Thermal Capacitance (Slug) Calorimeter Theory including Heat Losses and other Decaying Processes

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Outline

- Arc jet description
- Slug calorimeter description
- Issue of heat losses from slug calorimeter cited in literature
- Idealized slug calorimeter theory – no losses and constant physical properties
- General slug calorimeter theory – with losses and variable heat capacity – Slug Loss Model
- Slug Loss Model applied to slug calorimeter data from one arc jet run
- Finite Element Analysis (FEA) model of slug calorimeter data from same arc jet run
- Comparison of all models
- Conclusions
Arc jet description

Reentry Flight Environment
100 cm Diameter Nose Cap
On a Reusable Vehicle

Arc Jet Facility Simulation

25x25 cm Test Body in the Arc Jet Plasma Stream
Slug calorimeter description

Schematic of a Thermal Capacitance (Slug) Calorimeter
Slug calorimeter description

Typical Temperature—Time Curve for Slug Calorimeter

\[ q = \frac{M c_p}{A} \frac{\Delta T_b}{\Delta t} = L \rho c_p \frac{\Delta T_b}{\Delta t} \]
Slug calorimeter description

Time Curve when heat & other losses are significant during heating phase
“The heat losses are usually hard to control in models with high-heat-flux conditions.”


“If more accurate results are required, the losses through the insulation layer should be modelled and accounted for by a correction term . . .”

Idealized slug calorimeter theory

- Right circular cylinder made of copper
- Insulated at back face & around circumferential area
- Slug initially at uniform temperature
- Starting at time $t = 0$, constant heat flux $q$ is applied to front face
- Coordinate $x$ defined as zero at front face and $L$ at back face
- Problem can be modeled as one dimensional unsteady state heat transfer
- Additional simplifying assumption: all physical properties are constant with temperature
Boundary value problem

PDE for one dimensional unsteady state heat transfer

\[ \frac{\partial^2 T}{\partial x^2} = \frac{\rho c_p}{k} \frac{\partial T}{\partial t} \]

Definition of thermal diffusivity

\[ \alpha = \frac{k}{\rho c_p} \]

Boundary conditions

\[ \frac{\partial T(0,t)}{\partial x} = -\frac{q}{k} \]
\[ \frac{\partial T(L,t)}{\partial x} = 0 \]

Initial condition

\[ T(x,0) = T_o \]
Solution to PDE boundary value problem

Overall solution = steady state solution + transient solution

\[ T(x,t) = v_{ss}(x,t) + w(x,t) \]

\[ v_{ss}(x,t) = T_o + \frac{qt}{L\rho c_p} + \frac{qL}{3k} + \frac{qx^2}{2Lk} - \frac{qx}{k} \]

\[ \frac{\partial T(x,t)}{\partial t} = \text{constant} \]

\[ w(x,t) = -2qL \sum_{n=1}^{\infty} \frac{1}{n^2} \cos \left( \frac{n\pi x}{L} \right) e^{-\alpha \left( \frac{n\pi}{L} \right)^2 t} \]

\[ T(x,t) = \left( T_o + \frac{qt}{L\rho c_p} + \frac{qL}{3k} + \frac{qx^2}{2Lk} - \frac{qx}{k} \right) - 2qL \sum_{n=1}^{\infty} \frac{1}{n^2} \cos \left( \frac{n\pi x}{L} \right) e^{-\alpha \left( \frac{n\pi}{L} \right)^2 t} \]
Animation of solution for copper slug
L= 1 cm, q= 2600 W/cm², elapsed t= 0.3 s

Steady state solution

Response slowed down by a factor of 33
Animation of solution for copper slug
$L = 1 \text{ cm}, q = 2600 \text{ W/cm}^2$, elapsed $t = 0.3 \text{ s}$

Transient solution

Response slowed down by a factor of 33
Animation of solution for copper slug
L = 1 cm, q = 2600 W/cm², elapsed t = 0.3 s

Overall solution

Response slowed down by a factor of 33
Setting $q_{\text{indicated}} = 0$ gives time for the heat to have just penetrated to the back side of the slug.

