Thermal Capacitance (Slug) Calorimeter Theory including Heat Losses and other Decaying Processes

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Thermal and Fluids Analysis Workshop 2008
San Jose State University, San Jose, CA
Presented: August 18, 2008

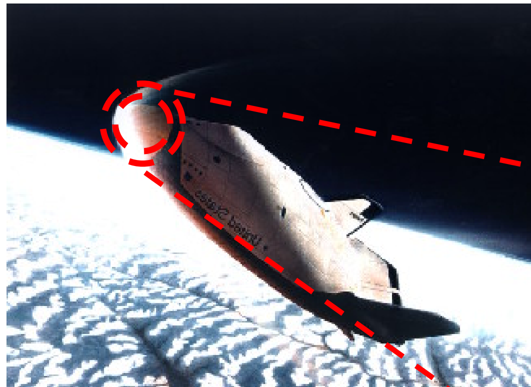
Outline

- Arc jet description
- Slug calorimeter description
- Issue of heat losses from slug calorimeter cited in literature
- Idealized slug calorimeter theory – no losses and constant physical properties
- General slug calorimeter theory – with losses and variable heat capacity – Slug Loss Model
- Slug Loss Model applied to slug calorimeter data from one arc jet run
- Finite Element Analysis (FEA) model of slug calorimeter data from same arc jet run
- Comparison of all models
- Conclusions

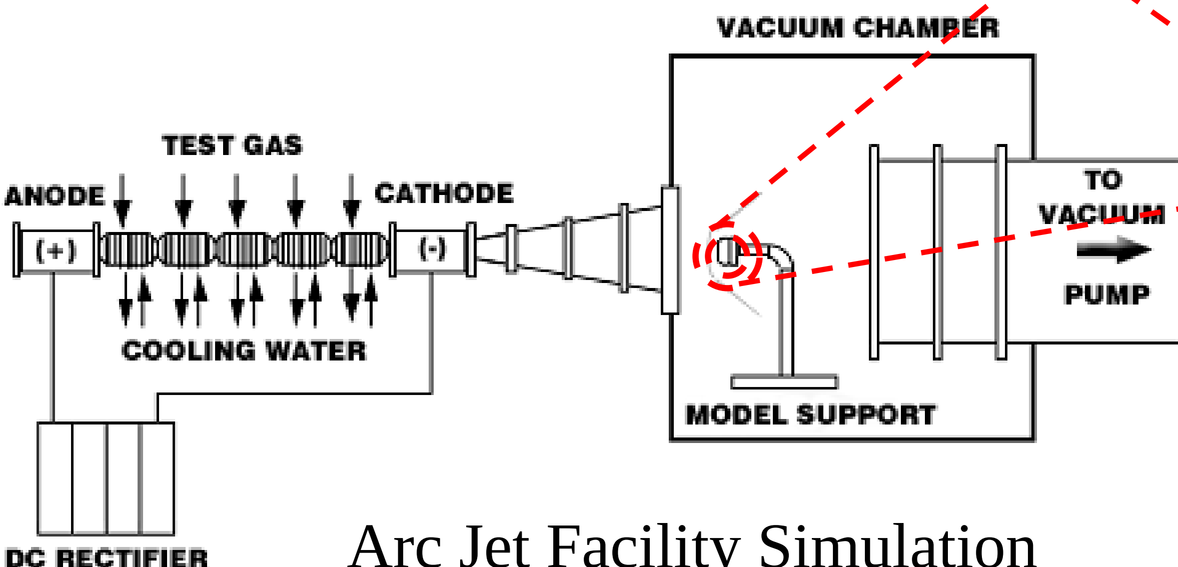
Arc jet description

Reentry Flight Environment

100 cm Diameter Nose Cap
On a Reusable Vehicle

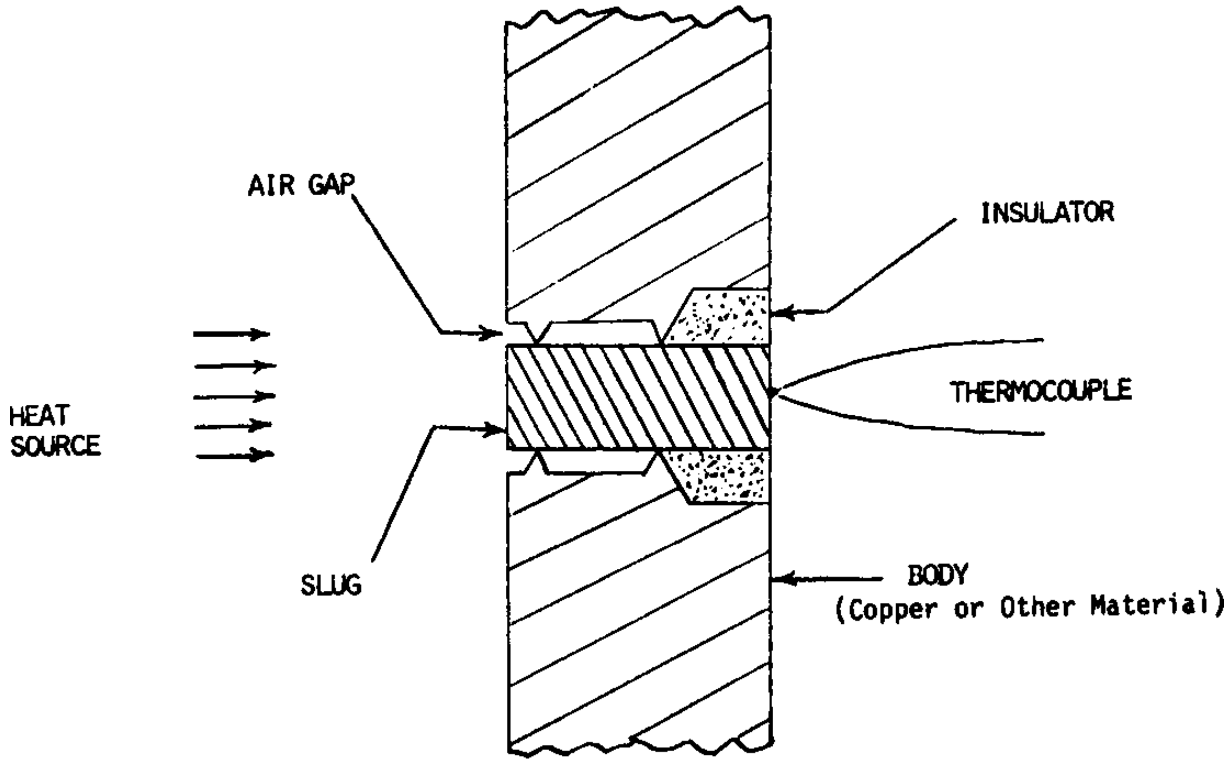


25x25 cm Test Body in the Arc Jet Plasma Stream



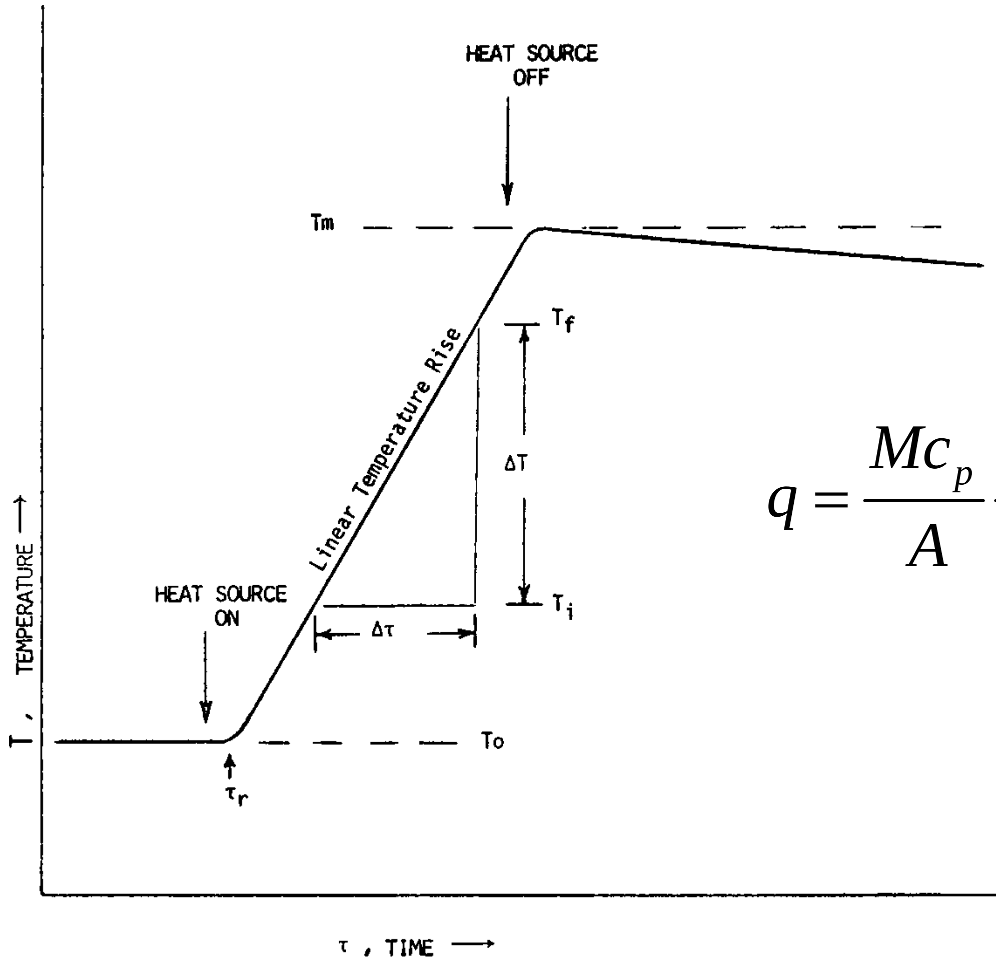
Arc Jet Facility Simulation

Slug calorimeter description



Schematic of a Thermal
Capacitance (Slug) Calorimeter

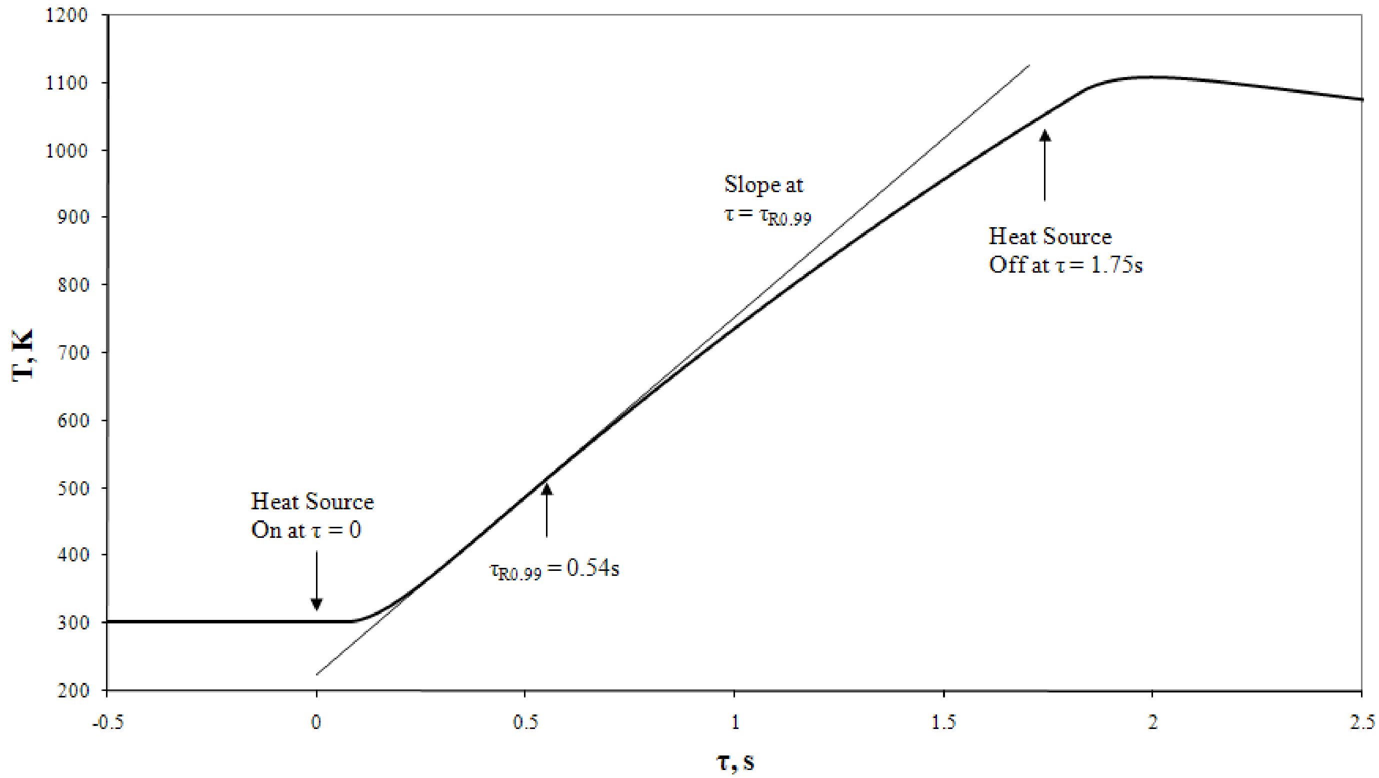
Slug calorimeter description



$$q = \frac{Mc_p}{A}\frac{\Delta T_b}{\Delta t} = L\rho c_p \frac{\Delta T_b}{\Delta t}$$

