DETERMINATION OF WALL HEAT LOADINGS FROM INTERNAL HEAT SOURCES FOR COMPLEX GEOMETRIES

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ABSTRACT

An algorithm for the determination of heat loadings on a containing wall due to heat sources internal to the wall is developed and presented, in parallel with an illustrative example involving the calculation of radiation shape factors for the distribution of radiation energy from a volumetric heat source located within a simple torus. The six major steps in this algorithm: development of a local coordinate system, numerical partitioning of the wall, numerical partitioning of the internal heat source, determination of distance formulas, determination of incidence angles, and, finally, application of appropriate wall blockage criteria are discussed. The illustrative example is developed in parallel with the discussion in that each individual step in the algorithm is applied to the example before proceeding to the next step, with the end result that poloidal variations in energy deposition for a basic toroidal geometry are determined for a variety of toroidal aspect ratios and two different source volumetric heat generation profiles.

INTRODUCTION

It is often desired to determine how the irregular distribution of a heat source internal to a containing wall affects the variation of heat loadings on the wall itself. Certainly, the loadings will not be uniform even in the case of a volumetric heat source with a regular symmetrical shape because of irregularities in the wall itself, and with non-symmetries in the internal heat generation volume the problem becomes even more complex. It is the purpose of this paper to present a systematic procedure to evaluate these wall loading variations which involves the following six steps:

(1) Coordinate System Selection

An appropriate coordinate system must be selected which reflects the primary geometric features of the heat source and the containing wall, additional features and complexities notwithstanding. For example, a cylindrical coordinate system would probably be used for a series of telescoping cylinders or a reactor cooling tower and a rectangular one for a house or a building. As will be shown later, a coordinate transformation to the rectangular system will need to be performed is this system is not the one initially chosen.

(2) Wall Partitioning

A numerical partitioning of the wall into a large number of cells which reflect the various locations on the wall will need to be accomplished. The wall can be divided into the different "parts" (e. g. a rectangular box and a round dome over the box), and then each of these parts can be divided into a large number of surface cells.

The rest of the procedure involves calculating "shape factors" for each of these individual cells. The "shape factor" represents the power loading (per unit area) for that individual cell divided by the average power loading on the wall, which is simply the total heat generated by the internal source divided by the total area of the wall.

(3) Heat Source Partitioning

A three-dimensional partitioning of the heat source into small cells needs to be conducted, and the contribution of the heat generated from each of these cells must be summed, for every individual wall cell, to obtain the total heat load on the cell.

The reader should note at this point that a computer program performing this will have five nested loops (3 nested loops for the source nested within 2 for the wall cells); with even 100 divisions at each coordinate this involves 10,000,000,000 loops. While this is certainly not beyond the capabilities of modern workstations, care must be taken to make the calculations performed within each loop as efficient as possible.

(4) Distance Formulas

Obviously the primary factor determining the contribution of a heat source cell to an individual wall cell is the distance to that cell, so formulas must be developed that determine these distances from every heat source cell to every possible wall cell. In this procedure a conversion from the working coordinate system to the rectangular coordinate system is first performed, and then the standard Cartesian distance formulas are used.

(5) Incidence Angles

Another factor would be the "slant" of the wall cell with respect to incoming energy from a cell in the heat generation source. If the direction of energy propagation is normal to the wall, contribution to the heat load on the wall will be a maximum; for larger incidence angles it will be less.

There may also be cases where the direction of propagation is parallel to the wall surface, or even coming in from "behind" the surface. Then the contribution would be zero, as discussed in the next paragraph.

(6) Blockage Criteria

Within the innermost loop, a test needs to be conducted to determine whether the heat source cell contributes to a certain individual wall cell at all. There are several reasons why it may not; one is if the wall cell is "facing away" from that part of the source, in which the situation discussed above prevents a contribution from that source cell. Another possibility is that the energy from the source hits another portion of the wall first; hence that section "blocks" the energy from being deposited on the section being considered.

The end result of all this should be a graphical representation of power loadings on all sections of the wall. This could either involved absolute power loadings, or shape factors as discussed earlier.

The basic approach of this paper will be to "show" rather than "tell" how this is done by applying this procedure to the calculation of radiation shape factors for internal volumes in toroidal geometries. When a simple toroidal heat source centered on the toroidal axis is bounded by a toroidal wall, the calculations are relatively straightforward, but have not yet been done, possibly because practical applications for this are somewhat limited. One place where some application was possible was in the conceptual tokamak thermonuclear fusion reactors [1, 2] designed in the 1980s (different designs are now being considered more seriously). Since these devices had planar walls, some modification would of course need to be made to the present analysis.

