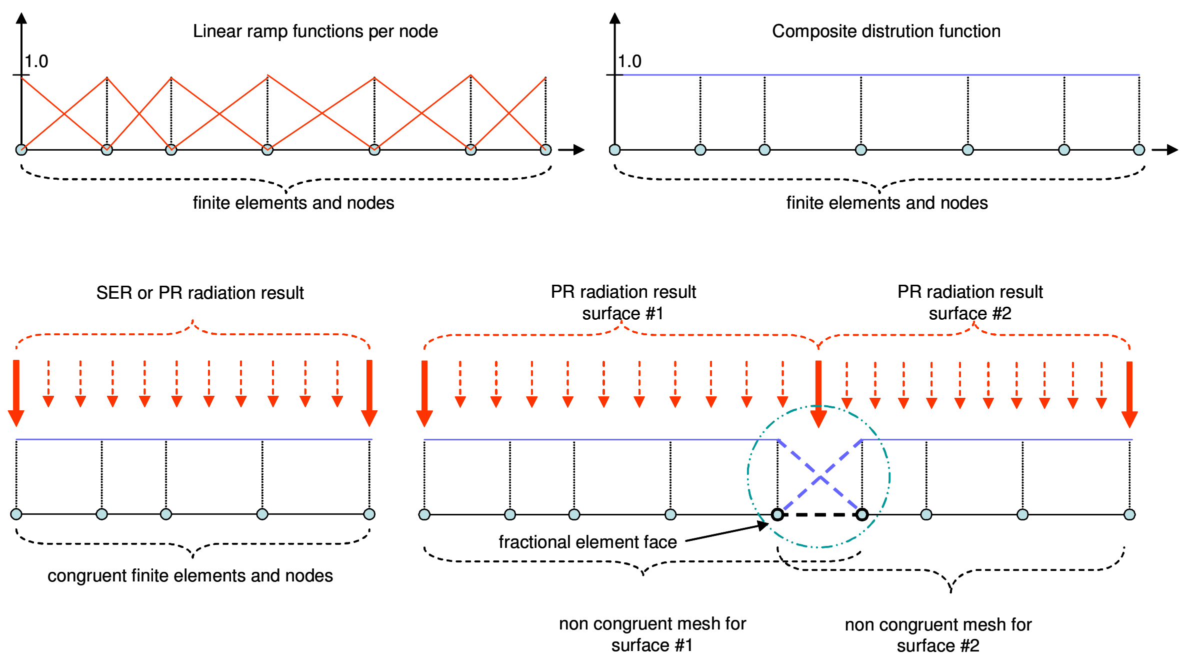
When dealing with SER and PR cases, SDB2SG must map the results obtained for the radiation mesh over presumably more finely meshed finite elements. The upper right figure shows a hemisphere modeled by 6x24 finite elements; the lower figure shows a primitive hemisphere modeled with AxB = 5x6. The upper figure could come from a FR or SER case, or could be the incongruent mesh corresponding to the 5x6 meshed primitive surface below.

When processing the results from the radiation solver, SDB2SG distributes the results to the nodes associated with the radiation surface. SDB2SG handles FR, SER, and PR with the same basic method, simplifying algorithm strategies for models consisting of a mixture of the three methods. This process begins by creating a nodal facet representation for each radiation surface in which each radiation surface is represented as a list of nodes, with a surface area a unit normal vector for each node. After solving the radiation model, nodal facet data are used to compute the weight factors that map the radiation results back to the nodes.

The nodal facet representation is crucial to the distribution and mapping of radiation results to the nodes. For SER and PR with congruent mesh relationships, this process is straightforward, involving no fractional element faces. With fully included element faces, nodal area and normal vector calculation is simple, because the nodal area for each vertex of the element "belongs" entirely to that surface.