Typical Temperature—Time Curve
for Slug Calorimeter

Slug calorimeter description



Time Curve when heat & other losses are significant during heating phase

Slug calorimeter heat losses & correcting for them (quotes from literature)

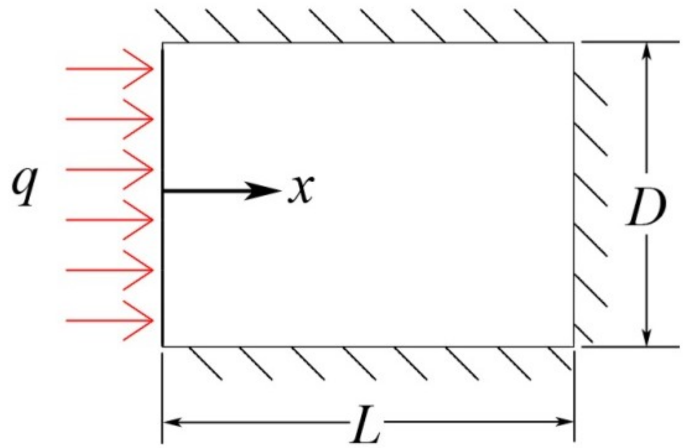
"The heat losses are usually hard to control in models with high-heat-flux conditions."

Diller, T. E., "Advances in Heat Flux Measurements," *Advances in Heat Transfer*, Vol. 23, Academic Press, 1993, pp. 307-311

"If more accurate results are required, the losses through the insulation layer should be modelled and accounted for by a correction term . . ."

Childs, P. R. N., Greenwood, J. R., and Long, C. A., "Heat flux measurement techniques," *Proceedings of the Institution of Mechanical Engineers*, Vol. 213, Part C, 1999, pp. 664-665.

Idealized slug calorimeter theory



•Right circular cylinder made of copper
•Insulated at back face & around circumferential area
•Slug initially at uniform temperature
•Starting at time = 0, constant heat flux q is applied to front face
•Coordinate x defined as zero at front face and L at back face
•Problem can be modeled as one dimensional unsteady state heat transfer
•Additional simplifying assumption: all physical properties are constant with temperature

Boundary value problem

PDE for one dimensional unsteady state heat transfer

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t}$$

Definition of thermal diffusivity

$$\alpha = \frac{k}{\rho c_p}$$

Boundary conditions

$$\frac{\partial T(0,t)}{\partial x} = -\frac{q}{k}$$

$$\frac{\partial T(L,t)}{\partial x} = 0$$

Initial condition

$$T(x,0) = T_o$$

Solution to PDE boundary value problem

$$Overall\ solution = steady\ state\ solution + transient\ solution$$

$$T(x,t) = v_{ss}(x,t) + w(x,t)$$

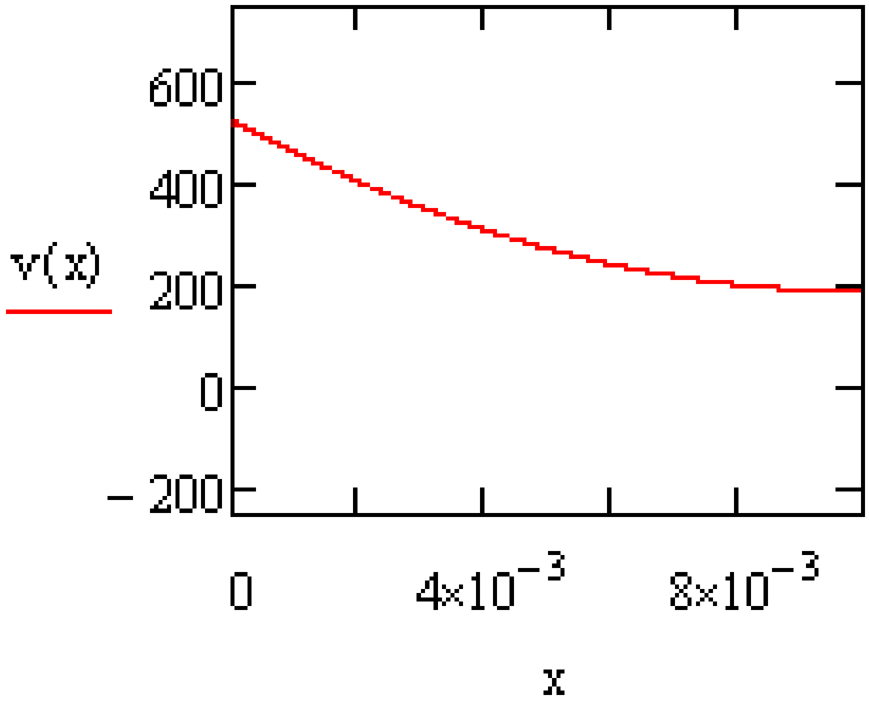
$$v_{ss}(x,t) = T_o + \frac{qt}{L\rho c_p} + \frac{qL}{3k} + \frac{qx^2}{2Lk} - \frac{qx}{k} \qquad \frac{\partial T(x,t)}{\partial t} = constant$$

$$w(x,t) = -\frac{2qL}{k\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left(\frac{n\pi x}{L}\right)e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t}$$

$$T(x,t) = \left(T_o + \frac{qt}{L\rho c_p} + \frac{qL}{3k} + \frac{qx^2}{2Lk} - \frac{qx}{k}\right) - \frac{2qL}{k\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left(\frac{n\pi x}{L}\right)e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t}$$

Animation of solution for copper slug
L= 1 cm, q= 2600 W/cm^2, elapsed t= 0.3 s

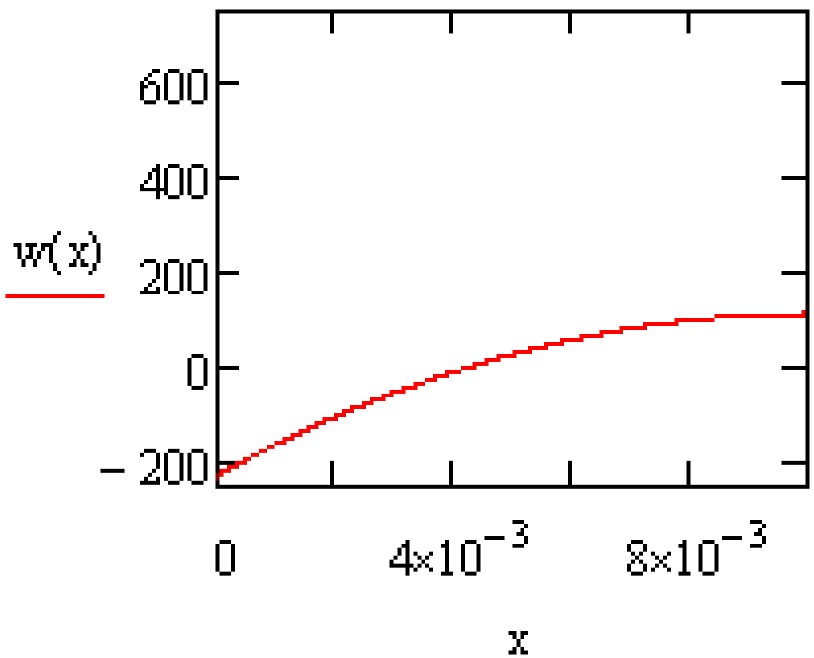
Steady state solution



Response slowed down by a factor of 33

Animation of solution for copper slug
L= 1 cm, q= 2600 W/cm^2, elapsed t= 0.3 s

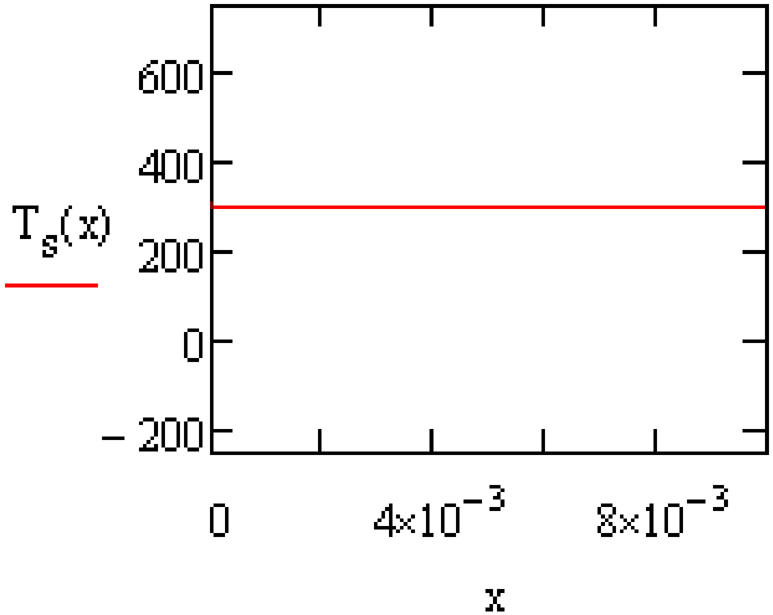
Transient solution



Response slowed down by a factor of 33

Animation of solution for copper slug
L= 1 cm, q= 2600 W/cm^2, elapsed t= 0.3 s

Overall solution



Response slowed down by a factor of 33

Response time equation

Setting $q_{indicated}$ = 0 gives time for the heat to have just penetrated to the back side of the slug.

$$t_R = \frac{L^2}{\alpha\pi^2}\ln\left(\frac{2}{1-\frac{q_{indicated}}{q_{input}}}\right)$$

For practical purposes, the response time calculated when $q_{indicated}/q_{input}$ = 0.99 should be sufficient elapsed time for the heat flux determination from the back face temperature to begin to be valid, and implies steady state.

$$t_{R0.99} = \frac{L^2}{\alpha\pi^2}\ln\left(\frac{2}{1-0.99}\right)$$

Other useful equations

$$T_b = T(L,t) = T_o + \frac{q\alpha t}{kL} - \frac{1}{6}\frac{qL}{k} - \frac{2qL}{k\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t}$$

$$T_f = T(0,t) = T_o + \frac{q\alpha t}{kL} + \frac{1}{3}\frac{qL}{k} - \frac{2qL}{k\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t}$$

$$T_{ave} = T_o + \frac{q\alpha t}{kL}$$

$$T_f - T_b = \frac{1}{2}\frac{qL}{k} - \frac{4qL}{k\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}e^{-\alpha\left(\frac{(2n-1)\pi}{L}\right)^2 t}$$

$$T_{ave} - T_b = \frac{1}{6}\frac{qL}{k} + \frac{2qL}{k\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t}$$

Summation terms approach zero with steady state

$$T_b = T(L,t) = T_o + \frac{q\alpha t}{kL} - \frac{1}{6}\frac{qL}{k}$$

$$T_f = T(0,t) = T_o + \frac{q\alpha t}{kL} + \frac{1}{3}\frac{qL}{k}$$

$$T_{ave} = T_o + \frac{q\alpha t}{kL}$$

$$T_f - T_b = \frac{1}{2}\frac{qL}{k}$$

$$T_{ave} - T_b = \frac{1}{6}\frac{qL}{k}$$

Accounting for heat losses – Slug Loss Model

A heat balance on the slug with losses gives

$$input - output(i.e.\ losses) = accumulation$$

$$qA - \frac{\left(T_{ave} - T_o\right)}{R_{la}} = Mc_{po}\frac{dT_{ave}}{dt}$$

$$T_{ave} = T_b + \frac{qL}{6k}$$

Getting equation in terms of T_b

$$\left(q\left(\frac{A}{Mc_{po}} - \frac{L}{6kR_{la}Mc_{po}}\right) + \frac{T_o}{R_{la}Mc_{po}}\right) - \frac{T_b}{R_{la}Mc_{po}} = \frac{dT_b}{dt}$$

