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LHP Stability Theory

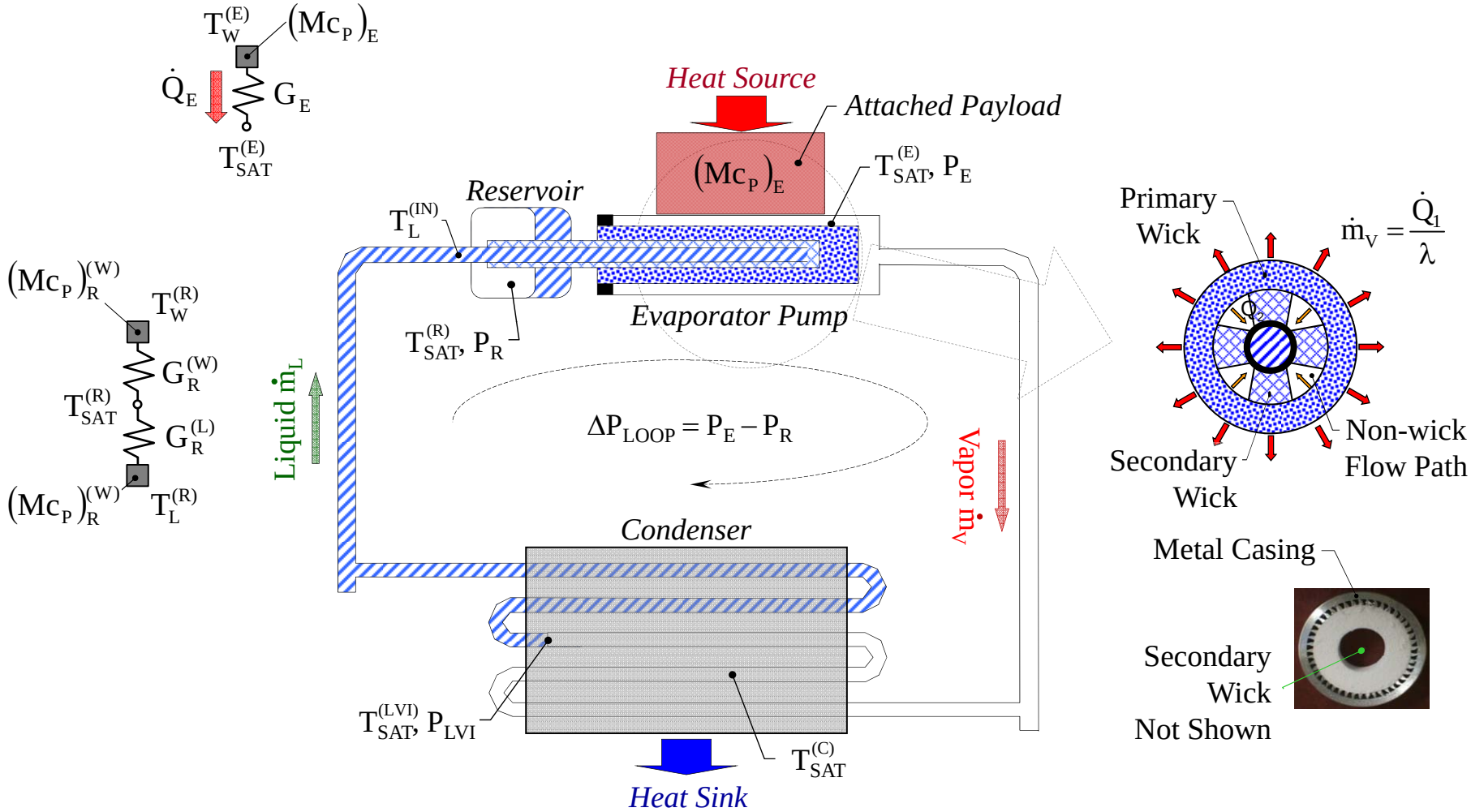
Triem T. Hoang
TTH Research Inc.