For non-congruent PR, fractional element faces tend to complicate things, though full element faces continue to be handled with simpler procedure used for SER and congruent PR cases. SDB2SG resorts to numerical integration for the fractional faces, and this integration process requires the selection of distribution functions. SDB2SG uses linear ramp distribution functions, though step functions were also tried with less accurate results.

SDB2SG divides fractional element faces into small areas (Gaussian integration method), and locates these integration sub-areas on the primitive surface relative to its AxB mesh and parametric coordinates. This process identifies the sub-area with a daughter surface, or multiple daughters if the point falls on AxB mesh gridlines. The distribution function then specifies how this bit of area is split to the nodal vertices of the element. Therefore, shared element faces cause associated nodes to include locations that actually fall outside the daughter boundary. In the nodal facet representation, the edges of the daughter surface will be "fuzzy" in the regions of shared element faces. The edge of one surface fades out while simultaneously the edge of the neighboring surface fades in. These points are illustrated in the following figure.



NUMERICAL METHODS

SDB2SG maps the radiation results to the nodes using the nodal facet representations of the surfaces. Orbital heating is the simplest case, being based on simple area-weighting of the results, according to the following equation.

(1) $$q_{nodal} = A_{nodal} Q_{total} / A_{total}$$

Surface to surface radiation distribution involves a more complex weighting scheme, though its basic inspiration comes from the classic integral equations used to compute view factors. The following equation shows this integration and how it is approximated from the nodal facet representation. The double summation clearly implies breaking the bulk radiation conductor down into a rank-2 matrix of size m by n; each element of the matrix being a nodal coupling, with all the coupling summing to the correct total radiation conductance.

(2) $$A_i F_{ij} = A_j F_{ji} = \int_{A_i}\int_{A_j} \frac{\cos(\theta_i)\cos(\theta_j)}{\pi L_{ij}^2} dA_j dA_i \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\cos(\theta_i)\cos(\theta_j)}{\pi L_{ij}^2} \Delta A_j \Delta A_i$$

The gray body radiation conductance G_{ij} between 2 surfaces is given by the following relationship. The summation will not be explicitly computed, since this result can be obtained directly from the radiation solver. Therefore, it is replaced by D_{ij} which represents reflected radiation component.

(3) $$G_{ij} = \varepsilon_i A_i B_{ij} = \varepsilon_i A_i \left(F_{ij}\varepsilon_j + \sum_{k=1}^{p} F_{ik}(1-\varepsilon_k) B_{kj} \right) = \varepsilon_i A_i \left(F_{ij}\varepsilon_j + D_{ij} \right)$$

The rightmost term in the above equation divides the conductance into the direct and reflected components, two conductors in parallel. Substituting (2) into (3) gives the following.

(4) $$G_{ij} = \varepsilon_i \varepsilon_j \int_{A_i}\int_{A_j} \frac{\cos(\theta_i)\cos(\theta_j)}{\pi L_{ij}^2} dA_j dA_i + \varepsilon_i A_i D_{ij}$$

For purposes of numerical weight calculation, putting D_{ij} on the same footing as F_{ij} is desirable, so we can rewrite D_{ij} as.

(5) $$D_{ij} = \varepsilon_i \int_{A_i}\int_{A_j} \Theta_{ij} dA_j dA_i = \varepsilon_i \Theta_o \int_{A_i}\int_{A_j} dA_j dA_i$$

In (5) Θ_{ij} is an unknown function. Like view factor integration kernel, Θ_{ij} is a function of angles, areas, and distances, but as implied in (3), the function involves all the other surfaces. Computationally, the cost of accurate Θ_{ij} determination exceeds the cost of the radiation solution, so an approximate function must be chosen. The following constant function was chosen: $\Theta_{ij} = \Theta_o$ = constant.

