



### Propellant Bulk Temperature Modeling for the Orion Launch Abort System





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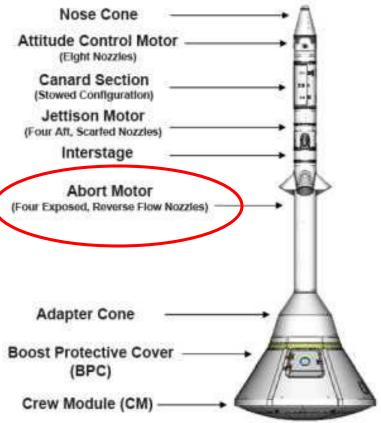
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- Solid Rocket Performance Is Highly Dependent on Propellant Temperature
- Our Task: Predict The Pre-Launch Temperature of the Orion Launch Abort System Main Abort Motor
- Environment Inside the Motor Precludes Direct Sensing

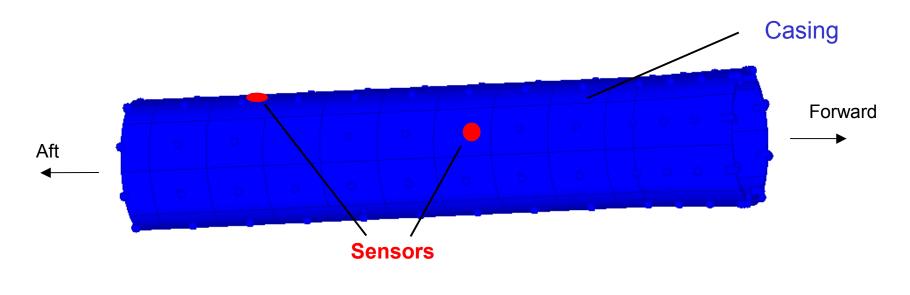








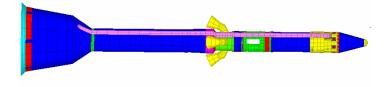
- External Case Temperature Sensors Available To Help Predict PMBT
  - More Reliable Than Measuring Various Environment Conditions To Predict PMBT
  - Must Account For Temperature Lag Between The Case And The PMBT
  - Correlate System-Level Model Or Develop New PMBT Model



# Two Modeling Approaches Compared



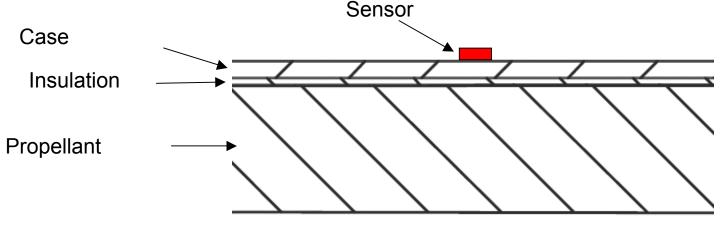
- Analytical Model
  - Inputs: External Temperature History
  - Provides:
    - Bulk Temperature And Temperature Lag If Given Sufficient External Temperature History
    - Insight To Allow Temperature Lag Estimation If Given Some External Temperature History
- System-Level Thermal Model
  - Inputs: External Environment Data
  - Provides:
    - Bulk Temperature and Temperature Lag If Given Actual External Environments
    - Temperature Lag Limits If Given Worst-Case External Environments







- 1-Dim Radial Model Focuses On Three Materials
  - Case, Insulation, and Propellant
  - Neglect Axial Conduction Into/Out Of AM
  - Neglect Contact Resistances Between Materials
  - Neglect Heat Capacity of Case and Insulation
  - Assume Sensor Temperature as Case OD Temperature
  - Model Propellant As Hollow Circular Cylinder (Use Min ID of Grain, But Equate Mass Using Density Multiplier)
- Solve PMBT As Function Of Time History Of Sensor Temperature
- Solve Analytically In Excel And Implement in Thermal Desktop As A Check







- Solution Steps, PMBT From External Temperature History
  - Set T<sub>init</sub> (Assumed Uniform) Equal To Initial Sensor Temperature
  - Solve For PMBT Response To Unit Step External Temperature Change
  - Solve Duhamel Superposition Using External Temperature History And PMBT Response To Unit Step External Temperature Change
  - Use Sufficient History of Sensor Temperatures To Damp Initial Transient

Step: 
$$T_{ext}(t) = \begin{cases} T_{init} & t < \tau_0 \\ T_{init} + \Delta T_{ext} & t \ge \tau_0 \end{cases}$$