Robert W. Baldauff
U.S. Naval Research Laboratory

Outline

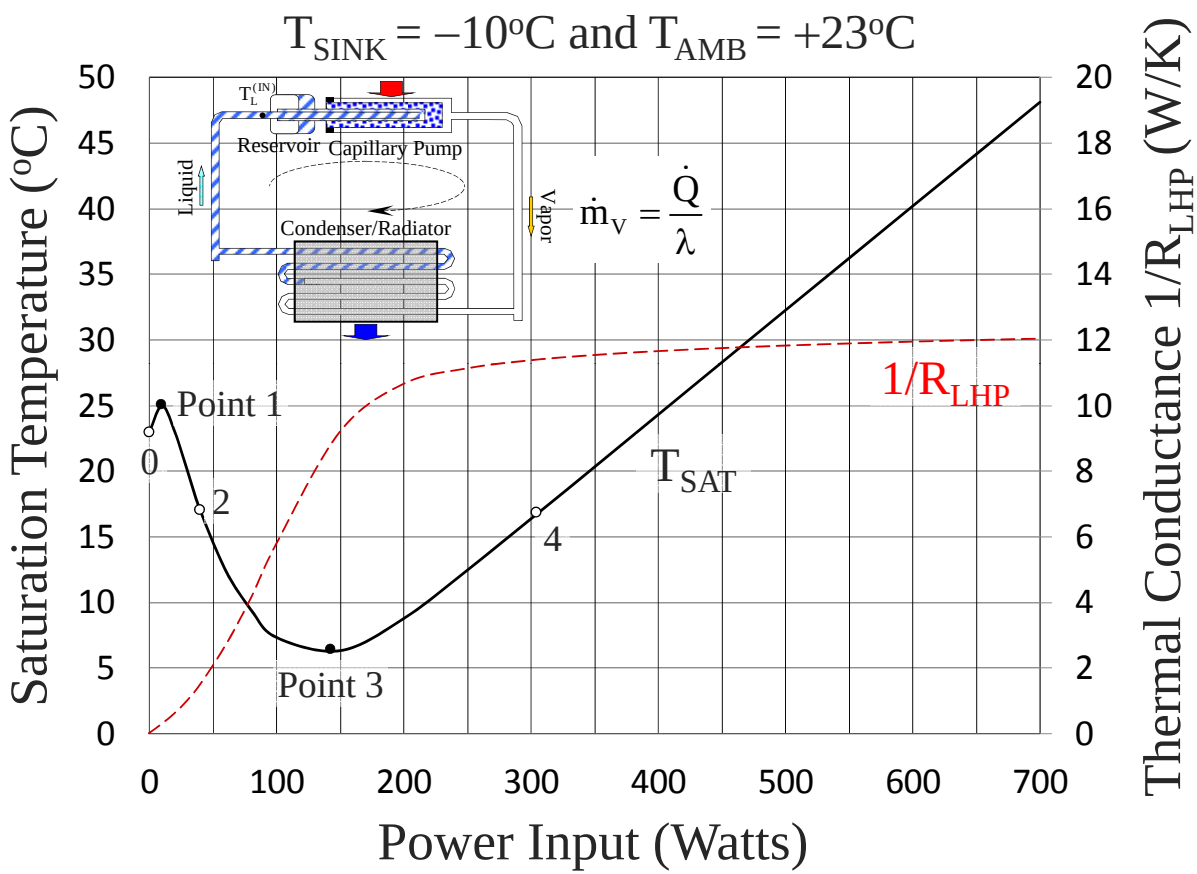
- Background of Loop Heat Pipe (LHP) Technology
- Nonlinear Dynamics in LHPs
- Oscillatory Behaviors in LHP Operation
- Linear Stability Theory
- *Instability* Criteria
- Discussion / Summary
- Path Forward