For practical purposes, the response time calculated when $q_{\text{indicated}}/q_{\text{input}} = 0.99$ should be sufficient elapsed time for the heat flux determination from the back face temperature to begin to be valid, and implies steady state.

$$t_R = \frac{L^2}{\alpha \pi^2} \ln \left( \frac{2}{1 - \frac{q_{\text{indicated}}}{q_{\text{input}}}} \right)$$

$$t_{R0.99} = \frac{L^2}{\alpha \pi^2} \ln \left( \frac{2}{1 - 0.99} \right)$$
Other useful equations

\[ T_b = T(L, t) = T_o + \frac{q\alpha t}{kL} - \frac{1}{6} \frac{qL}{k} - \frac{2qL}{k\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-\alpha \left(\frac{n\pi}{L}\right)^2 t} \]

\[ T_f = T(0, t) = T_o + \frac{q\alpha t}{kL} + \frac{1}{3} \frac{qL}{k} - \frac{2qL}{k\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\alpha \left(\frac{n\pi}{L}\right)^2 t} \]

\[ T_{ave} = T_o + \frac{q\alpha t}{kL} \]

\[ T_f - T_b = \frac{1}{2} \frac{qL}{k} - \frac{4qL}{k\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} e^{-\alpha \left(\frac{(2n-1)\pi}{L}\right)^2 t} \]

\[ T_{ave} - T_b = \frac{1}{6} \frac{qL}{k} + \frac{2qL}{k\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-\alpha \left(\frac{n\pi}{L}\right)^2 t} \]
Summation terms approach zero with steady state

\[ T_b = T(L, t) = T_o + \frac{q \alpha t}{kL} - \frac{1}{6} \frac{qL}{k} \]

\[ T_f = T(0, t) = T_o + \frac{q \alpha t}{kL} + \frac{1}{3} \frac{qL}{k} \]

\[ T_{ave} = T_o + \frac{q \alpha t}{kL} \]

\[ T_f - T_b = \frac{1}{2} \frac{qL}{k} \]

\[ T_{ave} - T_b = \frac{1}{6} \frac{qL}{k} \]
A heat balance on the slug with losses gives

\[ \text{input} - \text{output} (i.e. \text{losses}) = \text{accumulation} \]

\[
qA - \frac{(T_{ave} - T_o)}{R_{la}} = Mc_{po} \frac{dT_{ave}}{dt}
\]

\[
T_{ave} = T_b + \frac{qL}{6k}
\]

Getting equation in terms of \( T_b \)

\[
\left( q \left( \frac{A}{Mc_{po}} - \frac{L}{6kR_{la}Mc_{po}} \right) + \frac{T_o}{R_{la}Mc_{po}} \right) - \frac{T_b}{R_{la}Mc_{po}} = \frac{dT_b}{dt}
\]
Defining two constants

\[
a = \left( q \left( \frac{A}{Mc_{po}} - \frac{L}{6kR_{la}Mc_{po}} \right) + \frac{T_o}{R_{la}Mc_{po}} \right)
\]

\[
b = \frac{1}{R_{la}Mc_{po}}
\]

Differential equation can be written as

\[
a - bT_b = \frac{dT_b}{dt}
\]

Which integrates to

\[
T_b = \left( T_{b1 \text{ fit}} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b}
\]
Accounting for heat losses – Slug Loss Model

Once data is fit to this

\[ T_b = \left( T_{b1_{\text{fit}}} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b} \]

Rearrange this

\[
a = \left( q \left[ \frac{A}{M_{cpo}} - \frac{L}{6kR_{la}M_{cpo}} \right] + \frac{T_o}{R_{la}M_{cpo}} \right)
\]

\[ b = \frac{1}{R_{la}M_{cpo}} \]

To solve for \( q \)

\[
q = \frac{M_{cpo}}{A} \left( \frac{a - bT_o}{1 - \frac{L}{6kR_{la}A}} \right)
\]
Once you have this fit

\[ T_b(t) = \left( T_{b1\text{ fit}} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b} \]

By this equation

\[ T_{ave} = T_b + \frac{qL}{6k} \]

you also have

\[ T_{ave}(t) = \left( \left( T_{b1\text{ fit}} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b} \right) + \frac{qL}{6k} \]

and

\[ \frac{dT_b(t)}{dt} = \frac{dT_{ave}(t)}{dt} = -b \left( T_{b1\text{ fit}} - \frac{a}{b} \right) e^{-b(t-t_1)} \]