Certainly similar problems have been considered and solved. Various modes of heat transfer in a cylindrical annulus have been investigated under a variety of different conditions. Conduction, convection, and radiation mechanisms between concentric cylinders have been investigated and discussed for both participating and non-participating media in the annulus itself. Combined convection and radiation heat transfer in a horizontal cylindrical annulus has been examined by Kuo, Morales, and Ball [3] who have built on the earlier work of Fernandes and Francis [4] and Pandey [5] in analyzing a two-dimensional horizontal annulus containing a radiatively participating gray medium.

In the present case, the cylindrical annulus is simply "bent" or "coiled" so that a toroidal, or ring, geometry is approximated. Essentially, this involves an inner ring where volumetric heat generation of heat is taking place. Heat loads on the outer ring are then determined, as a function of poloidal angle (this poloidal angle corresponds to the sweep of the minor radius of the torus, just as a toroidal angle relates to the major radius). The procedure applies because unlike the case of the cylindrical annulus, these loads will by no means be uniform over this angle, because of toroidal effects. As will be shown, only for very large "aspect ratios" (the ratio of the major to minor radii) is the cylindrical condition of uniformity approached.

For a vacuum or radiatively non-participating medium in the region between the inner ring, which constitutes the heat source, and the outer toroidal wall, polodial variations in radiation energy deposition are considered and "poloidal radiation shape factors" are developed. As discussed earler, a poloidal radiation shape factor is defined as the ratio of the local radiation power loading at a specific poloidal angle on the torus to the radiation power loading averaged over the entire surface of the torus under the assumption of toroidal symmetry. Hence, a shape factor of 1.00 would mean an average loading at that particular angle and a shape factor of 1.25 would include a local loading 25% above the average.

COORDINATE SYSTEM SELECTION

For analysis a perfect torus was chosen with major and minor radii of R and a, respectively. Toroidal coordinates were laid out such that the major plane of the torus coincides with the x-y plane. For a fixed major toroidal radius R, the radial coordinate r would be the distance from the center line of the torus, where this center line would be described by the equations:

$$(1)\quad (x^2 + y^2)^{1/2} = R \; ; \; z = 0$$

The polodial coordinate θ was chosen such that $\theta = 0^o$ corresponds to the outer edge of the torus, and the toroidal coordinate ϕ was chosen such that the point on the center line where $\phi = 0^o$ lies on the positive x axis. A transformation back to rectangular coordinates, as will be necessary to calculate distances and angles, would then be given by:

$$(2)\qquad \begin{aligned} x &= (R + r\cos(\theta))\cos(\phi) \\ y &= (R + r\cos(\theta))\sin(\phi) \\ z &= r\sin(\theta) \end{aligned}$$

WALL PARTITIONING

Toroidal symmetry was assumed, and the toroidal cross-section corresponding to $\phi = 0^o$ was arbitrarily chosen for the analysis. For purposes of discussion, the outer surface of the torus, described by the equation r = a, is referred to as the "wall," although it is recognized that the results would apply to any surface described by this equation. Points on this wall corresponding to various polodial angles ranging from 0^o to 180^o were considered, and variations in the local loadings caused by the heat source within the torus were determined and plotted. Symmetry about the major plane of the torus was assumed, so that the local loadings at angles θ_w between 180^o and 360^o correspond to those at angles $360^o - \theta_w$. Contributions to wall loadings from the heat source will also be symmetrical about the torus cross-sectional surface $\phi = 0^o$ so that they need only be calculated from one toroidal direction and then multiplied by a factor of 2.

Note that in most other problems much more complexity would be involved in this step. The containing wall, for example, could have several sections all of different geometries.

HEAT SOURCE PARTITIONING

For determination of heat loads caused by an isotropic non-attenuating volumetric heat source within the torus, a three-dimensional partitioning of the source was carried out, with r ranging from 0 to ρ, the cross-sectional radius of the source, and with ϕ ranging from 0^o to 180^o because of the symmetry condition listed above. Note, however, that contributions from the cells are not symmetrical about the major plane of the torus (except in the special cases $\theta = 0^o$ and $\theta = 180^o$

on the wall) so that θ for the source must range from 0^o to 360^o. The differential volume for each cell will be

$$(3)\quad dV = (dr)(r\, d\theta)([R + r \cos(\theta)]\, d\phi) = (rR + r^2 \cos(\theta))\, dr\, d\theta\, d\phi$$