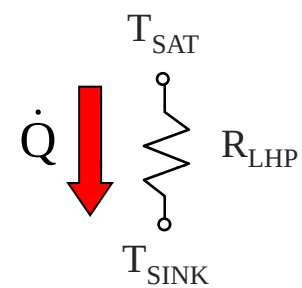
Loop Heat Pipe



LHP is a capillary-pumped device having no mechanical moving part

LHP Steady State Operation

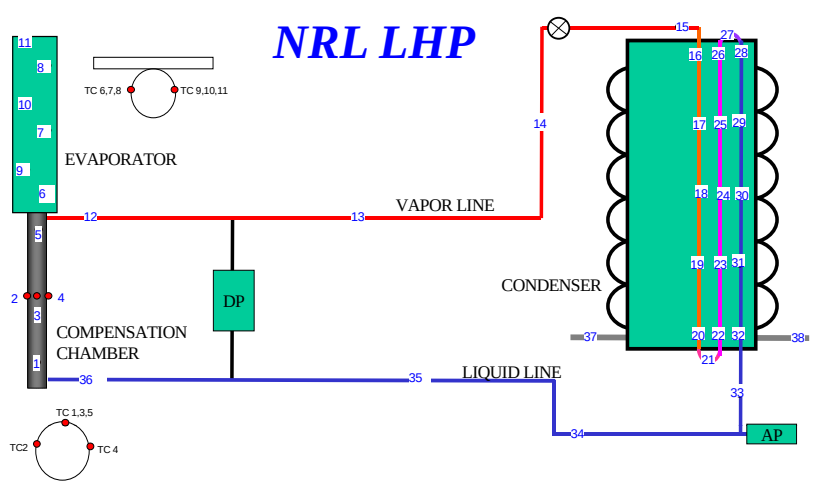


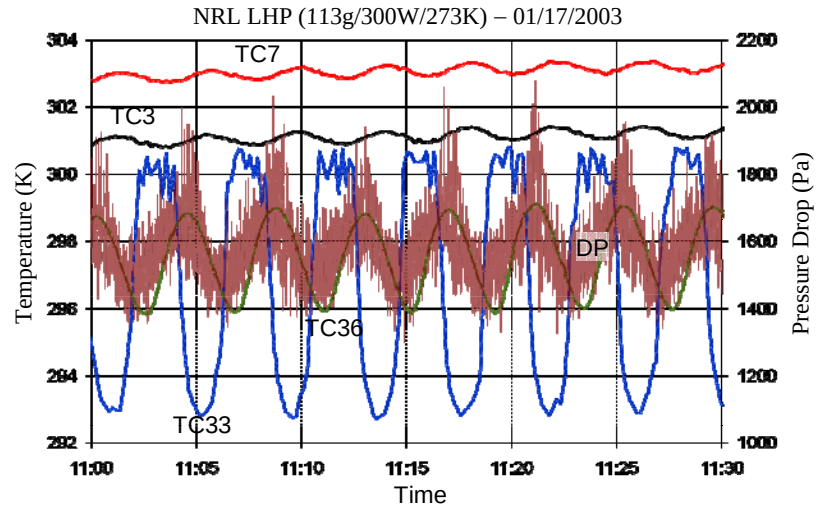


$$T_{SAT} - T_{SINK} \equiv R_{LHP}\,\dot{Q}$$

LHP is a nonlinear heat transport system

Temperature Oscillations in LHPs





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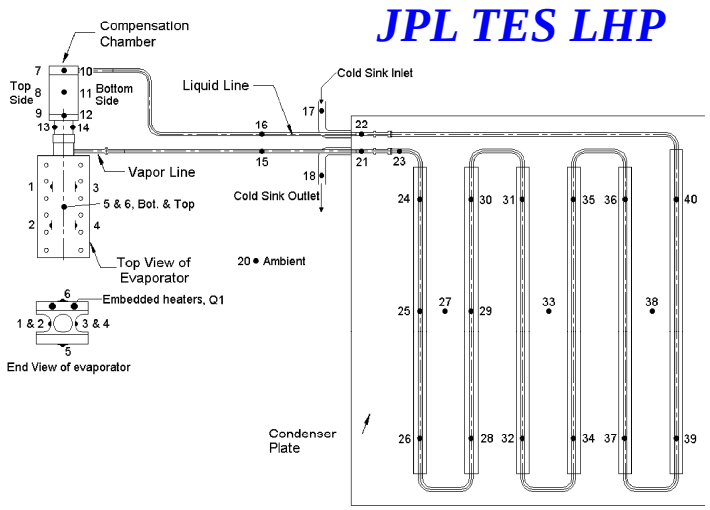

