



## Modeling Membrane Filtration for Water Recovery

Paul Boyle, Brent Houchens,  
Albert Kim

Presented By  
Paul Boyle

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# Potable Water Production



- Potable water quality/availability is a serious issue
  - 1 billion without reliable access to clean drinking water
  - up to 5 million die yearly of water borne diseases [1]
- Long duration space flight will require recycling of water to decrease liftoff weight
- Membrane filtration (RO) shows promise
  - semi-permeable membrane
  - removes foulants, including bacteria, viruses and salts



# Limitations of Membrane Filtration



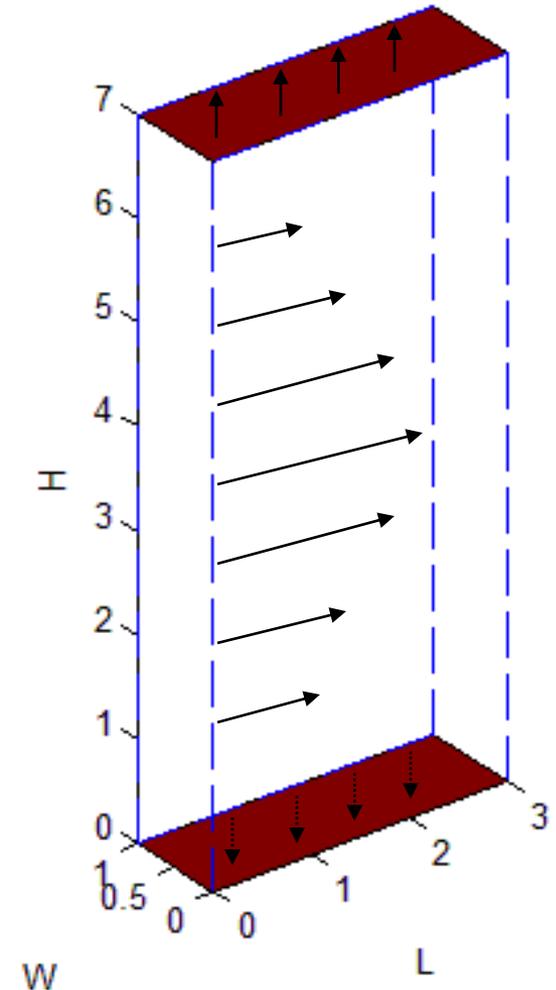
- Organic and Colloidal Fouling
  - fouling results in decreased permeate flux
  - combined fouling results from different foulants interacting to foul at a greater than additive rate [2]
- Maintenance
  - crossflow temporarily aids in membrane cleaning
  - fouling leads to costly maintenance requiring system shut down
- Power consumption
  - higher pressure required to maintain permeate flow rate



# Solution Domain



- Cell dimensions scale with volume fraction and particle radius
- Continuum flow field:
  - one dimensional crossflow
  - quasi-steady
- Particle boundary conditions:
  - axial and no-flow directions (L and W) have periodic boundary conditions
  - reflective boundary condition at membrane





- Probability and move acceptance

$$P_x = v_x(\sigma) / v_x|_{\max}$$

$$P_z = \min \left[ 1, \exp \left[ -\beta \Delta E - \beta \lambda \mathbf{F}_h \cdot \Delta \mathbf{r} \right] \right]$$

$$\mathbf{F}_h = \frac{k_b T v_z \sigma K^{-1} \phi}{D_B + D_{SI}} \hat{\mathbf{z}}$$