The total heat source power generation will be

$$(4)\quad P = \int_V G(r)\, dV = \int_r \int_\theta \int_\phi G(r)\, (rR + r^2 \cos(\theta))\, d\phi\, d\theta\, dr = 4\, \pi^2\, R \int_r r\, G(r)\, dr$$

where it is recognized that the integral of the second term in the above expression is zero. Note that heat source power density is allowed to vary as a function of the radial coordinate. Since the total wall surface area is

$$(5)\quad A = (2\pi a)(2\pi R) = 4\pi^2 aR$$

the average wall power loading will be

$$(6)\quad p_{av} = 4\pi^2 R \int_r r\, G(r)\, dr\, /\, 4\pi^2 aR = (1/a) \int_r r\, G(r)\, dr$$

For the case of spatially uniform power generation G(r) = G within the heat source, this reduces to

$$(7)\quad p_{av} = G\rho^2 / 2a$$

For a given point on the wall, the contribution to the local power loading by an individual differential cell in the heat source is given by

$$(8)\quad dp = G(r) \cos(\beta)\, dV / 4\pi D^2$$

where D is the distance between the cell and the point of the wall and β is the angle the inward-facing normal at the wall point makes with the line joining this point to the cell. This expression is then integrated over the entire volume of the heat source to determine the local power loading, where it is recognized that all of the variable quantities above are functions of the cell location within the heat source.

DISTANCE FORMULAS AND INCIDENCE ANGLES

The determinations of D and cos β for each of the cells is relatively straightforward. Given the wall point coordinates $[x_w, y_w, z_w]$ and the cell coordinates $[x_c, y_c, z_c]$, the standard Cartesian distance formula for D can be applied with rectangular coordinates for a cell with toroidal coordinates $[r, \theta, \phi]$ given by Eq. (2) and those for the wall point given by

$$x_w = R + a\cos(\theta_w)$$
$$(9) \qquad y_w = 0$$
$$z_w = a\sin(\theta_w)$$

cos β can be determined by taking the scalar product of the inward-facing normal vector and the distance vector to the cell and dividing by the magnitudes. The result is

$$(10) \quad \cos(\beta) = (1/D)\,[(x_w - x_c)\cos(\theta_w) + (z_w - z_c)\sin(\theta_w)]$$

BLOCKAGE CRITERIA

Two additional requirements, however, add complexity to the analysis. The first, which is significant for wall poloidal angles greater than 90^o, is that the source element cannot be "behind" the wall. This corresponds to the case cos $\beta < 0$ and must be checked for each cell. The second requirement is that the source emission cannot be "blocked" by another part of the wall. To determine whether or not this is the case, the line joining the wall point and the source element is projected into the x-y plane and a focal point is determined. The x and y coordinates of this point are those of the point of nearest approach of this line projection to the overall origin of the torus. The z coordinate is the height of the joining line as it passes over this point. The coordinates of this focal point are thus

$$x_f = x_w y_c^2 / [(x_c - x_w)^2 + y_c^2]$$
$$(11) \qquad y_f = x_w y_c (x_w - x_c) / [(x_c - x_w)^2 + y_c^2]$$
$$z_f = z_w + (z_c - z_w)(x_f - x_w) / (x_c - x_w)$$

where it is noted that $y_w = 0$ for all points on the wall. Note that at this height z_f the inner wall of the torus has a major radius (distance to overall origin projected into the x-y plane) of

$$(12) \quad R_w = R - (a^2 - z_f^2)^{1/2}$$

Two criteria are then established. If either of these is met, no wall blockage will occur and power radiated by the source element will contribute to the wall local power loading.

The first criterion is that

$$(13) \quad [(x_c - x_w)^2 + y_c^2]^{1/2} < [(x_f - x_w)^2 + y_f^2]^{1/2}$$

which corresponds to the condition that the source element is nearer to the wall point than the focal point, implying that the joining line reaches the source element before it could possibly intersect the inside wall of the torus.

The second criterion is given by the equation

$$(14) \quad R_w = R - (a^2 - z_f^2)^{1/2} < (x_f^2 + y_f^2)^{1/2}$$

This corresponds to the case where the joining line passes outside of the inner surface of the torus without intersecting it, and then reaches the source element.

In this paper, a non-attenuating volumetric heat source is assumed. For a volumetric heat source which reabsorbs and thus attenuates some of its own radiation, a third requirement must also be met whereby source radiation is not attenuated or blocked by the source itself. Analysis of this attenuation will not be described here, but could be done by geometrically determining the point of intersection of the joining line with the surface of the heat source and the length of this line within the source. Appropriate linear attenuation coefficients would need to be supplied based on physical considerations.

Final determination of local power loadings for a volumetric source can thus be determined by summing up the individual contributions of all source elements for which the first two requirements listed above are met.