Accounting for heat losses – Slug Loss Model

Defining two constants

$$a = \left(q \left(\frac{A}{Mc_{po}} - \frac{L}{6kR_{la}Mc_{po}} \right) + \frac{T_o}{R_{la}Mc_{po}} \right)$$

$$b = \frac{1}{R_{la}Mc_{po}}$$

Differential equation can be written as

$$a - bT_b = \frac{dT_b}{dt}$$

Which integrates to

$$T_b = \left(T_{b1\,fit} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b}$$

Accounting for heat losses – Slug Loss Model

Once data is fit to this

$$T_b = \left(T_{b1\,fit} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b}$$

Rearrange this

$$a = \left(q \left(\frac{A}{Mc_{po}} - \frac{L}{6kR_{la}Mc_{po}} \right) + \frac{T_o}{R_{la}Mc_{po}} \right)$$

$$b = \frac{1}{R_{la}Mc_{po}}$$

To solve for q

$$q = \frac{Mc_{po}}{A} \frac{\left(a - bT_o \right)}{\left(1 - \frac{L}{6kR_{la}A} \right)}$$

Accounting for heat losses – Slug Loss Model

Once you have this fit

$$T_b(t) = \left(T_{b1\,fit} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b}$$

By this equation

$$T_{ave} = T_b + \frac{qL}{6k}$$

you also have

$$T_{ave}(t) = \left(\left(T_{b1\,fit} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b} \right) + \frac{qL}{6k}$$

and

$$\frac{dT_b(t)}{dt} = \frac{dT_{ave}(t)}{dt} = -b \left(T_{b1\,fit} - \frac{a}{b} \right) e^{-b(t-t_1)}$$

All analytical expressions

Accounting for heat losses – Slug Loss Model

Now write energy balance equation with actual loss resistance and variable heat capacity

$$qA - \frac{\left(T_{ave}(t) - T_o\right)}{R_l} = Mc_p(T_{ave}(t))\frac{dT_{ave}(t)}{dt}$$

Variable heat capacity with T is obtained from the Shomate equation for copper

$$c_p(T_{ave}) = A + BT_{ave} + CT_{ave}{}^2 + DT_{ave}{}^3 + \frac{E}{T_{ave}{}^2}$$

where

$$A = 2.789933x10^2 \frac{J}{kgK} \qquad B = 4.421789x10^{-1} \frac{J}{kgK^2}$$

$$C = -4.918152x10^{-4} \frac{J}{kgK^3} \qquad D = 2.19879x10^{-7} \frac{J}{kgK^4}$$

$$E = 1.079706x10^6 \frac{JK}{kg}$$

Accounting for heat losses – Slug Loss Model

Solve energy balance equation for R_l, the actual loss resistance

$$R_l(t) = \frac{\left(T_{ave}(t) - T_o\right)}{\left(qA - Mc_p(T_{ave}(t))\frac{dT_{ave}(t)}{dt}\right)}$$

Other useful equations

$$q_{slopeTb}(t) = \frac{Mc_p(T_b(t))}{A}\frac{dT_b(t)}{dt}$$

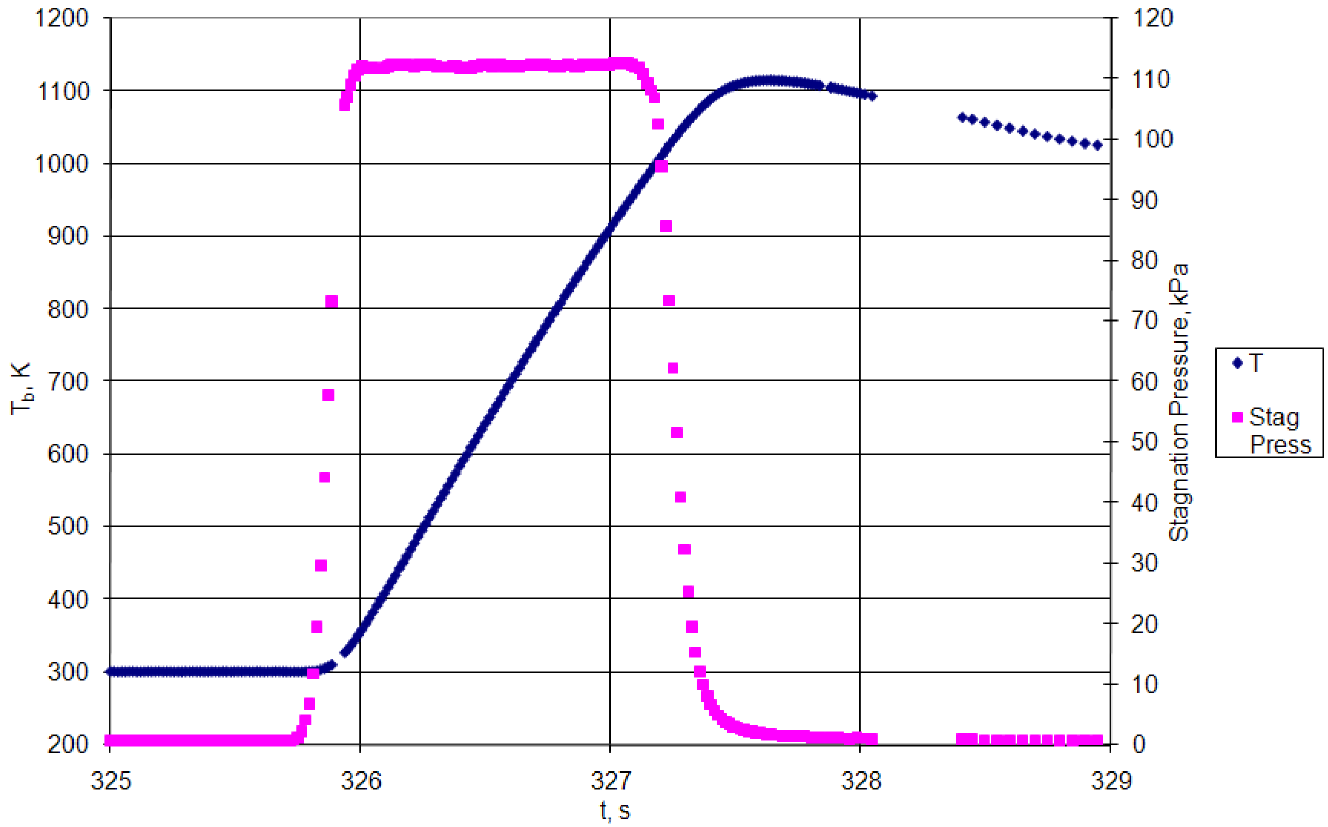
$$q_{slopeTave}(t) = \frac{Mc_p(T_{ave}(t))}{A}\frac{dT_{ave}(t)}{dt}$$

$$q_{loss}(t) = q - q_{slopeTave}(t) = q - \frac{Mc_p(T_{ave}(t))}{A}\frac{dT_{ave}(t)}{dt}$$

$$FracLoss(t) = 1 - \frac{Mc_p(T_{ave}(t))}{qA}\frac{dT_{ave}(t)}{dt}$$

Slug Loss Model (SLM) applied to one arc jet run (IHF187R025)

Back Face Temperature & Stagnation Pressure versus Time.



SLM applied to one arc jet run (IHF187R025)

$$\rho = 8{,}925.7 \frac{kg}{m^3} \qquad c_{po} = 385.615 \frac{J}{kgK} \qquad k = 385.2 \frac{W}{mK}$$

$$M = 0.004529\, kg \qquad D = 0.00781\, m$$

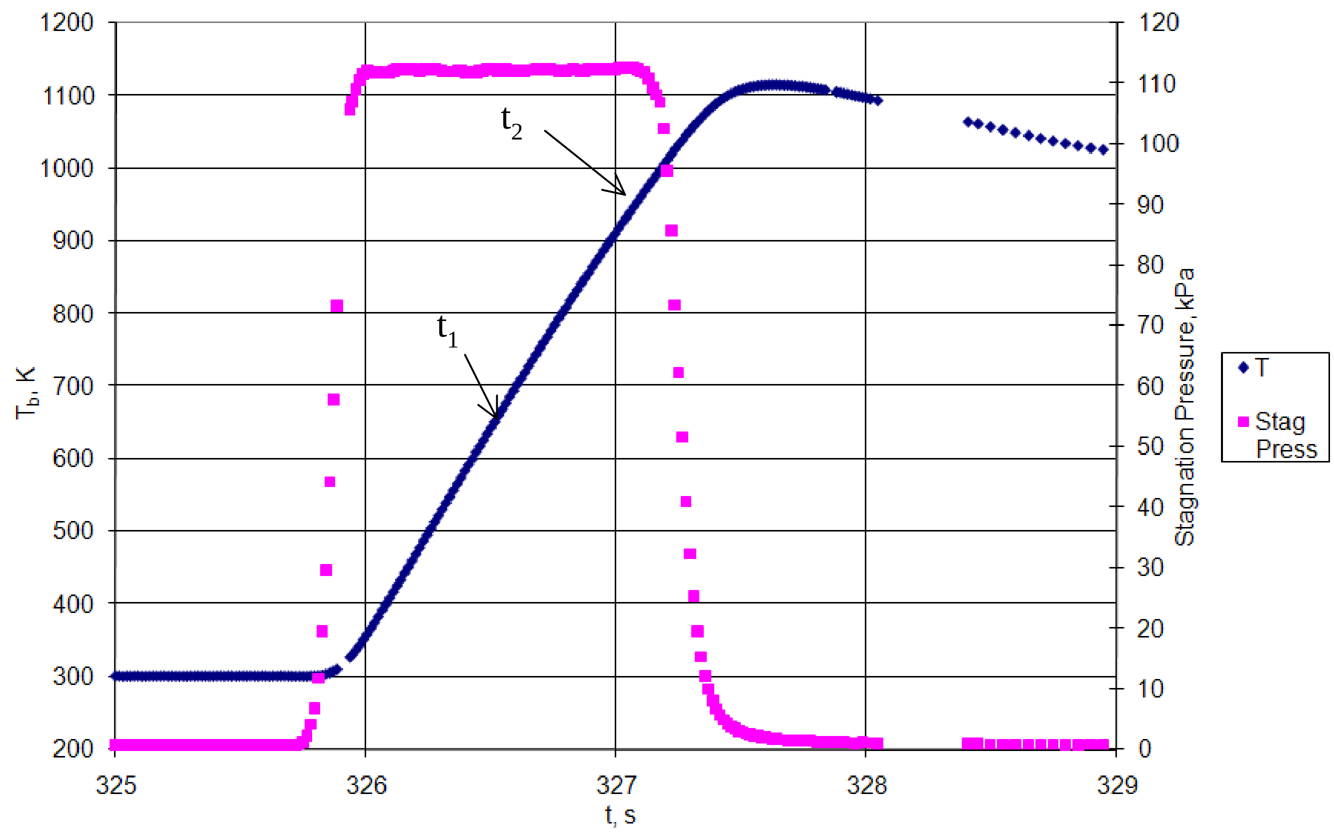
$$A = 0.25 \pi D^2 = 0.000047906\, m^2$$

$$L = \frac{M}{\rho A} = 0.010592\, m$$

$$t_{R0.99} = \frac{\rho c_{po} L^2}{k \pi^2} \ln\left(\frac{2}{1-0.99}\right) = 0.538\, s$$

SLM applied to one arc jet run (IHF187R025)

Back Face Temperature & Stagnation Pressure versus Time.