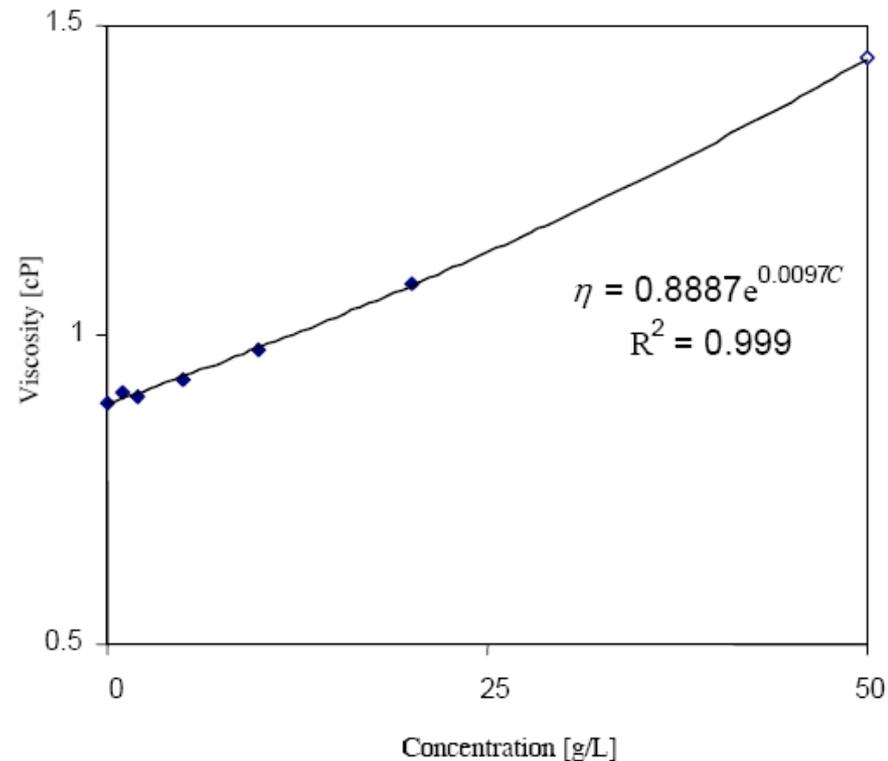
- Probability of movement to lower energy state is 1
  - otherwise the exponential is used