PDE for Step Response : 
$$(\rho c_p)_{prop} \frac{\partial T_{prop}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( k_{prop} r \frac{\partial T_{prop}}{\partial t} \right)$$

$$t \leq \tau_{0}: \quad T_{prop} = T_{init}$$

$$r = r_{prop,ID}: \quad -k_{prop} \frac{\partial T_{prop}}{\partial r} = 0$$

$$r = r_{prop,OD}: \quad -k_{prop} \frac{\partial T_{prop}}{\partial r} = h_{eff} \left( T_{prop} - T_{ext}(t) \right)$$
where  $(hA)'_{eff} = \left( \left( (hA)'_{eff,case} \right)^{-1} + \left( (hA)'_{eff,ins} \right)^{-1} \right)^{-1} = \left( \frac{\ln(r_{case,OD}/r_{case,ID})}{2\pi k_{case}} + \frac{\ln(r_{ins,OD}/r_{ins,ID})}{2\pi k_{ins}} \right)^{-1}$ 

## **Analytical Solution, Unit Step Responses**



- Series Solution Using Separation Of Variables
- Converges Well
  - Best At Large  $t^+$  And Small  $r^+$
  - Bulk Temperature Response Converges Even Better

$$\Psi^{+}(r^{+},t^{+}) = 1 - \Phi^{+}(r^{+},t^{+}) = \sum_{n=1}^{\infty} A_{n} \left( J_{0}(\lambda_{n}^{+}r^{+}) - \frac{J_{1}(\lambda_{n}^{+}r_{i}^{+})}{Y_{1}(\lambda_{n}^{+}r_{i}^{+})} Y_{0}(\lambda_{n}^{+}r^{+}) \right) \exp\left(-\lambda_{n}^{+2}t^{+}\right)$$

$$\Psi_{b}^{+}(t^{+}) = 1 - \Phi_{b}^{+}(t^{+}) = \frac{2}{r_{o}^{+2} - r_{i}^{+2}} \int_{x^{+}=r_{i}^{+}}^{x^{+}=r_{o}^{+}} \Psi^{+}(x^{+},t^{+})x^{+}dx^{+} = \frac{2r_{o}^{+}}{r_{o}^{+2} - r_{i}^{+2}} \sum_{n=1}^{\infty} \frac{B_{n}}{\lambda_{n}^{+}} \exp\left(-\lambda_{n}^{+2}t^{+}\right)$$

where

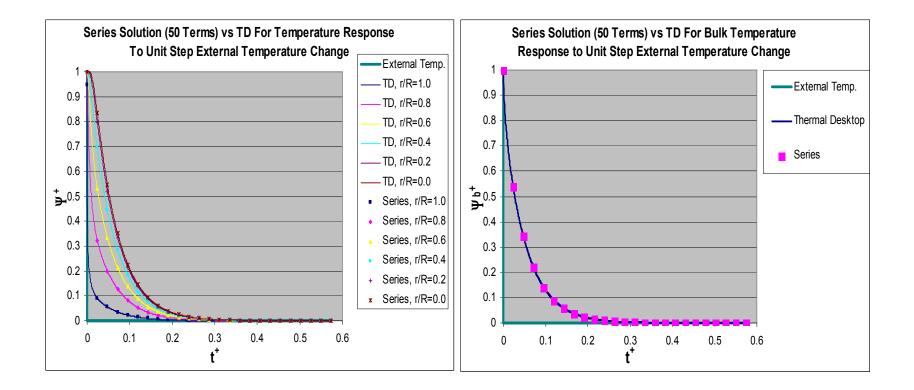
$$\begin{split} A_{n} &= \frac{2\Psi_{0}^{+}\text{Bi}\,r_{o}^{+}Y_{1}(\lambda_{n}^{+}r_{i}^{+})\left(J_{0}(\lambda_{n}^{+}r_{o}^{+})Y_{1}(\lambda_{n}^{+}r_{i}^{+}) - J_{1}(\lambda_{n}^{+}r_{i}^{+}) - J_{1}(\lambda_{n}^{+}r_{o}^{+})Y_{0}(\lambda_{n}^{+}r_{o}^{+})\right)}{\left(\lambda_{n}^{+2} - \text{Bi}^{2}\right)r_{o}^{+2}\left(J_{0}(\lambda_{n}^{+}r_{o}^{+})Y_{1}(\lambda_{n}^{+}r_{i}^{+}) - J_{1}(\lambda_{n}^{+}r_{i}^{+})Y_{0}(\lambda_{n}^{+}r_{o}^{+})\right)^{2} - \lambda_{n}^{+2}r_{i}^{+2}\left(J_{0}(\lambda_{n}^{+}r_{i}^{+})Y_{1}(\lambda_{n}^{+}r_{i}^{+}) - J_{1}(\lambda_{n}^{+}r_{i}^{+})Y_{0}(\lambda_{n}^{+}r_{o}^{+})\right)^{2}}{B_{n}} \\ B_{n} &= A_{n}\left(J_{1}(\lambda_{n}^{+}r_{o}^{+}) - \frac{J_{1}(\lambda_{n}^{+}r_{i}^{+})}{Y_{1}(\lambda_{n}^{+}r_{o}^{+})}Y_{1}(\lambda_{n}^{+}r_{o}^{+})\right) \end{split}$$