Assuming Θ_{ij} is similar to the kernel of the integral in (4), this effectively makes the cosine and the length terms constant, physically implying reflected radiation strikes the surface at some constant average angle and travels the same average distance. Replacing the integrals with summations and inserting (5) into (4) gives the following, which is the equation SDB2SG uses to determine the surface to surface nodal weighting factors.

$$G_{ij} = \varepsilon_i \varepsilon_j \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\cos(\theta_i)\cos(\theta_j)}{\pi L_{ij}^2} \Delta A_j \Delta A_i + \varepsilon_i \Theta_o \sum_{i=1}^{m} \sum_{j=1}^{n} \Delta A_j \Delta A_i \qquad (6)$$

The Θ_o constant is chosen to ensure the second summation completes the conductance balance on G_{ij}. Equation (6) plainly shows G_{ij} can be divided into two matrices (size m by n) of nodal coupling – one matrix for direct and the other for reflected radiation. When no reflections take place ($F_{ij}\varepsilon_j = B_{ij}$) only the direct weights in the left summation are used. When radiation is purely by reflection ($F_{ij} = 0$, but $B_{ij} > 0$) only the right summation is used. When both modes of radiation are present ($B_{ij} > F_{ij}\varepsilon_j$), both summations are used in proper proportions.

The following three REF Distribution Method options provide controls in PATRAN for this method.

- Full (default) – automatically proportions the two weighting method according to B_{ij} and F_{ij} from the radiation solver
- Area – uses only the left summation in (6), distance and viewing angle are ignored, factored to ensure the full G_{ij} value is conserved; radiation will be spread evenly over the entire surfaces, including regions that are not mutually visible
- Direct – uses only the right summation in (6) , factored to ensure the full G_{ij} value is conserved; all radiation will be distributed over the mutually viewable regions of the surfaces

When dividing PR and SER radiation to the finite element mesh, self-radiation is not ignored. SDB2SG computes the self radiation for each surface by summing all conductors to that surface, including the conductors to space. SDB2SG then performs the standard surface to surface algorithm described above to interconnect the mesh on this surface.

After all surface to surface radiation terms (including self-radiation) have been converted to inter-nodal couplings, a radiation conductance balance is performed on each node, with the remaining balance counted as conductance to space. Other than its role in computing self-radiation, the space conductors determined by the radiation solver are not used.