# Continuum Flow Representation

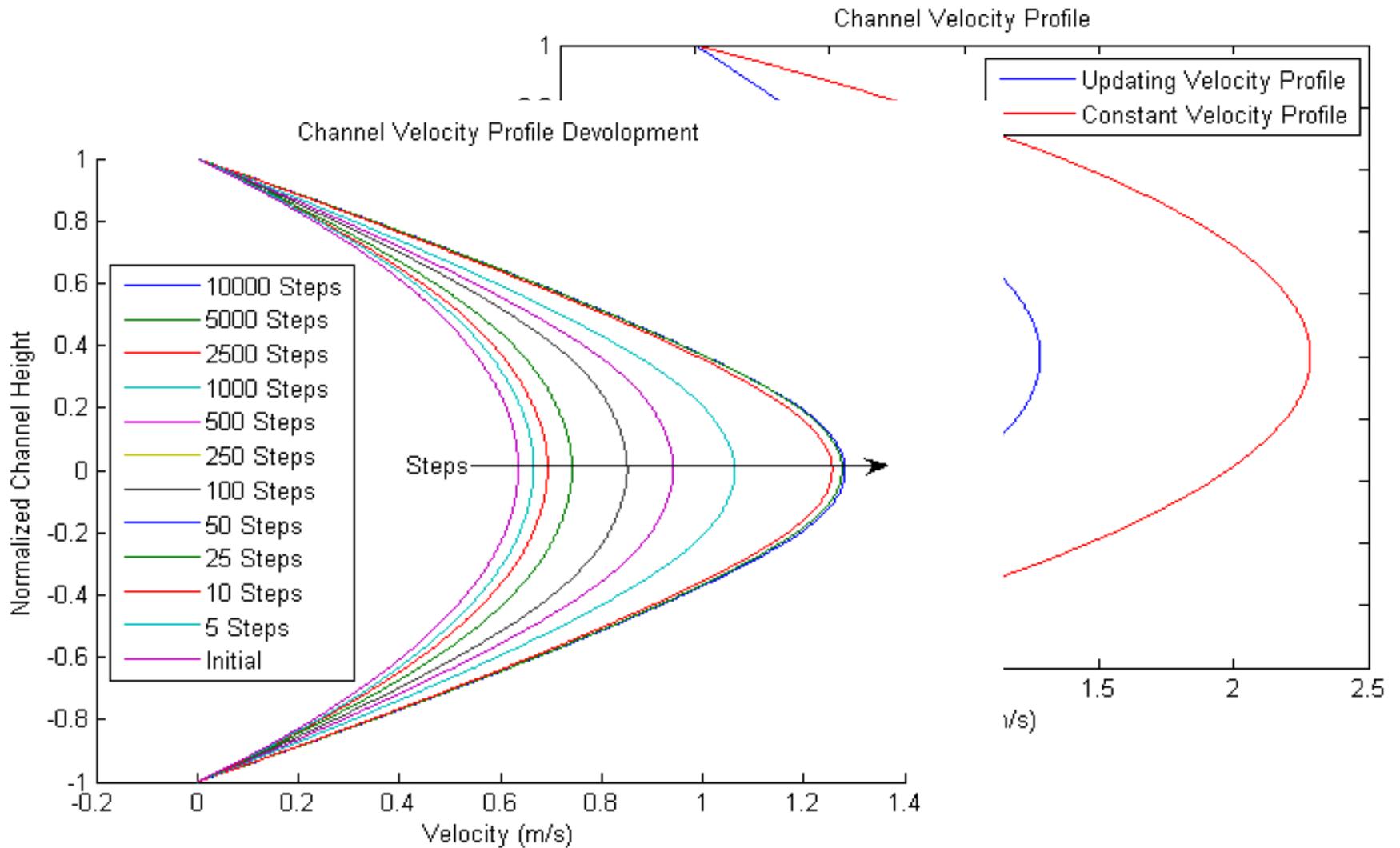


- **Viscosity**  $\eta C = \eta_0 \exp \alpha C$ 
  - generalized Newtonian
  - experimentally determined coefficients [3]
  - higher volume fractions shown to follow exponential trend
- **Boundary conditions at membrane**
  - no slip
  - $v_w \square 0$
- **Spectral collocation solution**
  - Chebyshev polynomials
  - Chebyshev-Gauss-Lobatto grid





# Dynamically Updating Velocity





# Particle-Particle Van der Waals



- Van der Waals potential
  - general form [4]

$$V_{VDW-SS} = -\frac{A_{H-SS}}{3} \left[ \frac{1}{s^2 - 4} + \frac{1}{s^2} + \frac{1}{2} \ln \left( 1 - \frac{4}{s^2} \right) \right]$$

- long-range interaction [4]

$$V_{VDW-SS} \quad s \quad = -A_{H-SS} \frac{16}{9} \frac{1}{s^6}$$

- Electrostatic double layer potential
  - close-range [5]

$$V_{EDL-SS} = \frac{32\pi R \epsilon_0 \epsilon_r k_b^2 T^2}{z^2 e^2} \gamma^2 \exp -\kappa l$$

$$\gamma = \tanh\left(\frac{ze\zeta}{k_b T}\right)$$

$$\kappa = \sqrt{\frac{2 \times 10^3 N_A e^2 z^2 C_{EL}}{\epsilon_0 \epsilon_r k_b T}}$$

- long-range [5]

$$V_{EDL-SS} = \frac{64\pi R^2 \epsilon_0 \epsilon_r k_b^2 T^2}{2R+l} \frac{\gamma^2}{z^2 e^2} \exp -\kappa l$$

[5] Elimelech, M., J. Gregory, X. Jia, and R.A. Williams, *Particle Deposition & Aggregation: Measurement, Modeling and Simulation*. Colloid and Surface Engineering: Applications in the process industries; Controlled Particle, Droplet and Bubble Formation, ed. R.A. Williams. 1995, Woburn, MA: Butterworth-Heinemann.

- Full range equation
  - Assume identical particles

$$V_{EDL-SS} = \frac{2\pi\epsilon_0\epsilon_r\zeta^2 R^{s-1}}{s} \ln \left[ 1 + \frac{\sqrt{e^{\kappa R} 2^{-s}}}{s-1} \right]$$

$$\kappa = \sqrt{\frac{2 \times 10^3 N_A e^2 z^2 C_{EL}}{\epsilon_0 \epsilon_r k_b T}}$$



- Van der Waals potential

$$V_{VDW-SP} = -\frac{A_{H-SP}}{6} \left[ \frac{1}{h} + \frac{1}{2+h} + \ln \left( \frac{h}{2+h} \right) \right]$$