SLM applied to one arc jet run (IHF187R025)

Back Face Temperature versus Time data from t_1 to t_2.

t, s	Tb, K	t, s	Tb, K	t, s	Tb, K	t, s	Tb, K
326.532	660.7955	326.682	744.1884	326.832	825.295	326.983	903.1487
326.547	668.8717	326.697	752.7801	326.847	833.1806	326.997	909.7945
326.562	677.0599	326.712	760.8678	326.862	841.5302	327.012	917.442
326.577	686.4049	326.727	768.6463	326.877	848.8967	327.027	925.1961
326.592	694.5348	326.742	777.3664	326.892	856.1385	327.042	932.1872
326.608	703.5074	326.757	785.3169	326.907	864.8431	327.057	939.6873
326.622	711.217	326.773	794.0334	326.922	872.3267	327.072	947.399
326.637	719.5663	326.787	801.3908	326.937	879.3349	327.087	954.5518
326.652	728.1516	326.802	809.1195	326.952	887.2804	327.102	961.6053
326.667	736.1388	326.818	818.2387	326.967	895.0791		

Fit to this equation

$$T_b(t) = \left(T_{b1\,fit} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b}$$

SLM applied to one arc jet run (IHF187R025)

Fitting this data to Slug Loss Model equation gives the best linear fit to T_b versus $e^{-b(t-t1)}$ when b = 0.29160 s^{-1}, where the R^2 value of the fit is maximized at 0.99999. The Solver function in an Excel spreadsheet was used to solve for b.

$$a = \text{intercept}\left(T_b \ vs \ e^{-b(t-t_1)}\right) b = 766.76 \frac{K}{s}$$

$$T_{b1\,fit} = \text{slope}\left(T_b \ vs \ e^{-b(t-t_1)}\right) + \frac{a}{b} = 660.32 \ K$$

SLM applied to one arc jet run (IHF187R025)

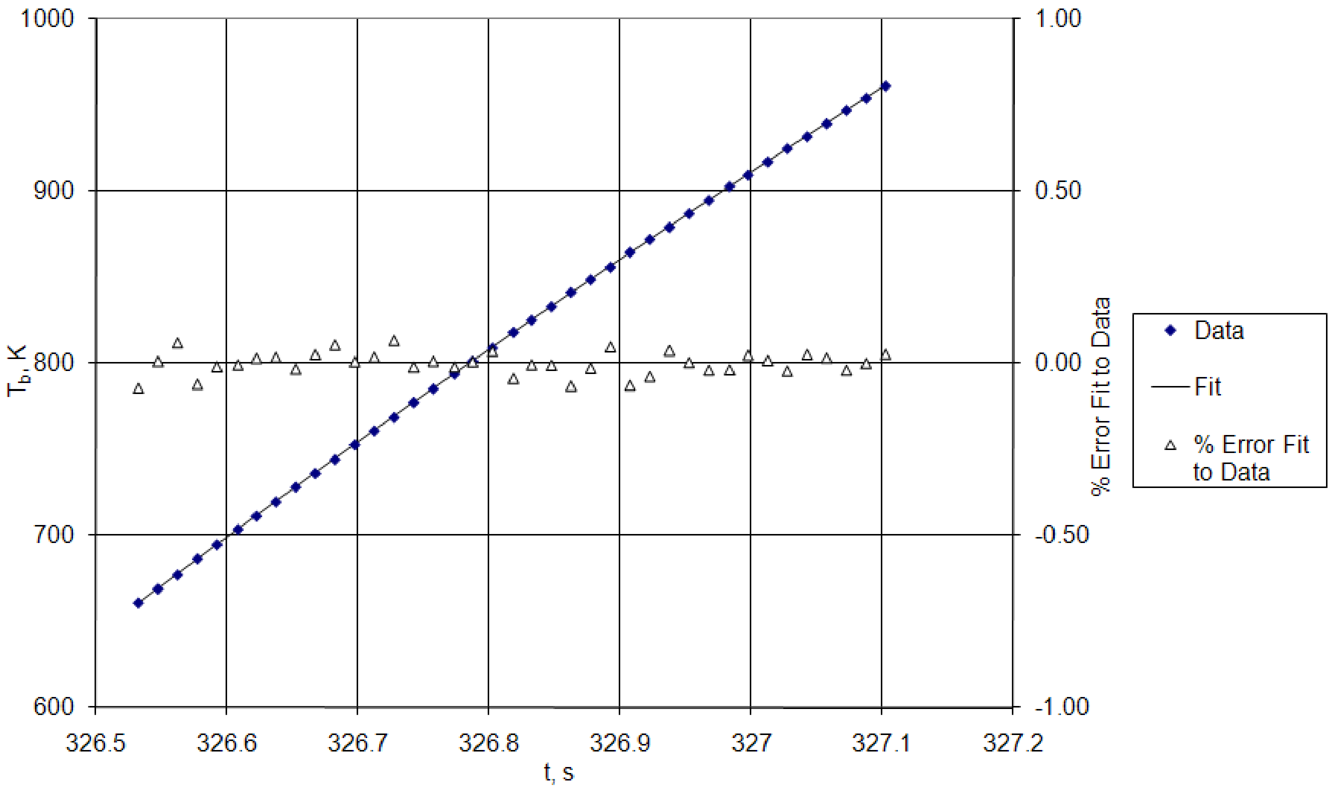
$$R_{la} = \frac{1}{bMc_{po}} = 1.964\frac{K}{W}$$

$$q = \frac{Mc_{po}}{A}\frac{\left(a - bT_o\right)}{\left(1 - \frac{L}{6kR_{la}A}\right)} = 26,005,000\frac{W}{m^2} = 2,600\frac{W}{cm^2}$$

This value is about 15% higher than the value of 2,250 W/cm^2 reported by the facility test engineers, where losses were not taken into account.

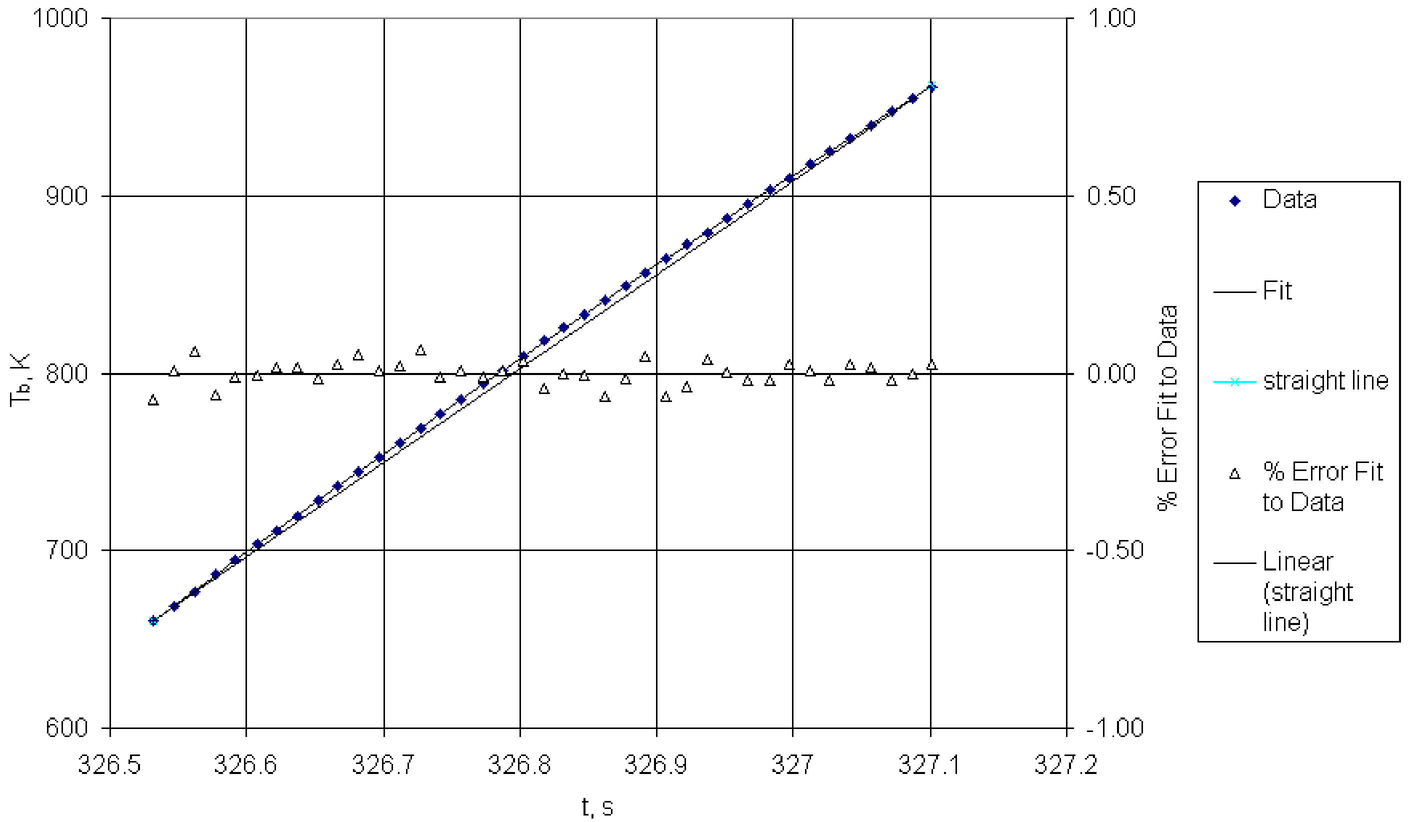
SLM applied to one arc jet run (IHF187R025)

Back Face Temperature – Fit Compared to Data.



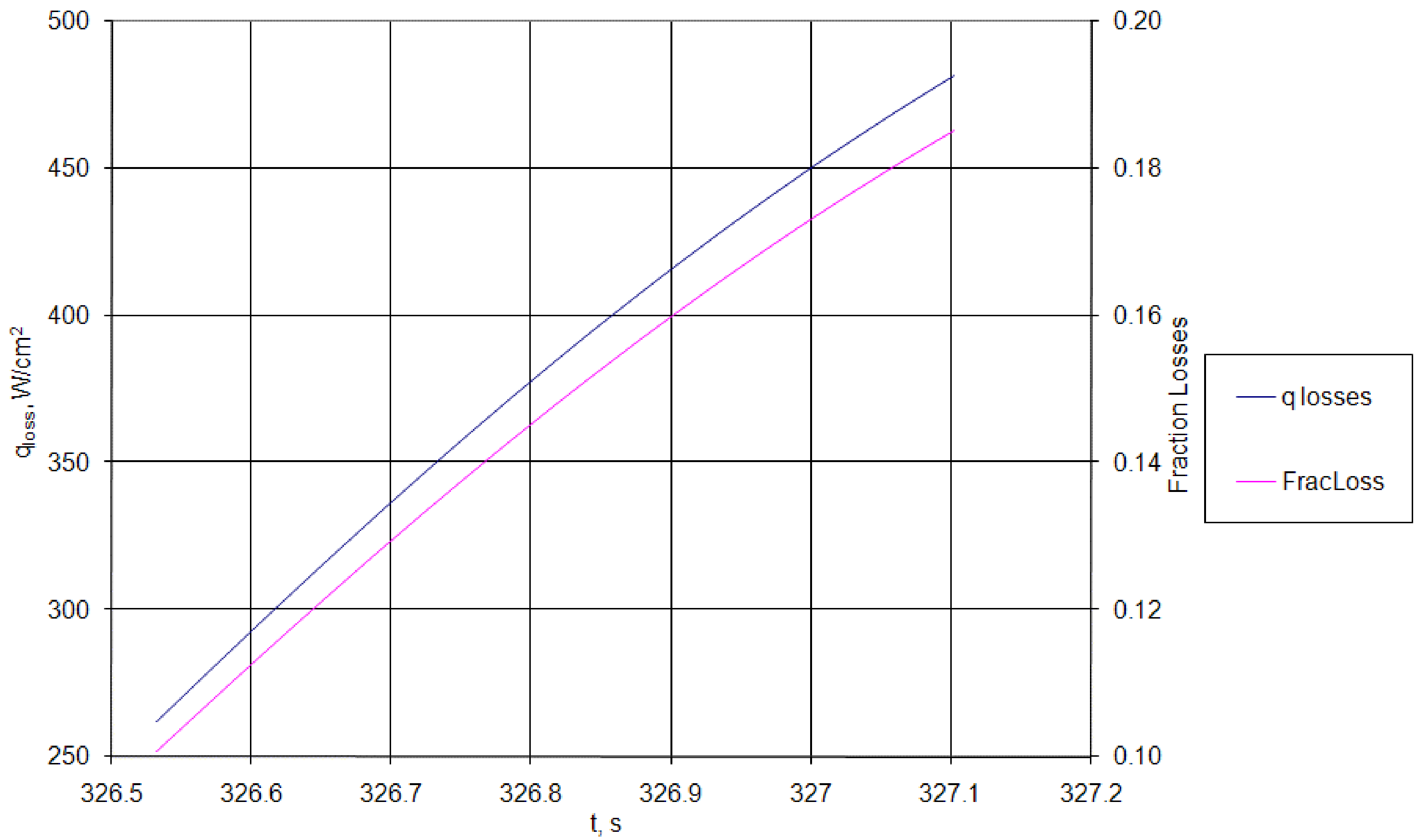
SLM applied to one arc jet run (IHF187R025)

Fit Compared to straight line.