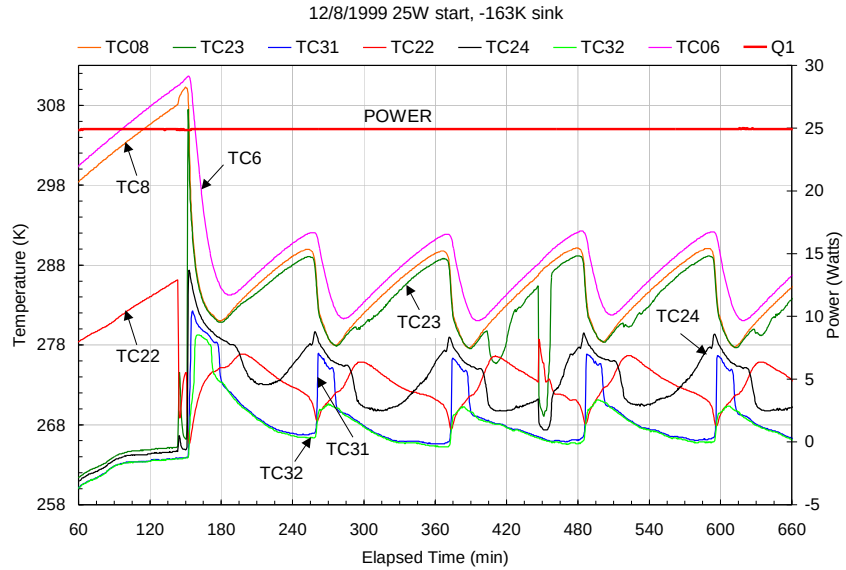

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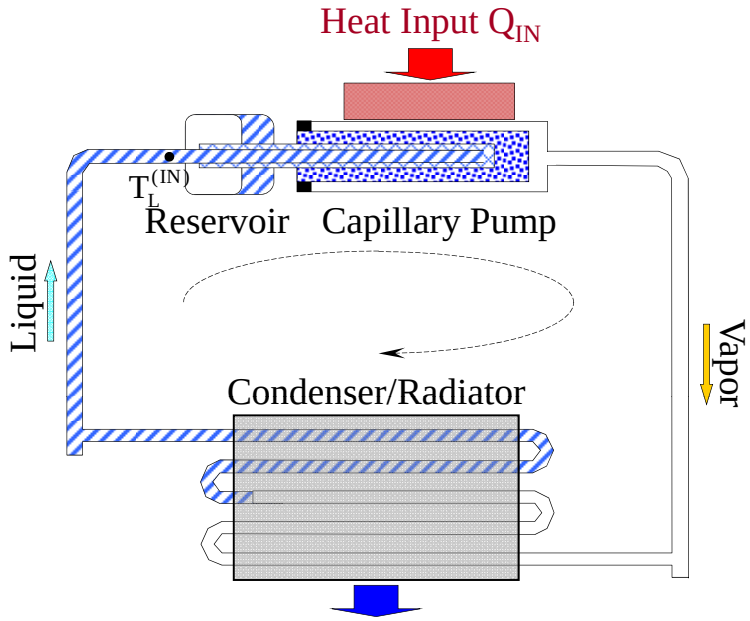
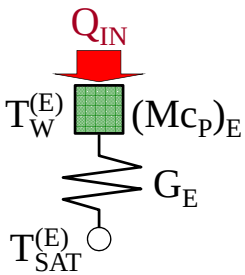
LHP Nonlinear Dynamics