and where  $\lambda_n^+$  are the positive roots of the characteristic equation  $J_1(\lambda_n^+ r_i^+) \Big( \text{Bi } Y_0(\lambda_n^+ r_o^+) - \lambda_n^+ Y_1(\lambda_n^+ r_o^+) \Big) - Y_1(\lambda_n^+ r_i^+) \Big( \text{Bi } J_0(\lambda_n^+ r_o^+) - \lambda_n^+ J_1(\lambda_n^+ r_o^+) \Big) = 0$ 

## Analytical Solution, Unit Step Responses, Cont.



- Series Solution Verified With Numerical Solution Obtained From Thermal Desktop
- 50 Terms Used, But Usually Only A Few Required







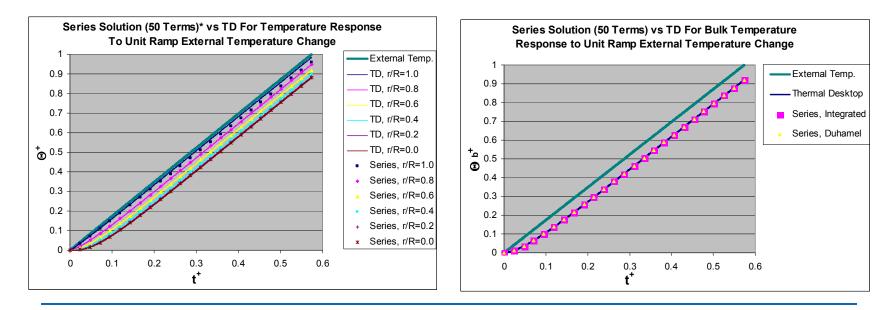
- Input: External Temperature History
- Output: PMBT For Times Up To Last External Temperature Input
- Accuracy
  - Depends On Replication Of True External Temperature History
  - Requires Sufficient Time History To Dampen Initial Transient In Order To Negate Assumption Of Uniform Initial Temperature

$$\theta_{b}(t^{+}) = T_{b}(t^{+}) - T_{init} = \int_{s^{+}=\tau_{0}^{+}=0}^{s^{+}=t^{+}} \theta_{ext}(s^{+}) \frac{d\Phi_{b}^{+}(t^{+}-s^{+})}{dt^{+}} ds^{+} = \frac{2r_{o}^{+}}{r_{o}^{+2}-r_{i}^{+2}} \int_{s^{+}=\tau_{0}^{+}=0}^{s^{+}=t^{+}} \theta_{ext}(s^{+}) \sum_{n=1}^{\infty} \lambda_{n}^{+} B_{n} \exp\left(-\lambda_{n}^{+2}\left(t^{+}-s^{+}\right)\right) ds^{+} ds^{+} = \frac{2r_{o}^{+}}{r_{o}^{+2}-r_{i}^{+2}} \int_{s^{+}=\tau_{0}^{+}=0}^{s^{+}=t^{+}} \theta_{ext}(s^{+}) \sum_{n=1}^{\infty} \lambda_{n}^{+} B_{n} \exp\left(-\lambda_{n}^{+2}\left(t^{+}-s^{+}\right)\right) ds^{+} ds^{+}$$



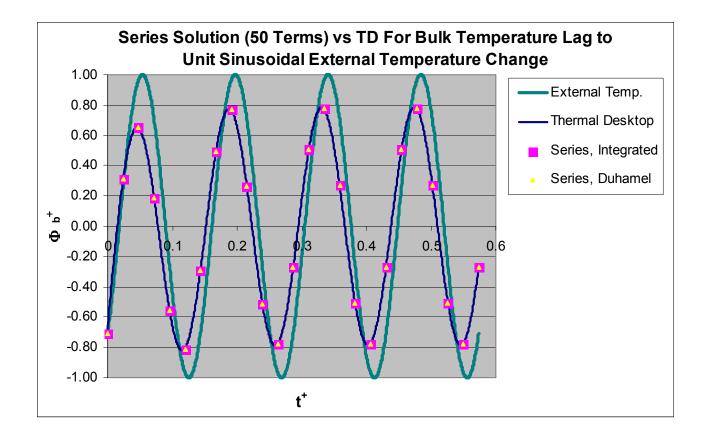


- Convergence
  - No problems With PMBT Response
  - Temperature Response Problematic At Large  $(t^+, r^+)$ , But Otherwise Good
- Developing Temperature Lag
  - Non-Dimensional Temperature Lag, 0.045648
  - Non-Dimensional Time Constant ~ $(\lambda_1^+)^{-2}$ , Using First-Term Approximation