APPLICATION – SG4PATRAN IN ACTION

THE MODELS

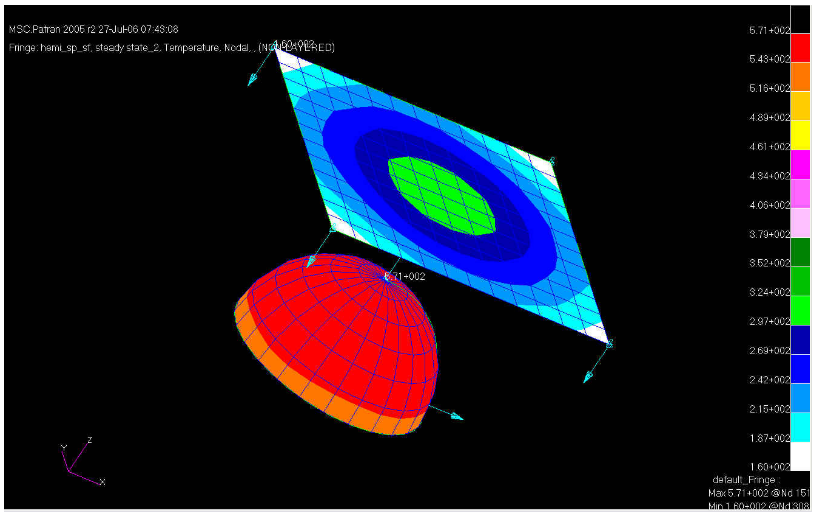


Figure 1: Small Facet Model

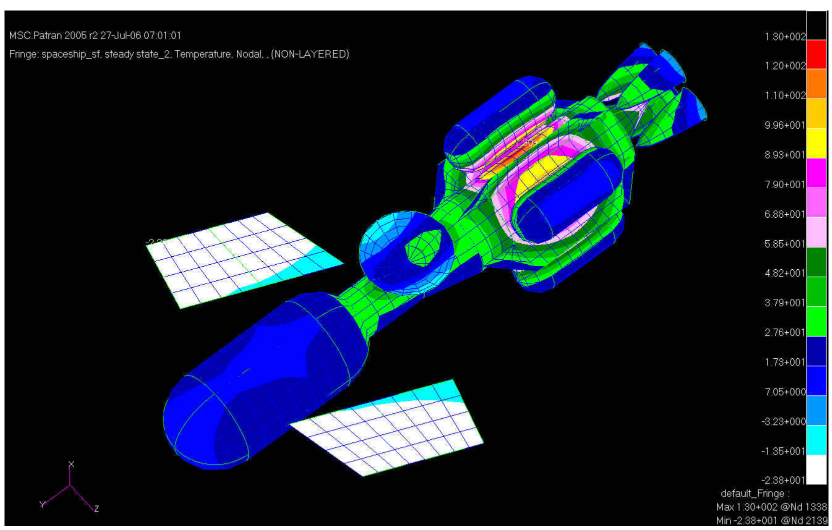


Figure 2: Small Faceted Spaceship Model

These show the small facet models of a hemisphere and plate that are radiating to each other and out to space, and a spaceship with complex radiation, as well. The small facet method is the slowest of the methods, but is the best to use where there are sharp gradients in either the temperature, or geometry.

The other two type of radiation that we used (SE and PR) look similar to the above figures, but the runtime to solve them and the final results were significantly different for each type of model. The table below shows the results from our studies.

	Time (sec)		Results		Percent Error	
	Rad Comp	Preprocess	High Temp	Low Temp	% Diff High from SF	% Diff Low from SF
Spaceship						
SF	68	15.07	130	-2.38	0.00	0.00
SE	10	15.68	125	-2.36	3.85	0.84
PR 1x1	3	14.68	129	-2.39	0.77	0.42
PR 4x4	48	19.304	134	-2.39	3.08	0.42
SF in orbit	160	14.9	130	-2.38	0.00	0.00
PR 4x4orbit	330	19.86	139	-2.48	6.92	4.2
Hemisphere						
SF	7	1.47	571	160	0.00	0.00
SE	0.6	1.4	570	165	0.18	3.13
PR 1x1	0.3	1.5	564	157	1.23	1.88
PR 4x4 mismatched	1	1.5	569	159	0.35	0.63
PR 6x6 matched	1.5	1.9	569	160	0.35	0.00

The percent error in the right columns relates to the small facet method of the specific model. The SE and PR methods are compared to the small facet results.

The 1x1 or 4x4 or 6x6 refers to the AB mesh of the primitive radiation load. The PR load that is "matched" states that the conduction mesh was aligned with the radiation mesh. "Mismatched" states that interpolation of the radiation results were needed on the conduction mesh.

The orbit that is seen in the last four spaceship models is a sun-synchronous orbit to show the effects of orbital heating on the radiation analysis. The primitive method with an AB mesh of 4x4 is similar to the SF method for the orbital analysis, which is what we would expect.

It is verified that the SF method is the slowest, although it is the most accurate of the methods. The primitive method, when using an AB mesh of 1x1 is the fastest running method, and does not have the worst error from the small facet method.

CONCLUSION

SINDA/G for PATRAN offers unique radiation modeling capabilities in a transparent and easy-to-use SINDA/G integration into PATRAN. SG4PATRAN is an open system concept that interfaces with all popular CAD systems or FEA programs and produces models that are thermal radiation/orbital heating code independent. The interface to radiation code can be in the form of flat plates created by the FEA, a grouping of small element faces into super elements, or primitive models that use true curved surfaces to represent the model. This interface also allows more coarse radiation meshed models to be overlaid on finer conduction models.

With these advances in radiation solutions, models take much less time to run, while holding a respectable accuracy level.