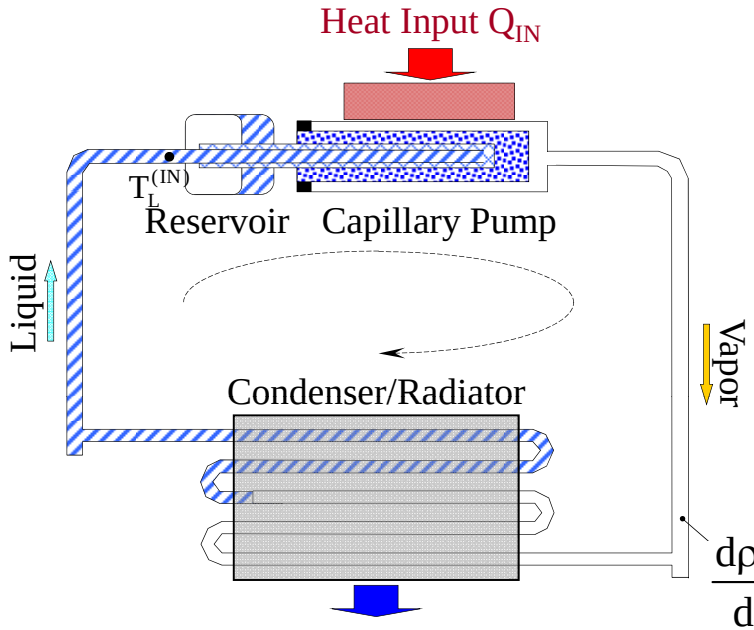


Heat Transfer Across Evaporator Wall



$$(Mc_P)_E \frac{dT_W^{(E)}}{dt} = \dot{Q}_{IN} - \dot{Q}_E$$

LHP Nonlinear Dynamics



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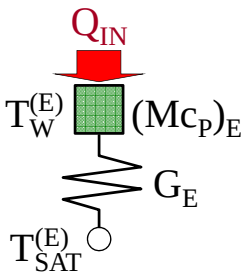


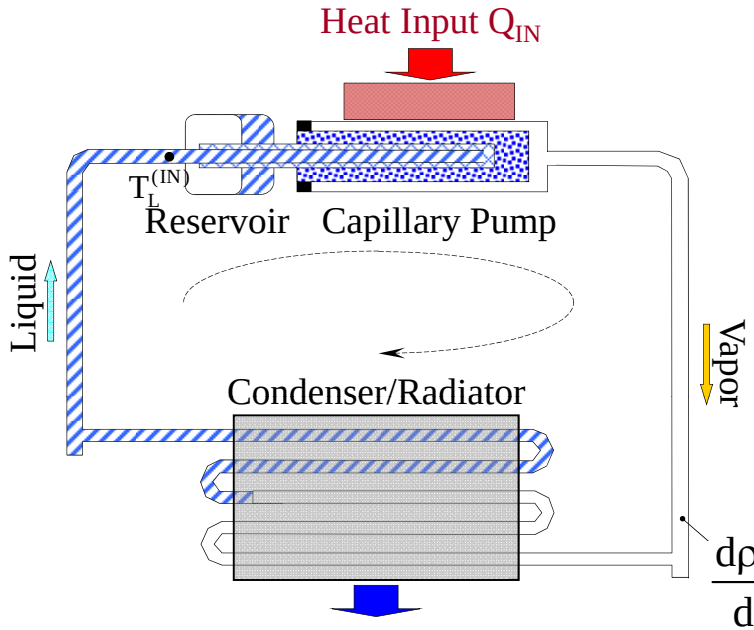
$$(Mc_P)_E \frac{dT_W^{(E)}}{dt} = \dot{Q}_{IN} - \dot{Q}_E$$

$$\frac{d\rho_V}{dt} = \frac{d}{dt}\left(\frac{M_V^{(VAPOR)}}{V_{VL} + V_C^{(2\phi)}}\right)$$

$$= \frac{1}{\lambda}\frac{\dot{Q}_1 - \dot{Q}_C^{(2\phi)}}{V_{VL} + V_C^{(2\phi)}} - \rho_V \frac{dV_C^{(2\phi)}}{dt}$$

$$\dot{Q}_C^{(2\phi)} = G_C^{(MAX)}(T_{SAT} - T_{SINK})\frac{V_C^{(2\phi)}}{V_C}$$