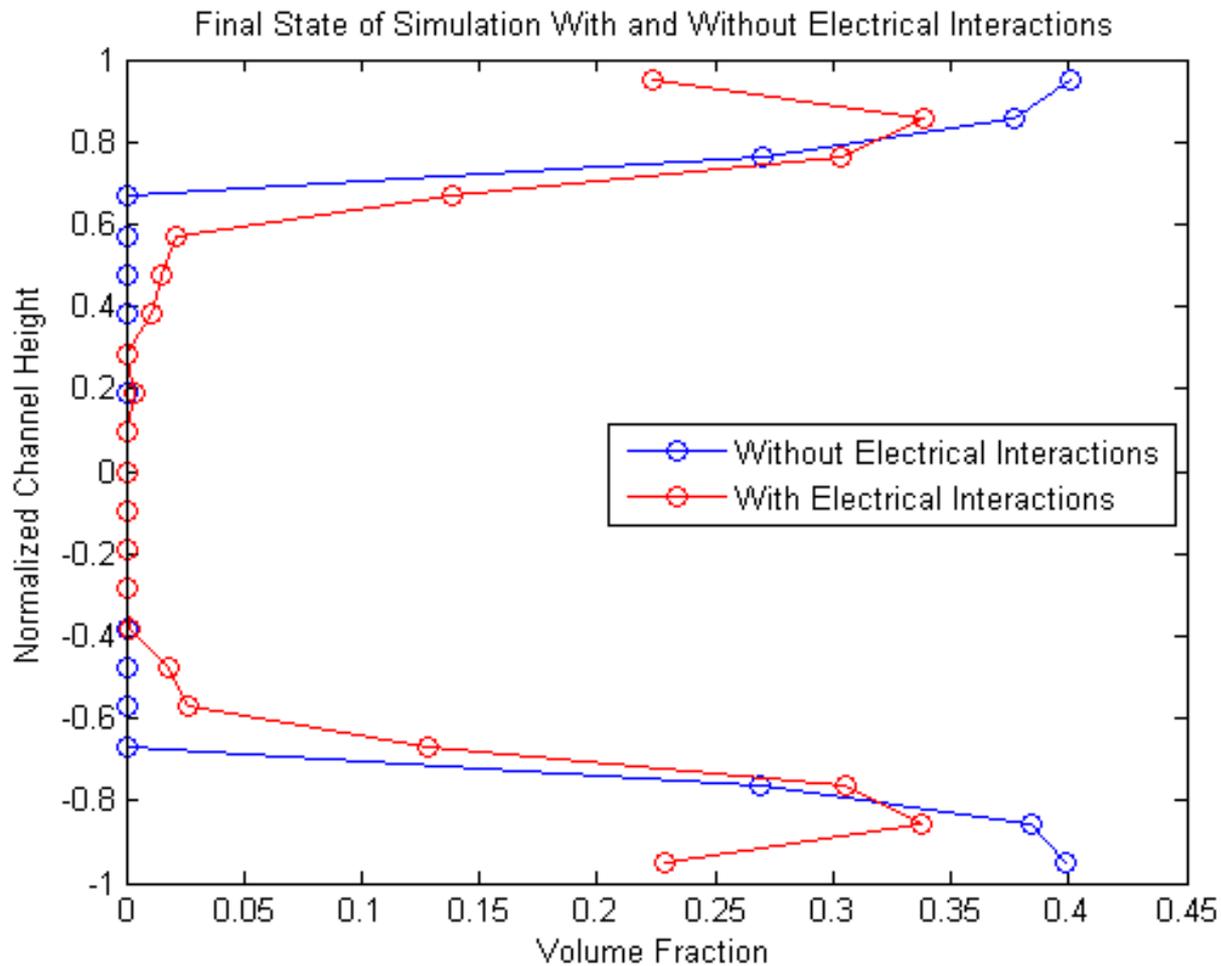
- Electrostatic double layer

$$V_{EDL-SP} = \frac{64\pi R \epsilon_0 \epsilon_r k_b^2 T^2}{z_{p1} z_{p2} e^2} \gamma_1 \gamma_2 \exp -\kappa l$$

– special case of sphere-sphere interaction



# Concentration Profile with and without Electric Interactions





# Surface Roughness

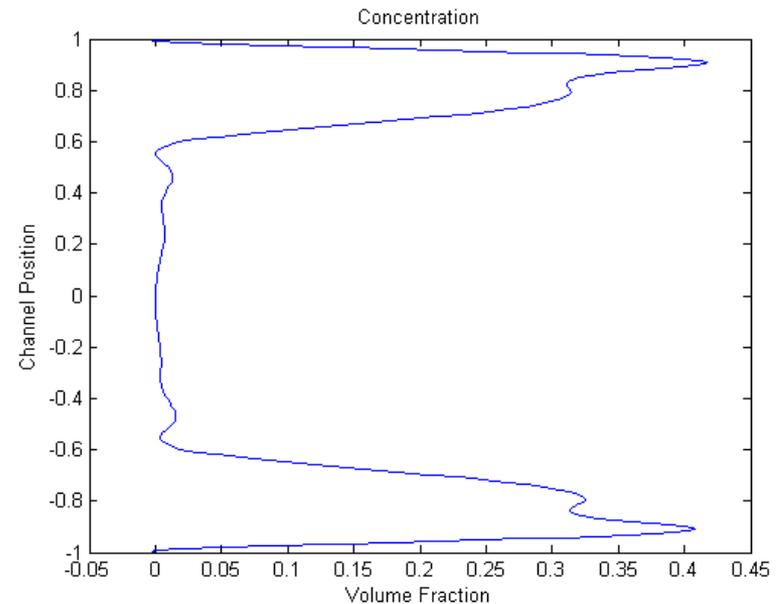


- Surface roughness of the membrane can substantially affect the motion of particles in the flow
- Fix particles to wall to approximate roughness
  - Allows a large variety of roughness possibilities
  - Key question: random distribution versus uniform distribution