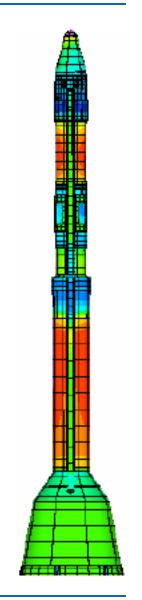
- Developing Amplitude And Phase Change
  - Non-Dimensional Amplitude, ~0.80
  - Phase Lag, slight







- LAS Modeled in Thermal Desktop v5.1
  - ~10k Nodes
  - 800 Surfaces/Solids
- Incorporates Environmental Effects
  - Sky (Sink) Radiation Temperature
  - Solar Radiation as a Function of Time of Day
  - Ambient Air Temperature
  - Wind
- Given Environmental Assumptions, This Model Predicts the Maximum Error Between the Sensors and the PMBT
  - Worst Case Error Arises During Worst Case Heating and Cooling Cases

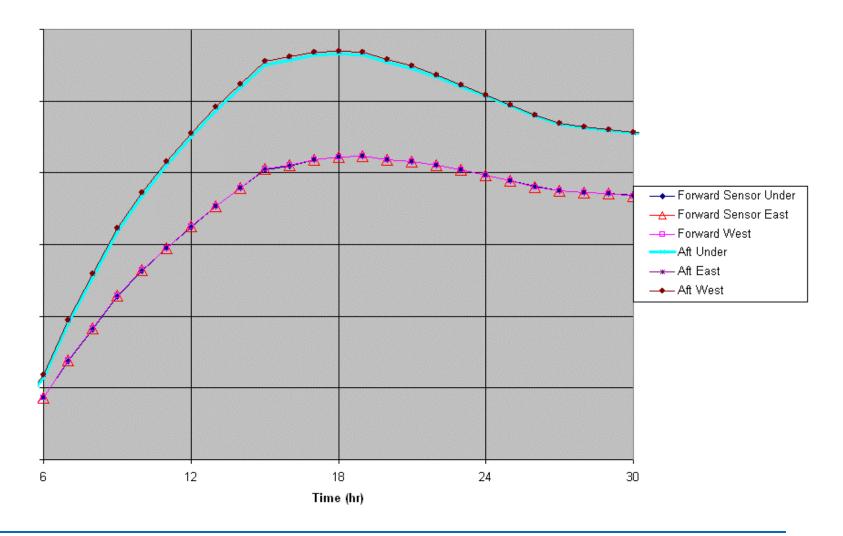


Worst Case Conditions – Sudden Heating Case

- Air Temperature
  - 99% Variation for Edwards AFB (i.e. WSMR)
    - Maximum temperature change in 3hrs = 20.7°F
    - Maximum temperature change in 6hrs = 33.7°F
    - Maximum temperature change in 12hrs = 37.8°F
  - Create a Worst Case Heating Condition
    - Run Cold Case for 4 days December Values
      - 1% Air Temperature Dec.
      - 1% Sky Temperature, 99% Radiation Dec.
      - 0 Wind Velocity
    - At 6AM on 5<sup>th</sup> day (just before sunrise), air temperature increases at maximum NEDD rate.
    - Temperature continues to rise until it intersects the 99% Hot Case Air Temperature