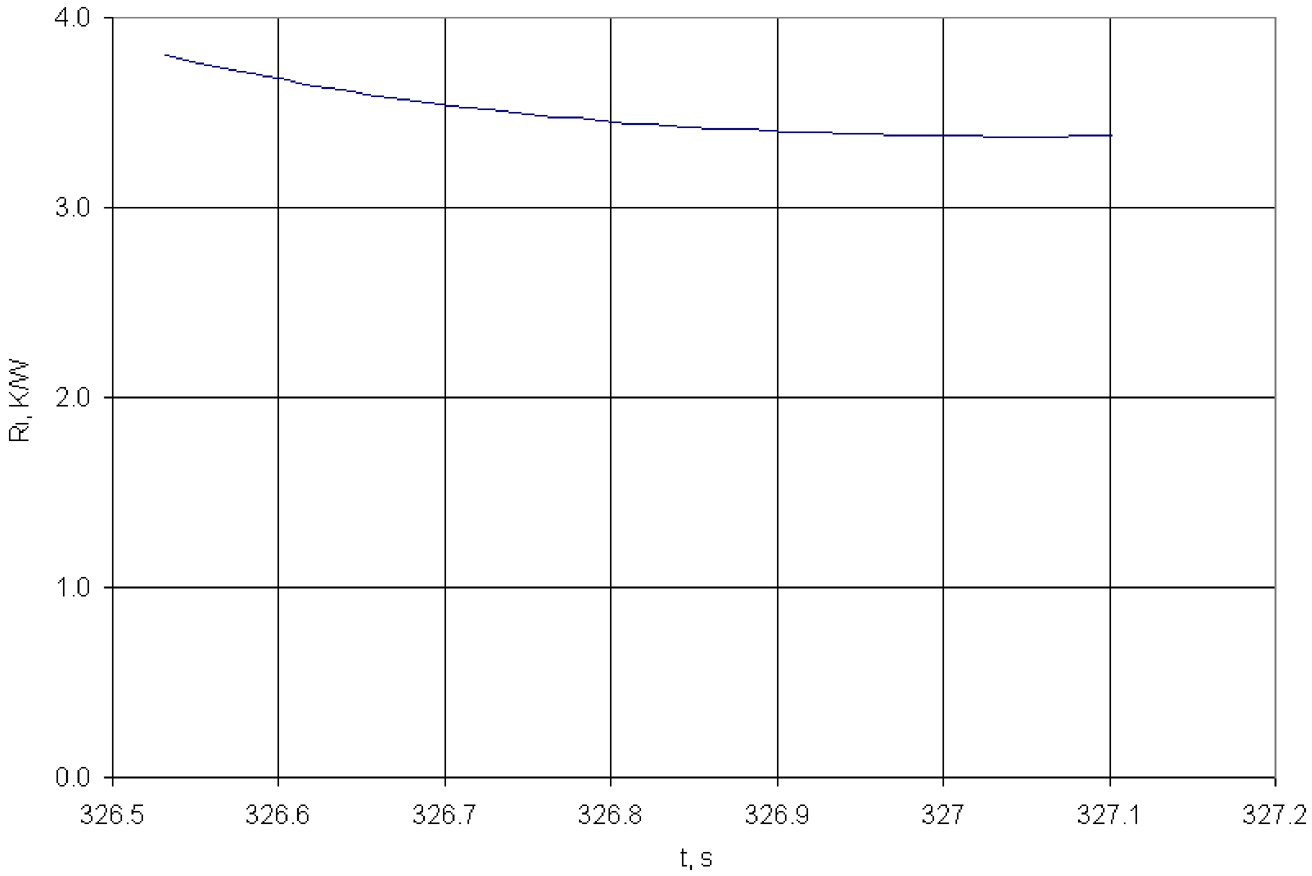
SLM applied to one arc jet run (IHF187R025)

Losses (per cm^2 slug frontal area) versus Time.



SLM applied to one arc jet run (IHF187R025)

Actual Loss Resistance versus Time.

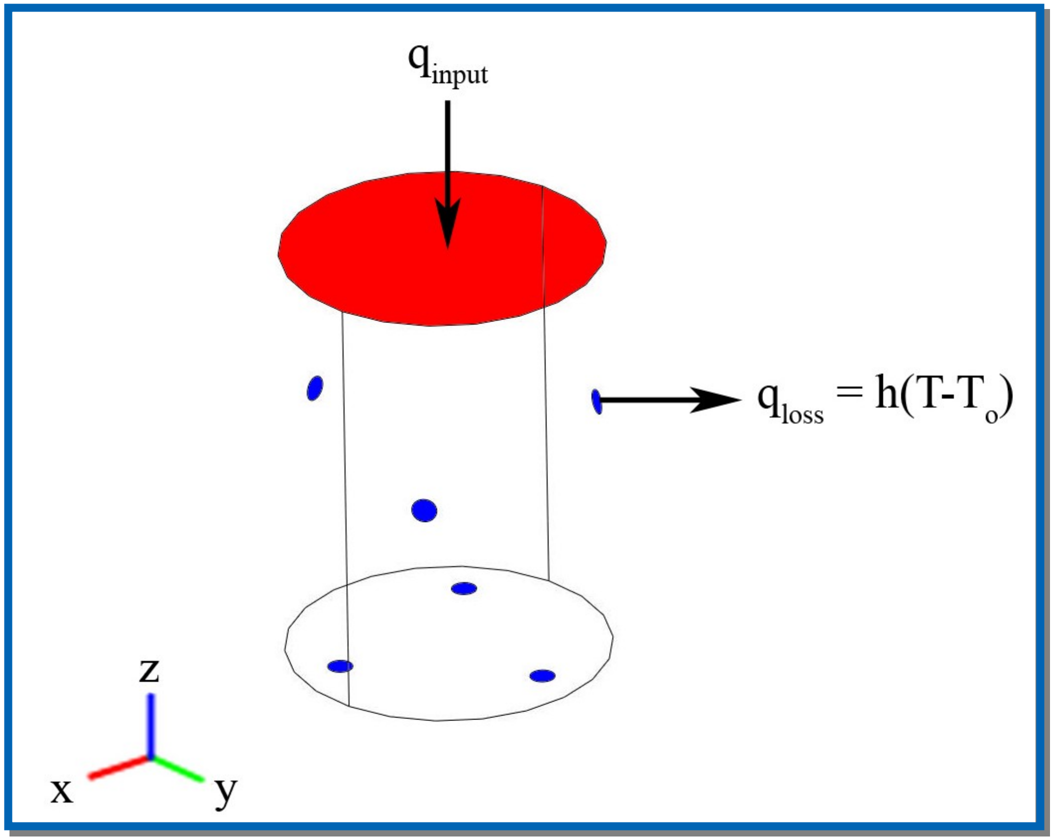


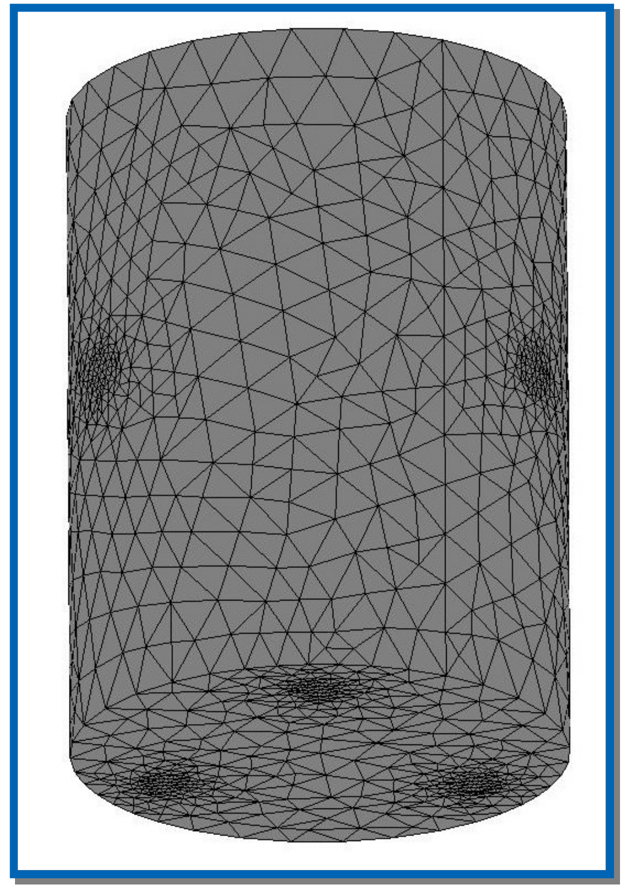
Animation of a 4” Hemi Slug Calorimeter

FEA Model

- A simple FEA model was created using COMSOL Multiphysics, COMSOL, Inc, Burlington, Massachusetts.
- The slug was modeled using 3D tetrahedral elements.
- Heat flux is applied to the top face using a smoothed Heaviside function (flc2hs) to create a ramp up and ramp down time.
- Losses occur through 0.6 mm diameter surface regions with a constant heat transfer coefficient *h*, to a constant holder temperature T_0.
- The material copper is used using temperature dependent properties of heat capacity and thermal conductivity.
- 3 second simulation time with a 0.01 second time step.