All analytical expressions
Accounting for heat losses – Slug Loss Model

Now write energy balance equation with actual loss resistance and variable heat capacity

\[ qA - \frac{(T_{ave}(t) - T_o)}{R_l} = Mc_p(T_{ave}(t)) \frac{dT_{ave}(t)}{dt} \]

Variable heat capacity with \( T \) is obtained from the Shomate equation for copper

\[ c_p(T_{ave}) = A + BT_{ave} + CT_{ave}^2 + DT_{ave}^3 + \frac{E}{T_{ave}^2} \]

where

\[ A = 2.789933 \times 10^2 \ \frac{J}{kgK} \quad B = 4.421789 \times 10^{-1} \ \frac{J}{kgK^2} \]

\[ C = -4.918152 \times 10^{-4} \ \frac{J}{kgK^3} \quad D = 2.19879 x 10^{-7} \ \frac{J}{kgK^4} \]

\[ E = 1.079706 \times 10^6 \ \frac{JK}{kg} \]
Accounting for heat losses – Slug Loss Model

Solve energy balance equation for $R_l$, the actual loss resistance

\[
R_l(t) = \frac{(T_{\text{ave}}(t) - T_o)}{qA - Mc_p(T_{\text{ave}}(t))} \frac{dT_{\text{ave}}(t)}{dt}
\]

Other useful equations

\[
q_{\text{slope}Tb}(t) = \frac{Mc_p(T_b(t))}{A} \frac{dT_b(t)}{dt}
\]

\[
q_{\text{slope}Tave}(t) = \frac{Mc_p(T_{\text{ave}}(t))}{A} \frac{dT_{\text{ave}}(t)}{dt}
\]

\[
q_{\text{loss}}(t) = q - q_{\text{slope}Tave}(t) = q - \frac{Mc_p(T_{\text{ave}}(t))}{A} \frac{dT_{\text{ave}}(t)}{dt}
\]

\[
\text{FracLoss}(t) = 1 - \frac{Mc_p(T_{\text{ave}}(t))}{qA} \frac{dT_{\text{ave}}(t)}{dt}
\]
Slug Loss Model (SLM) applied to one arc jet run (IHF187R025)

Back Face Temperature & Stagnation Pressure versus Time.
SLM applied to one arc jet run (IHF187R025)

\[
\rho = 8,925.7 \, \frac{\text{kg}}{\text{m}^3} \quad c_{po} = 385.615 \, \frac{\text{J}}{\text{kgK}} \quad k = 385.2 \, \frac{\text{W}}{\text{mK}}
\]

\[
M = 0.004529 \, \text{kg} \quad D = 0.00781 \, \text{m}
\]

\[
A = 0.25 \pi D^2 = 0.000047906 \, \text{m}^2
\]

\[
L = \frac{M}{\rho A} = 0.010592 \, \text{m}
\]

\[
t_{R0.99} = \frac{\rho c_{po} L^2}{k \pi^2} \ln \left( \frac{2}{1 - 0.99} \right) = 0.538 \, \text{s}
\]
SLM applied to one arc jet run (IHF187R025)

Back Face Temperature & Stagnation Pressure versus Time.
Back Face Temperature versus Time data from $t_1$ to $t_2$. 

<table>
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<th>$T_b$, K</th>
<th>$t$, s</th>
<th>$T_b$, K</th>
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Fit to this equation 

$$T_b(t) = \left( T_{b1fit} - \frac{a}{b} \right) e^{-b(t-t_1)} + \frac{a}{b}$$
SLM applied to one arc jet run (IHF187R025)

Fitting this data to Slug Loss Model equation gives the best linear fit to $T_b$ versus $e^{-b(t-t_1)}$ when $b = 0.29160 \text{ s}^{-1}$, where the $R^2$ value of the fit is maximized at 0.99999. The Solver function in an Excel spreadsheet was used to solve for $b$.