NUMERICAL IMPLEMENTATION AND RESULTS

The numerical implementation of this analysis is straightforward and easily programmed. Model parameters and the fineness of the mesh can be easily entered, and a special provision was made in the program to vary the fineness of the mesh in different toroidal regions so that accurate results can be obtained on individual computer workstations. The volumetric results presented in this article were based on a partitioning into 18 toroidal regions, with a relatively modest 100 X 100 X 100 mesh refinement employed for the five nearest regions and a 50 X 50 X 50 refinement for the others. The author is presently performing calculations on a Dell Precision T5400 workstation using an Absoft Windows-based FORTRAN compiler.

The author considered cases of volumetric source emission for toroidal aspect ratios (R/a) of 2.0, 4.0, and 10.0 and radial ratios (ρ/a) of 0.8 and 0.9. These results are for the sake of brevity not presented here, but will be presented at the workshop. It was expected, and confirmed, that deviations of the shape factor from 1.0 are larger for small aspect ratios where the differences between a toroidal and a cylindrical geometry are more pronounced. For all cases shape factors tended to be the highest at $\theta_w = 0^o$ and monotonically declined as wall polodial angle was increased. Apparently, the ability of the outer wall sections to "see" more of the volumetric source more than compensated for the tendency of the source to wrap around, and thus become nearer, the inner wall surface. The sinusoidal shape of the curves suggested that a curve of the form b + c cos(θ) could be fitted to the results, and this in fact was done, with fitting coefficients b and c for each case listed in Table 1 below:

Table 1. Fitting Coefficients for Volumetric Radiation Shape Factors

	$(\rho/a) = 0.8$	$(\rho/a) = 0.9$
R/a = 2.0	b = 0.9806757 c = 0.0812896	b = 0.9791952 c = 0.0885426
R/a = 4.0	b = 0.9917716 c = 0.0733846	b = 0.9916573 c = 0.0768915
R/a = 10.0	b = 0.9980446 c = 0.0496696	b = 0.9981883 c = 0.0510130

Note that the constant b, which corresponds to the unweighted average of the shape factors, is less than one. This is because the wall areas corresponding to the outer wall angles, where the shape factors are the highest, are larger than those corresponding to the inner angles. When the factors are weighted according to the wall areas corresponding to each angle, the expected weighted average of 1.0 is achieved.

Results were nearly identical for the same aspect and radial ratios and an inverse parabolic power density profile of the form $G(r) = G_0(1 - (r/\rho)^2)$. Shape factors were slightly less pronounced at the inner wall angles for an aspect ratio of 2.0.

CONCLUSIONS

A systematic procedure has been developed for the determination of heat loadings on a containing wall due to heat sources internal to the wall. This procedure was then applied to a toroidal geometry involving a ring-shaped radiative heat source transferring heat through a vacuum or radiatively non-participating medium. Radiation shape factors were developed, where a shape factor has been defined as the local radiation power loading at a certain point on a toroidal surface, resulting from the distribution of radiation energy from a surface or volumetric source located within the torus, to the average power loading over the entire surface.

Poloidal variations in energy deposition are thus analyzed on the assumption of toroidal symmetry for several different toroidal aspect ratios, and for isotropic and inverse parabolic volumetric radiation sources. It was found that in many cases these variations can be significant (between 10% and 20% in a number of cases), and thus warrant consideration in future analyses and designs.

ACKNOWLEDGEMENTS

The author wishes to acknowledge the valuable support received from his present institution, Texas A & M University at Galveston, and for the helpful comments of the technical reviewers for this paper. This work has been supported by a summer research grant that was a part of the author's startup package from Texas A & M University.

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NOMENCLATURE

Symbols adopted as part of the standard nomenclature (such as x, y, z, and r) are not included.

A	outer toroidal ("wall") surface area
a	minor radius of torus
D	distance between wall point and heat source element
e	source surface emissive power
F	poloidal shape factor ($= p/p_{av}$)
F_R	source reabsorption fraction
P	total heat source power generation
p	local wall power loading
p_{av}	average wall power loading
R	major radius of torus
R_w	projected major radius of toroidal inner surface
r	radial coordinate (toroidal coordinates)
β	angle between inward wall normal and line joining wall point to source element
θ	poloidal coordinate (toroidal coordinates)
θ_w	wall poloidal coordinate
ξ	azimuthal angle (surface emission)

ρ	minor radius of heat source
ϕ	toroidal coordinate (toroidal coordinates)
ψ	angle to outward normal (surface emission)
Ω	solid angle (for surface emission)

Subscripts

c	heat source element (volumetric or surface cell)
d	diffuse emission
f	focal point
R	reabsorption
w	wall point
y	cyclotron emission
z	charged particle emission