FEA Model

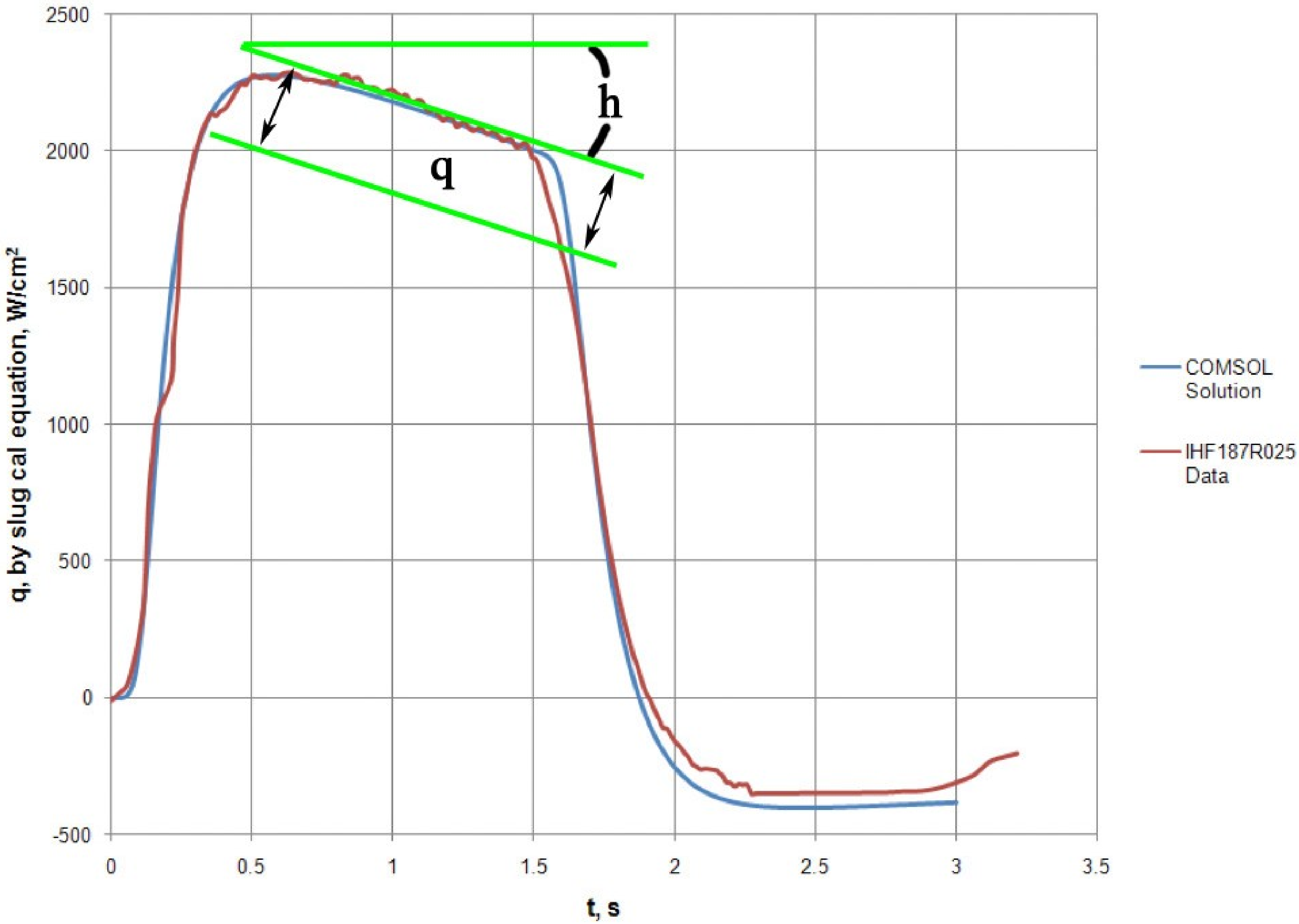




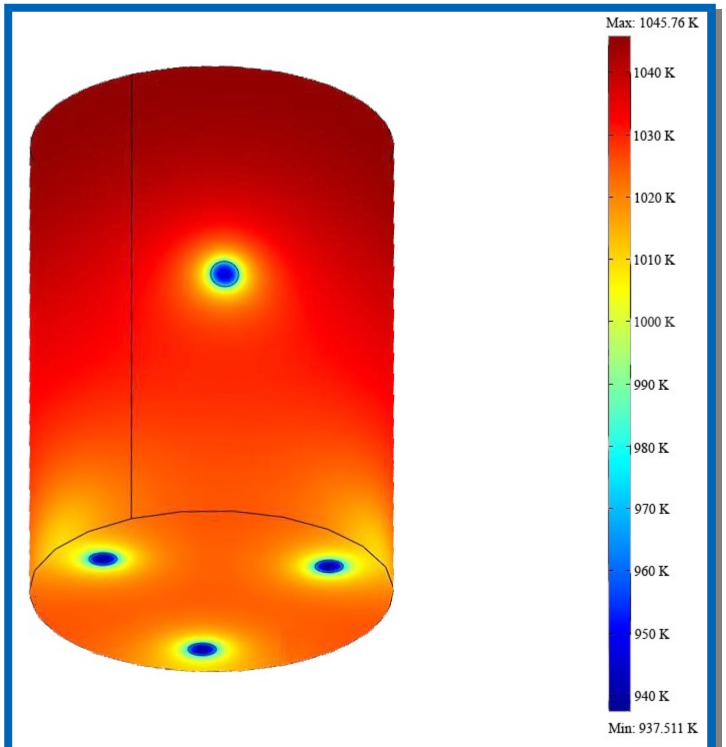
FEA Model

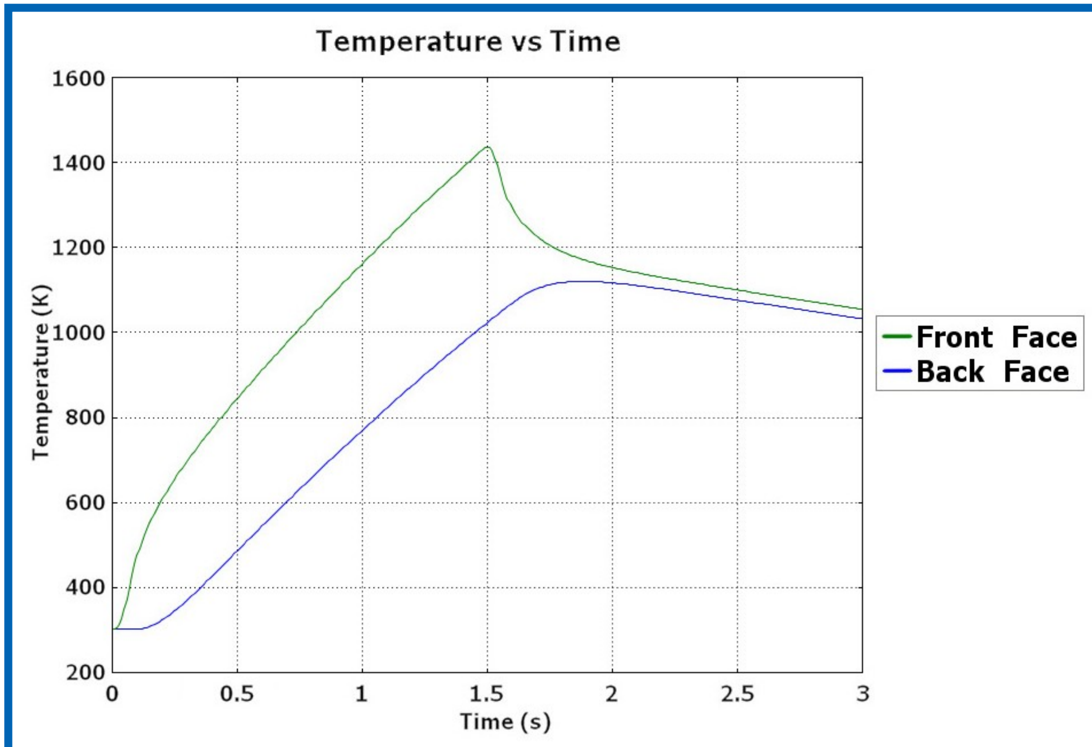
- Various runs were performed by varying q_{input}, *h* and the duration of the pulse in order to match the data.
- A unique solution of q_{input}= 2,600 W/cm^2 was found where the COMSOL solution closely agreed with the actual data. Sensitivity analysis showed this q value to be determinable to +/- 1%.
- A fringe plot of the temperature at t = 3 seconds was plotted to show the paths of the heat flow.
- Temperature was plotted versus time for the centers of the front and back faces of the slug.

Adjusting the results to actual data

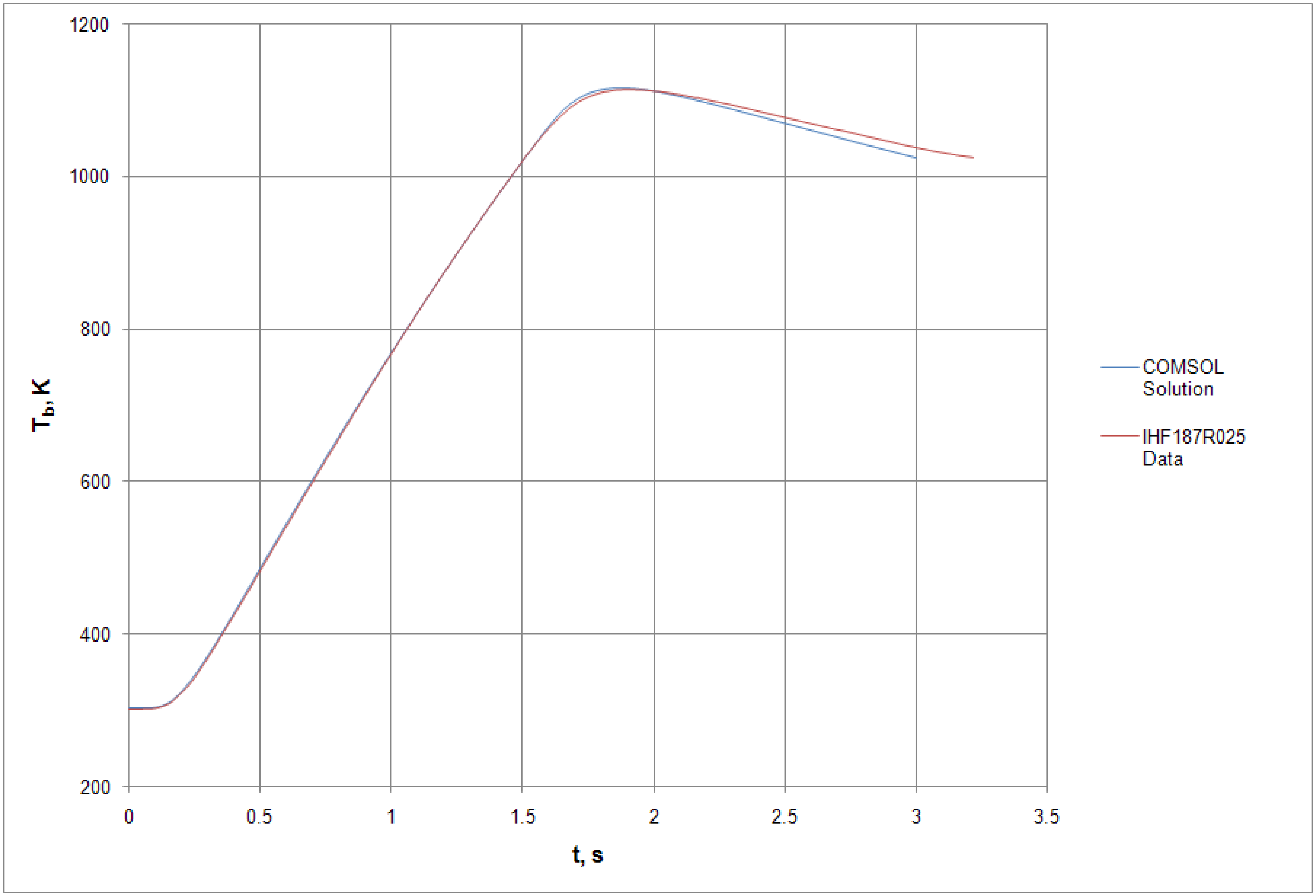


FEA Model

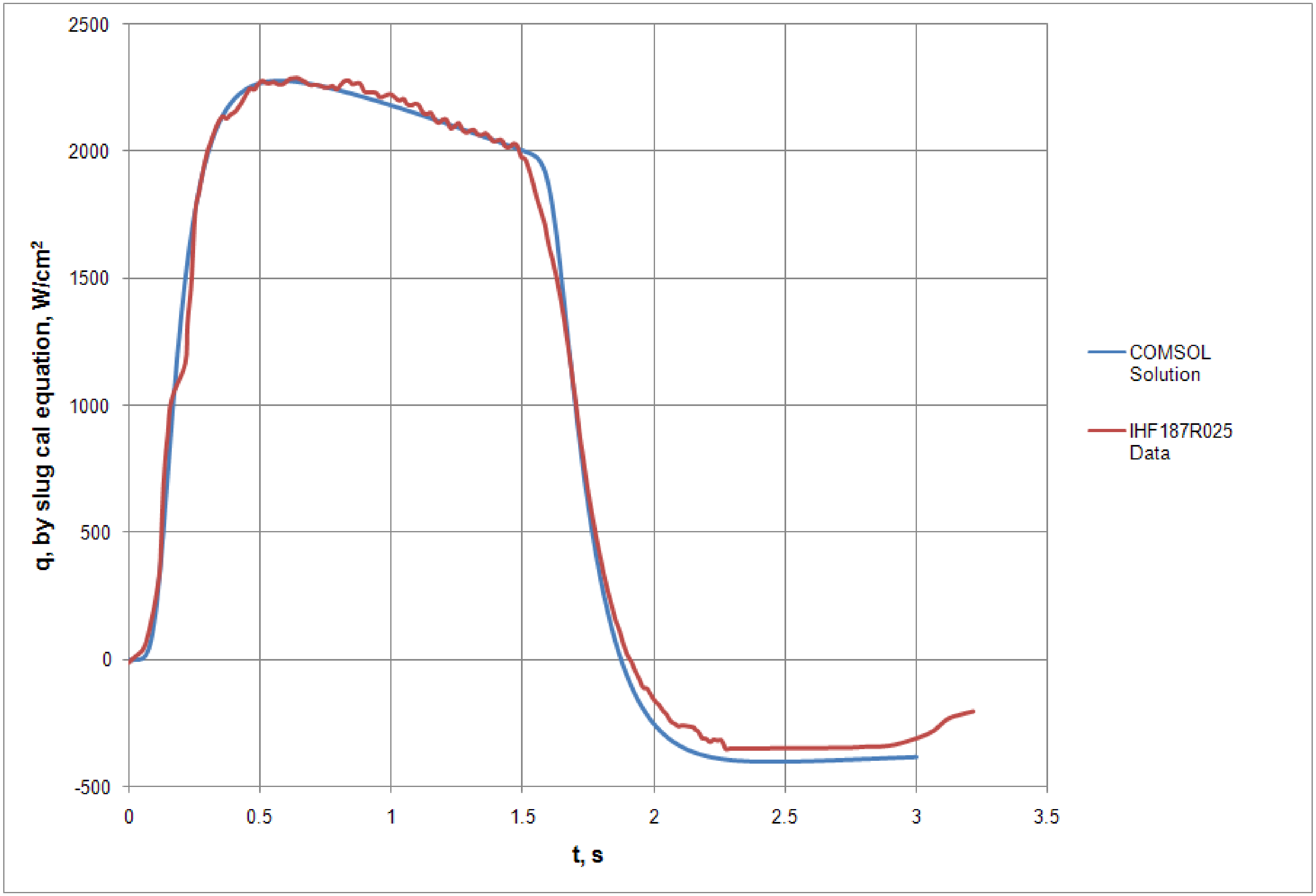

Max: 1045.76 K
1040 K
1030 K
1020 K
1010 K
1000 K
990 K
980 K
970 K
960 K
950 K
940 K
Min: 937.511 K