$$a = \text{intercept}\left(T_b \text{ vs } e^{-b(t-t_1)}\right)b = 766.76 \frac{K}{s}$$

$$T_{b1,\text{fit}} = \text{slope}\left(T_b \text{ vs } e^{-b(t-t_1)}\right) + \frac{a}{b} = 660.32 \text{ K}$$
\[ R_{la} = \frac{1}{bM \rho_{po}} = 1.964 \frac{K}{W} \]

\[ q = \frac{M \rho_{po}}{A} \left( \frac{a - bT_o}{1 - \frac{L}{6kR_{la}A}} \right) = 26,005,000 \frac{W}{m^2} = 2,600 \frac{W}{cm^2} \]

This value is about 15% higher than the value of 2,250 W/cm² reported by the facility test engineers, where losses were not taken into account.
SLM applied to one arc jet run (IHF187R025)

Back Face Temperature – Fit Compared to Data.
SLM applied to one arc jet run (IHF187R025)

Fit Compared to straight line.
SLM applied to one arc jet run (IHF187R025)

Losses (per cm$^2$ slug frontal area) versus Time.

![Graph showing losses versus time with q_loss and Fraction Losses plotted against time in seconds.](image-url)
Actual Loss Resistance versus Time.
Animation of a 4” Hemi Slug Calorimeter
FEA Model

- A simple FEA model was created using COMSOL Multiphysics, COMSOL, Inc, Burlington, Massachusetts.
- The slug was modeled using 3D tetrahedral elements.
- Heat flux is applied to the top face using a smoothed Heaviside function (flc2hs) to create a ramp up and ramp down time.
- Losses occur through 0.6 mm diameter surface regions with a constant heat transfer coefficient $h$, to a constant holder temperature $T_0$.
- The material copper is used using temperature dependent properties of heat capacity and thermal conductivity.
- 3 second simulation time with a 0.01 second time step.
FEA Model

\[ q_{\text{input}} \quad q_{\text{loss}} = h(T-T_0) \]
FEA Model

- Various runs were performed by varying $q_{\text{input}}$, $h$ and the duration of the pulse in order to match the data.
- A unique solution of $q_{\text{input}} = 2,600 \text{ W/cm}^2$ was found where the COMSOL solution closely agreed with the actual data. Sensitivity analysis showed this $q$ value to be determinable to +/- 1%.
- A fringe plot of the temperature at $t = 3$ seconds was plotted to show the paths of the heat flow.
- Temperature was plotted versus time for the centers of the front and back faces of the slug.
Adjusting the results to actual data
FEA Model

Temperature vs Time

- Front Face
- Back Face
COMSOL model with $q = 2600 \text{ W/cm}^2$ and actual data compared.
COMSOL model with $q = 2600 \text{ W/cm}^2$ and actual data compared
Ideal PDE & COMSOL No Loss Const Phys Props

COMSOL VS Ideal PDE Comparison

Time (s)

Ideal PDE Solution
COMSOL Constant k & Cp
% Difference
Ideal PDE & COMSOL No Loss Const Phys Props

The graph shows the temperature ($T_b$) over time ($t$) for two different cases:

- **Ideal PDESoln Perf Step** (light blue line)
- **No Loss Const Phys Props** (dark blue line)

The x-axis represents time ($t$, s) ranging from 0 to 3, and the y-axis represents temperature ($T_b$, K) ranging from 100 to 1500.
COMSOL No Loss Const Phys Prop compared to Loss Const Phys Prop & No Loss Var. Phys Prop
Loss Const Phys Prop & No Loss Var. Phys Prop compared to COMSOL Loss Var. Phys Prop

![Graph showing temperature vs time for different conditions.](image)
COMSOL Loss Var. Phys Prop compared to Slug Loss Model

![Graph showing comparison between Loss Var Phys Props and Slug Loss Model over time (t, s) and temperature (T, K).]
All Six cases

Graph showing temperature $T$, K, over time $t$, s for different cases:
- Ideal PDE Soln Perf Step
- No Loss Const Phys Props
- No Loss Var Phys Props
- Loss Const Phys Props
- Loss Var Phys Props
- Slug Loss Model
Conclusions

- A mathematical model, The Slug Loss Model, was developed, which takes into account losses, where the temperature time slope takes the mathematical form of exponential decay.
- The Slug Loss Model was applied to slug calorimeter data from a high heat flux arc jet run.
- A FEA Model was also developed and run for various cases.
- Good agreement was shown between the Slug Loss Model and the FEA Model.