Sensor Error Vs. Time



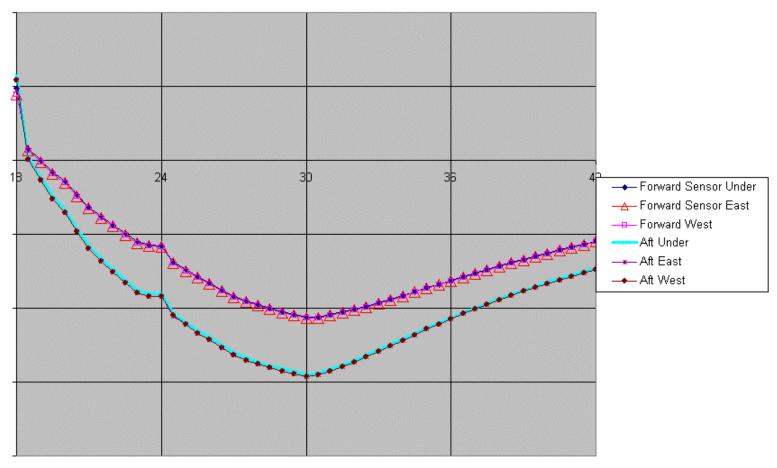




- Air Temperature
  - NEDD 99% Variation for Edwards AFB
    - Maximum temperature change in 3hrs = 20.7°F
    - Maximum temperature change in 6hrs = 33.7°F
    - Maximum temperature change in 12hrs = 37.8°F
  - Create a Worst Case Cooling Condition
    - Run Hot Case for 4 days December Values
      - 99% Air Temperature Dec.
      - 1% Sky Temperature, 99% Radiation During the Day
      - 99% Sky Temperature at Night
      - 0 Wind Velocity
    - At 6PM on 5<sup>th</sup> day (just before sunset), air temperature decreases at maximum NEDD rate.
    - Temperature continues to fall until it intersects the 1% Cold Case Air Temperature



Sensor Error Vs. Time

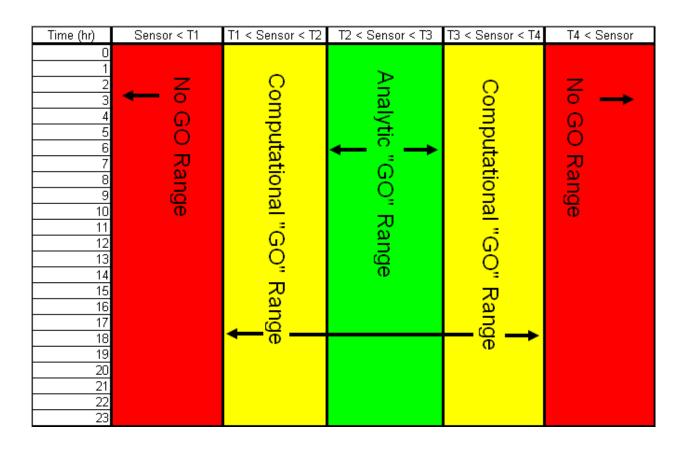


Time (hr)





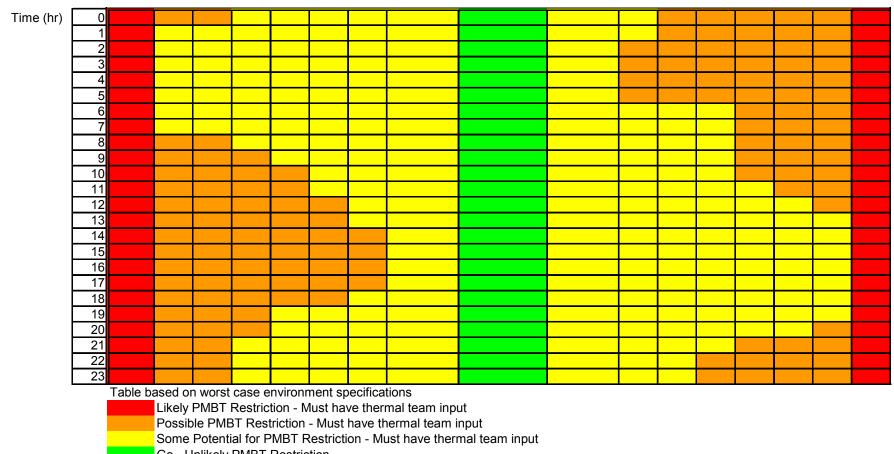
- Worst Case Day-Of-Flight Placarding
  - Analytic Range = Physical Limit
  - Computational Range = Limit Given Assumed Environments







#### Could Also Provide Ranges Vs. Time of Day



Go - Unlikely PMBT Restriction





- Two Models Have Been Developed to Predict the LAS Abort Motor Propellant Temperature Based on External Sensor Readings Only
  - Analytic Model Could Provide Conservative Day-of-Flight Ranges
  - Computational Model Can Incorporate Environmental Assumptions to Provide a Less-Conservative Range
- Both Models Could Be Used in Near Real Time Along with Recent Data to Predict PMBT if Operators So Chose
- Orbital Sciences and Its Thermal Analysis Team Are Proud to Continue their Support of Lockheed Martin and NASA's Ambitious Manned Spaceflight Objectives