Temperature vs Time
1600
1400
1200
1000
800
600
400
200
Temperature (K)
0
0.5
1
1.5
2
2.5
3
Time (s)
Front Face
Back Face

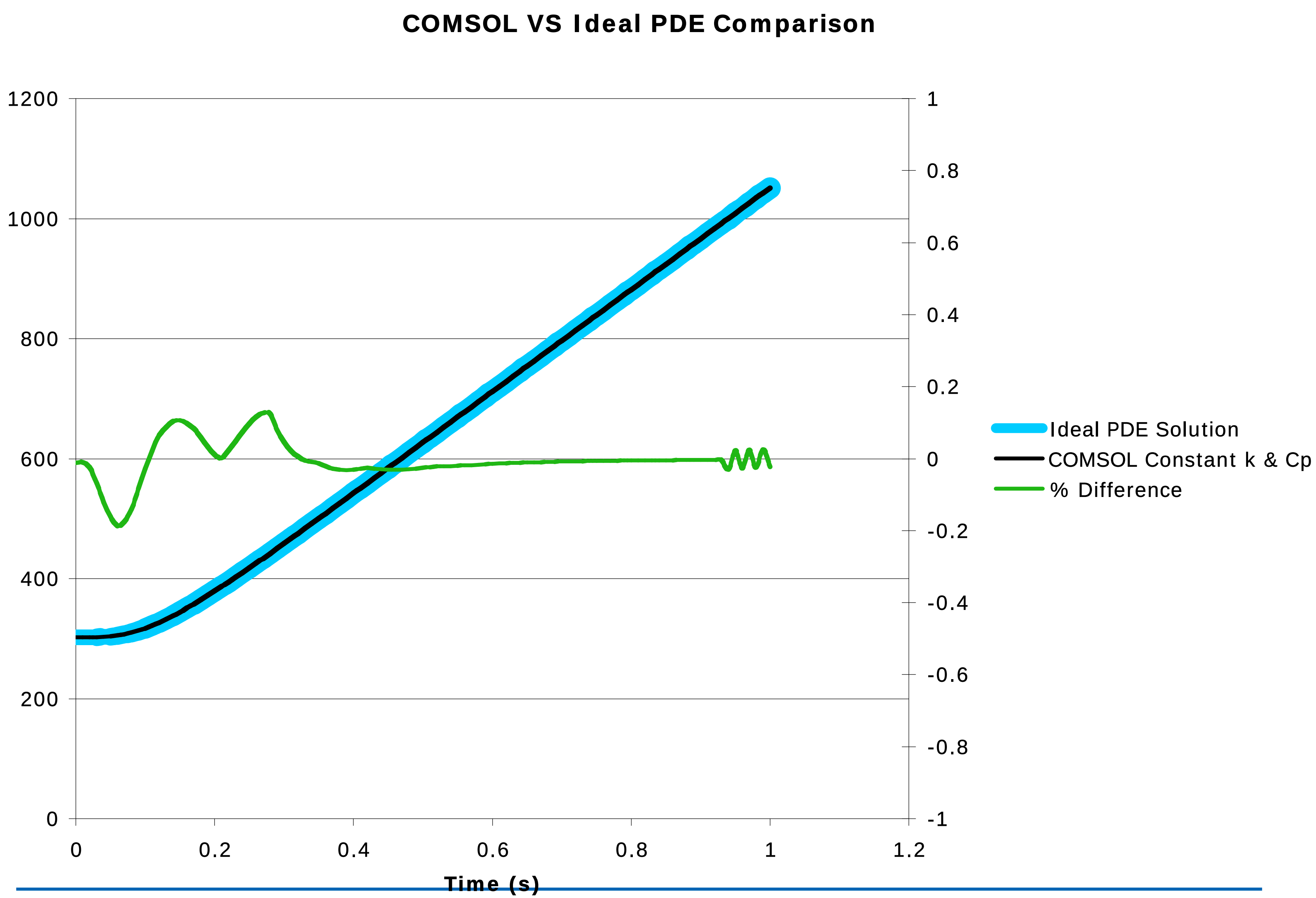
COMSOL model with q = 2600 W/cm^2 and actual data compared



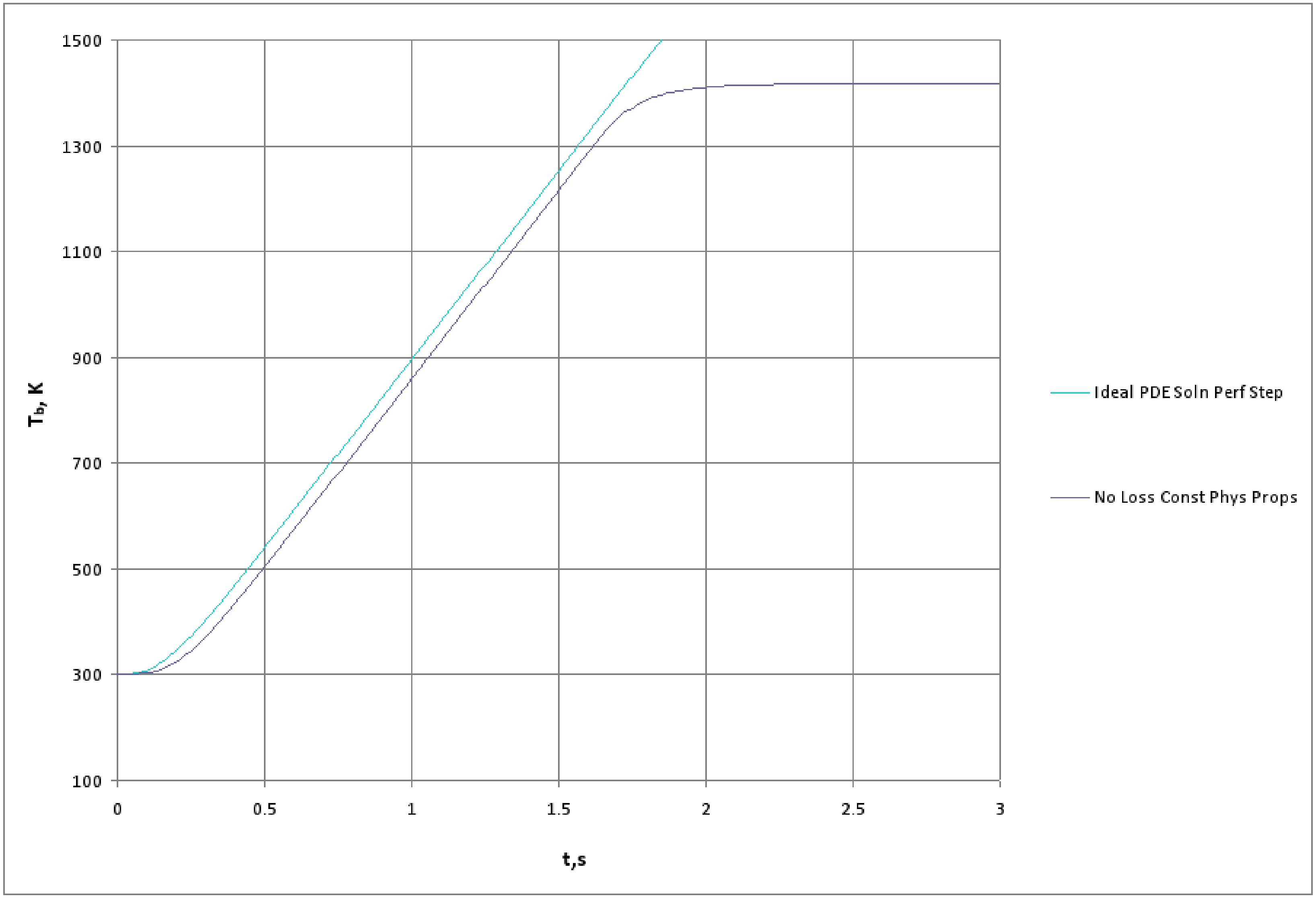
COMSOL model with q = 2600 W/cm² and actual data compared



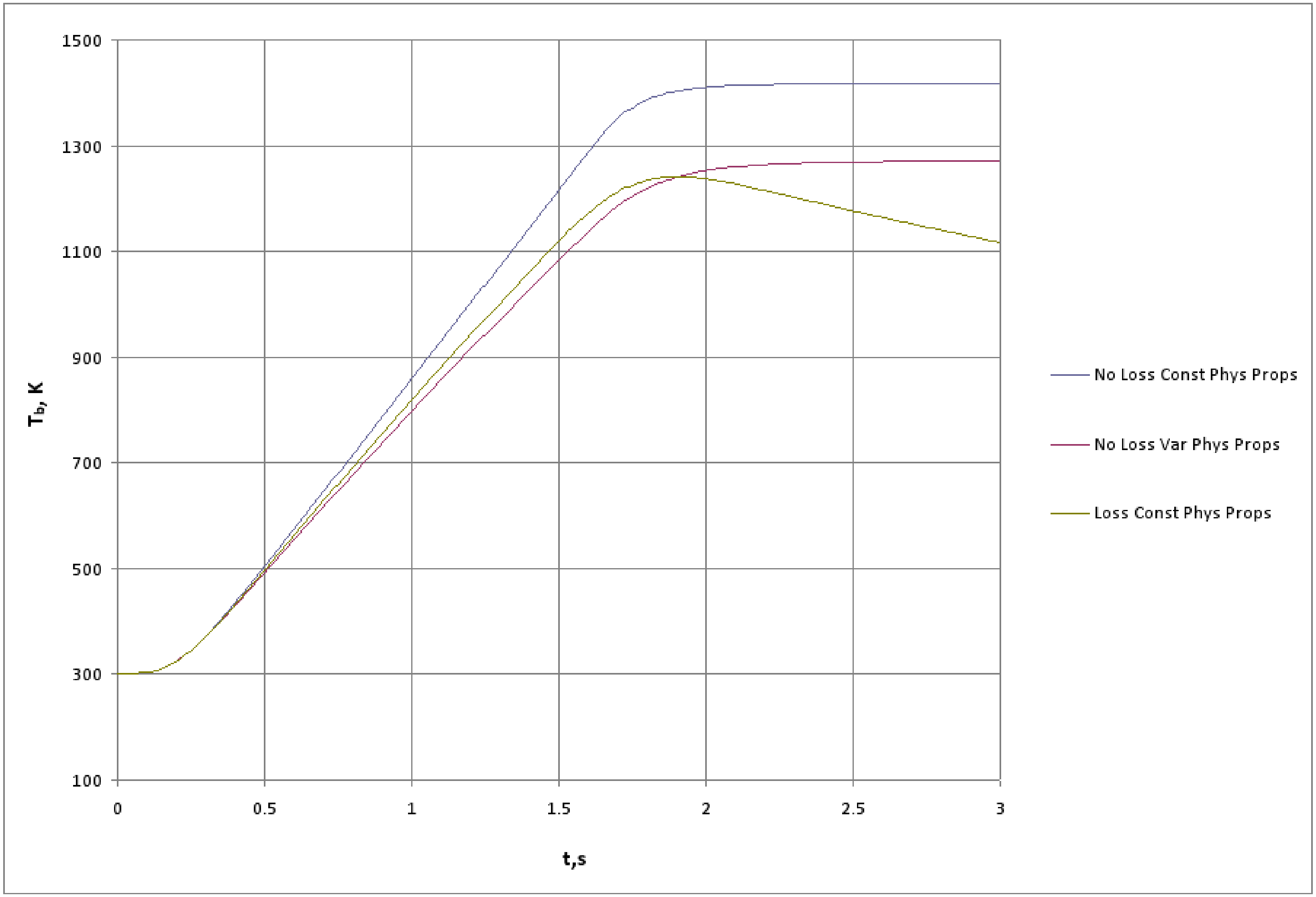
Ideal PDE & COMSOL No Loss Const Phys Props