# Numerical Refinement

- Wall phenomena
  - Artifact of concentration calculation
  - Change concentration calculation (volume)
- Limited effect on the flow field
- Actually improved speed





# N-Body/Parallel Optimization



- Previous work had moved particle by particle, evaluating a movement probability independent of other particle motion
- New movement process attempts new position before calculating move acceptance probability
- N-body algorithms exist
  - Lump particles in far field for faster evaluation
  - Fast Multipole Method (FMM) best candidate [6]
- Requires a new data handling structure
- Designed for “large” systems



# Parallel Optimization



- Change many small passes to a few large passes
- Each time a potential evaluation is carried out we need a sync, broadcast, and gather
- New data handling allows for N-body algorithms
- Exploit symmetry not previously available

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ E_{1,2} & 0 & 0 & 0 & 0 & 0 \\ E_{1,3} & E_{2,3} & 0 & 0 & 0 & 0 \\ E_{1,\dots} & E_{2,\dots} & E_{3,\dots} & 0 & 0 & 0 \\ E_{1,N-1} & E_{2,N-1} & E_{3,N-1} & E_{\dots,N-1} & 0 & 0 \\ E_{1,N} & E_{2,N} & E_{3,N} & E_{\dots,N} & E_{N-1,N} & 0 \end{bmatrix}$$

$$F = H + H^T = \begin{bmatrix} 0 & E_{1,2} & E_{1,3} & E_{1,\dots}^T & E_{1,N-1} & E_{1,N} \\ E_{1,2} & 0 & E_{2,3} & E_{2,\dots}^T & E_{2,N-1} & E_{2,N} \\ E_{1,3} & E_{2,3} & 0 & E_{3,\dots}^T & E_{3,N-1} & E_{3,N} \\ E_{1,\dots} & E_{2,\dots} & E_{3,\dots} & 0 & E_{\dots,N-1}^T & E_{\dots,N}^T \\ E_{1,N-1} & E_{2,N-1} & E_{3,N-1} & E_{\dots,N-1} & 0 & E_{N-1,N} \\ E_{1,N} & E_{2,N} & E_{3,N} & E_{\dots,N} & E_{N-1,N} & 0 \end{bmatrix}$$



## Future Work



- Determine appropriate number of particles fixed to wall to approximate real world surface roughness
- Increase simulation scale (more particles, longer channel)
- Application of FMM to large systems



# References



- [1] *Water, sanitation and hygiene links to health*. 2004 [cited 2008; Available from: [http://www.who.int/water\\_sanitation\\_health/publications/facts2004/en/index.html](http://www.who.int/water_sanitation_health/publications/facts2004/en/index.html)].
- [2] Harris, A., *A Mechanistic Study on the Coupled Organic and Colloidal Fouling of Nanofiltration Membranes*, in *Civil and Environmental Engineering*. 2008, Rice University: Houston, TX.
- [3] Hale, J.S., A. Harris, Q. Li, and B.C. Houchens. *The fluid mechanics of membrane filtration*. in *2007 ASME International Mechanical Engineering Congress and Exposition*. 2007. Seattle, WA.
- [4] Parsegan, V.A., *Van der Waals Forces: A Handbook for Biologists, Chemists, Engineers, and Physicists*. 2006, New York, NY: Cambridge University Press.
- [5] Elimelech, M., J. Gregory, X. Jia, and R.A. Williams, *Particle Deposition & Aggregation: Measurement, Modeling and Simulation*. Colloid and Surface Engineering: Applications in the process industries; Controlled Particle, Droplet and Bubble Formation, ed. R.A. Williams. 1995, Woburn, MA: Butterworth-Heinemann.
- [6] Greengard, L., *The Rapid Evaluation of Potential Fields in Particle Systems*. ACM Distinguished Dissertations. 1988, Cambridge, MA: Massachusetts Institute of Technology.