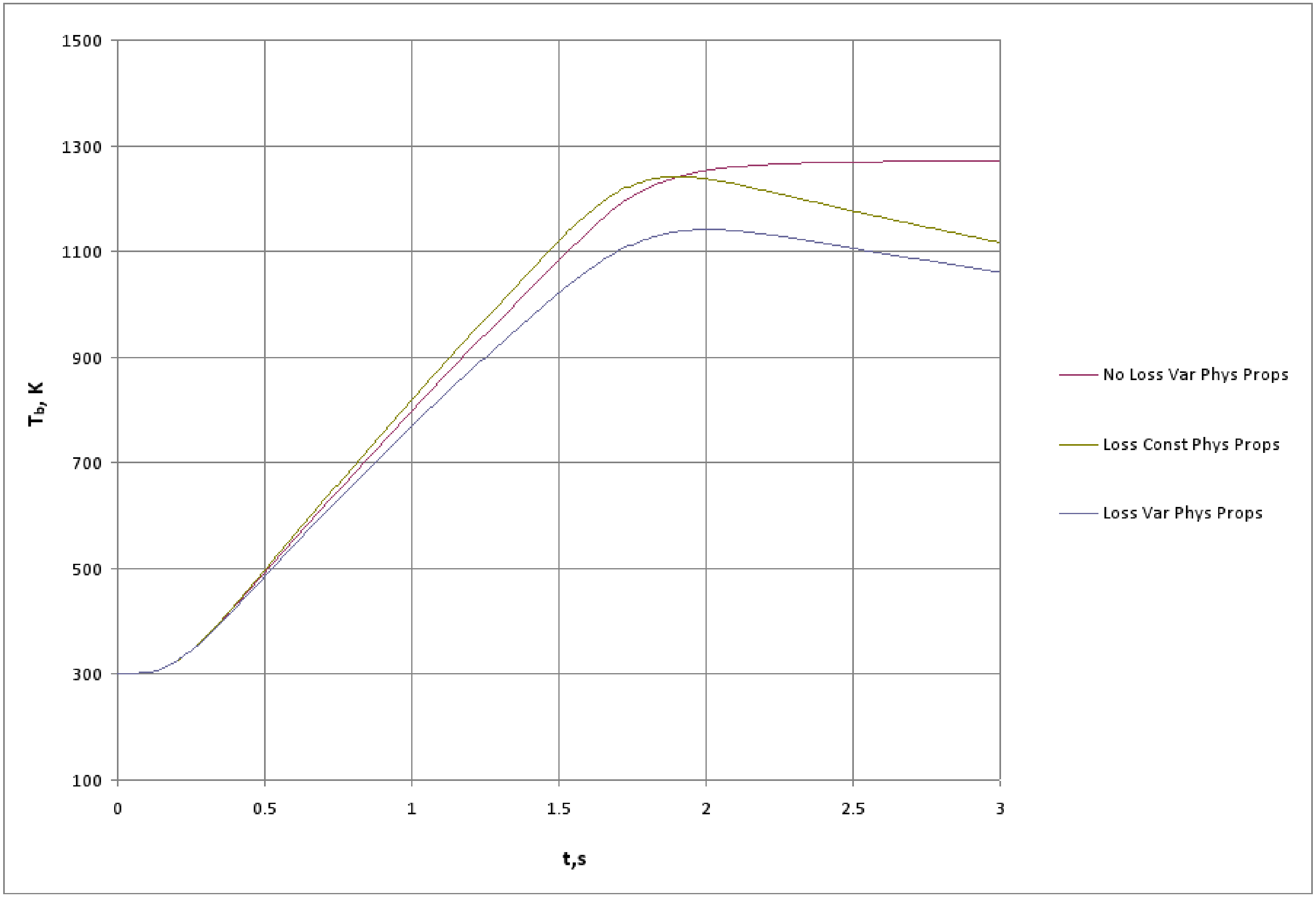
Ideal PDE & COMSOL No Loss Const Phys Props



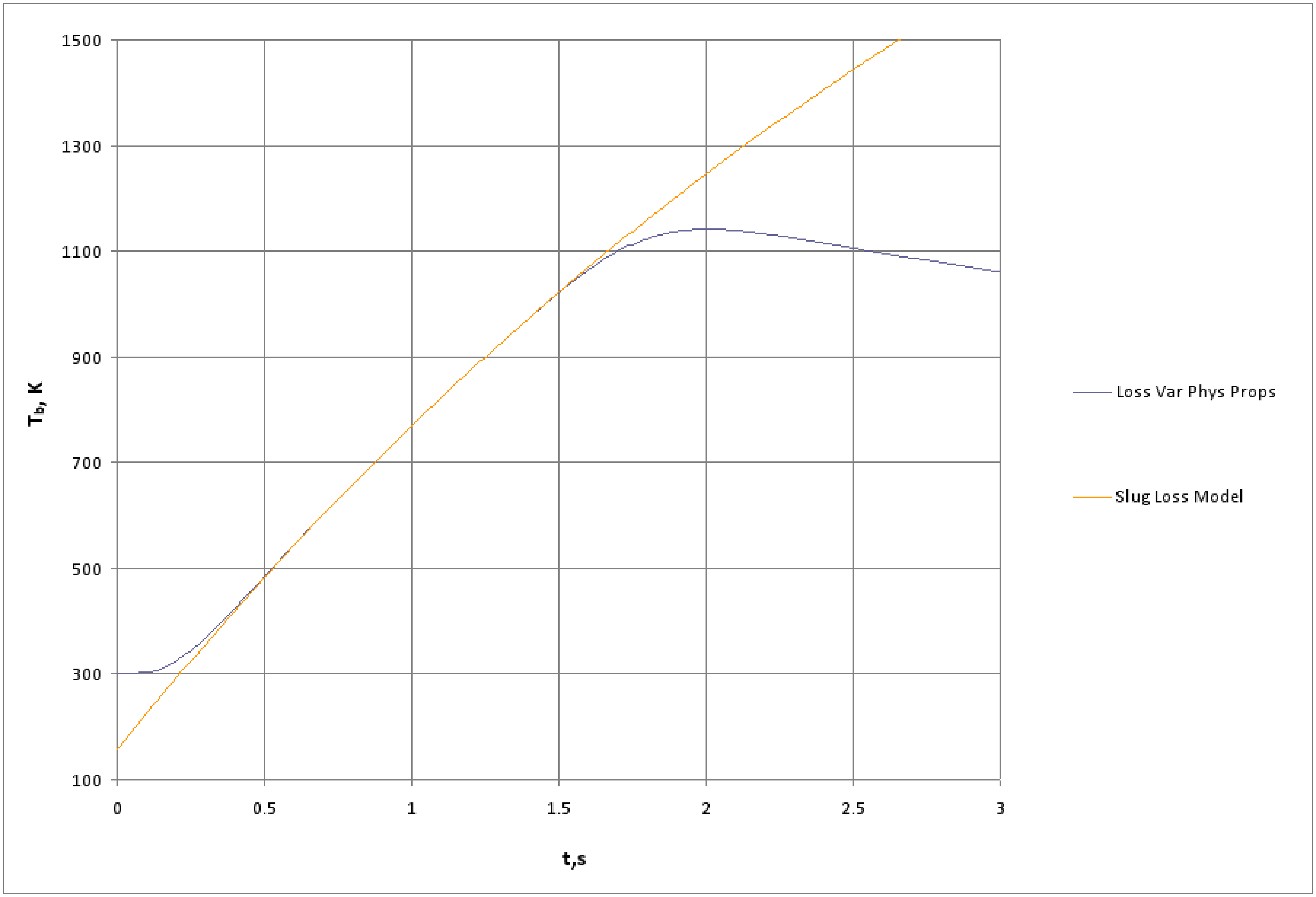
COMSOL No Loss Const Phys Prop compared to Loss Const Phys Prop & No Loss Var. Phys Prop



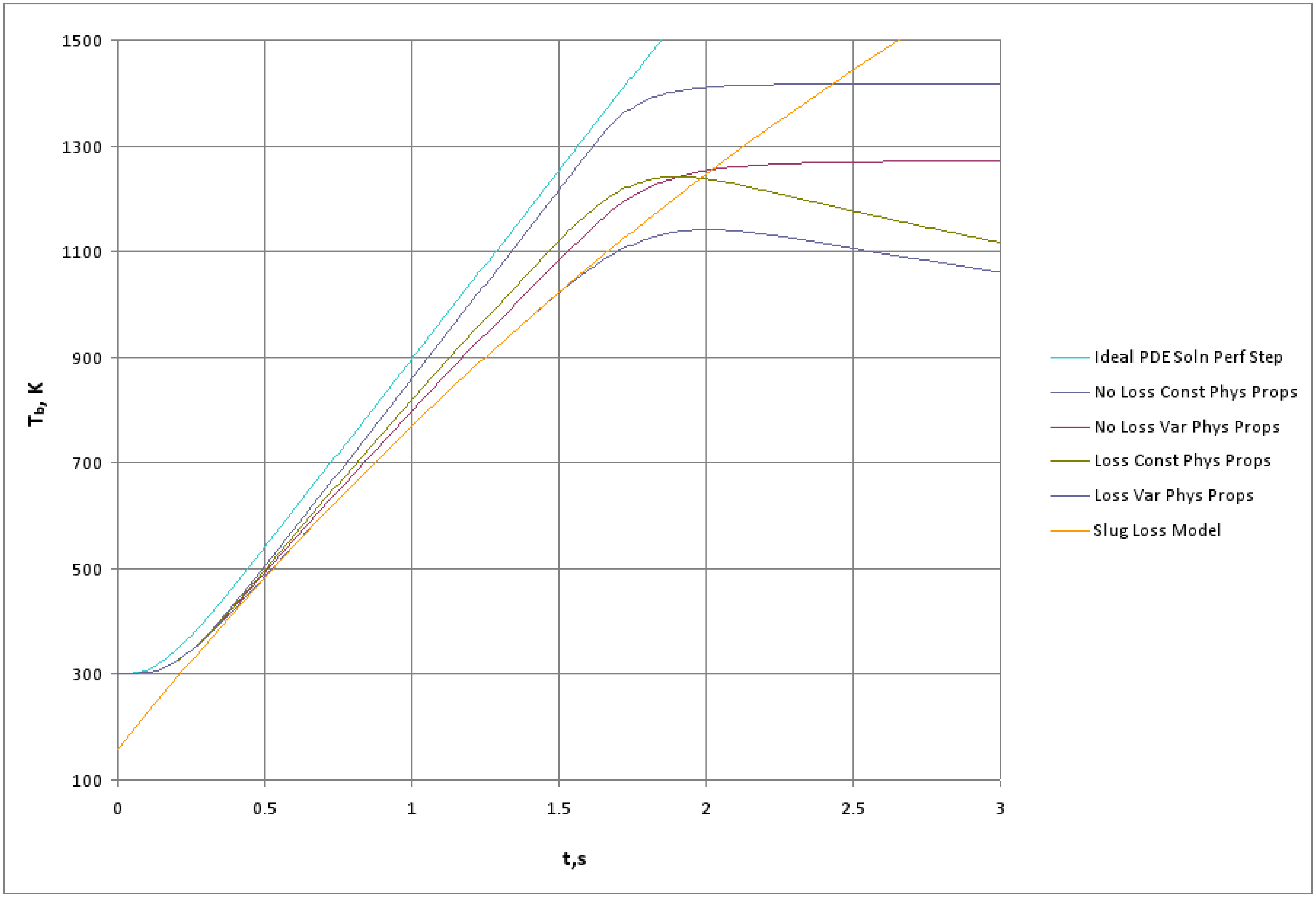
Loss Const Phys Prop & No Loss Var. Phys Prop compared to COMSOL Loss Var. Phys Prop



COMSOL Loss Var. Phys Prop compared to Slug Loss Model



All Six cases



Conclusions

- A mathematical model, The Slug Loss Model, was developed, which takes into account losses, where the temperature time slope takes the mathematical form of exponential decay.
- The Slug Loss Model was applied to slug calorimeter data from a high heat flux arc jet run.
- A FEA Model was also developed and run for various cases.
- Good agreement was shown between the Slug Loss Model and the FEA Model.