LHP Nonlinear Dynamics



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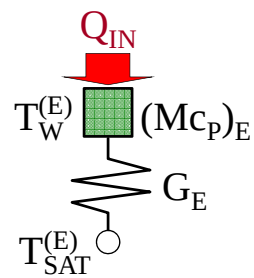

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$$\frac{d\rho_V}{dt} = \frac{d}{dt}\left(\frac{M_V^{(VAPOR)}}{V_{VL} + V_C^{(2\phi)}}\right) = \frac{1}{\lambda}\frac{\dot{Q}_1 - \dot{Q}_C^{(2\phi)}}{V_{VL} + V_C^{(2\phi)}} - \rho_V \frac{dV_C^{(2\phi)}}{dt}$$

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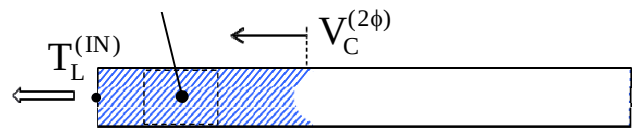
$$\frac{\partial T_L}{\partial t} + \frac{4\dot{m}_L}{\rho_L}\frac{\partial T_L}{\partial \mathcal{T}_L} + \frac{4h_{L-\infty}}{D\rho_L c_P^{(L)}}(T_L - T_\infty) = 0$$



$$\dot{m}_V = \frac{\dot{Q}_1}{\lambda}$$

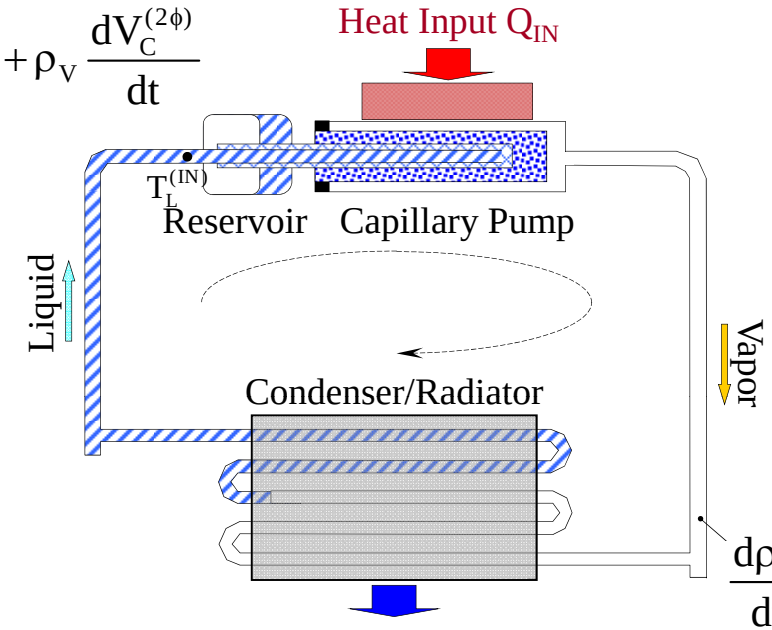
$$\dot{m}_L = \rho_L \frac{dV_C^{(2\phi)}}{dt} + \dot{m}_V$$

$$\frac{d\dot{m}_L}{dt} = \frac{(P_{LVI} - P_R) - \left[R_B^{(L)} + R_{LL}^{(L)} + R_{CL}^{(L)}\left(1 - \frac{V_C^{(2\phi)}}{V_C}\right)\right]\dot{m}_L}{\left[\frac{L_B}{A_B} + \frac{L_{LL}}{A_{LL}} + \frac{L_{CL}}{A_{CL}}\left(1 - \frac{V_C^{(2\phi)}}{V_C}\right)\right]}$$

LHP Nonlinear Dynamics

$$\frac{d\rho_V}{dt} = \frac{-\eta\dot{Q}_{SC} + \dot{Q}_2 + \dot{Q}_R^{(W)} + \dot{Q}_R^{(L)}}{\lambda V_R^{(VAPOR)}} + \rho_V \frac{dV_C^{(2\phi)}}{dt}$$

$$\dot{Q}_{SC} = \dot{m}_L c_P (T_{SAT} - T_L^{(IN)})$$



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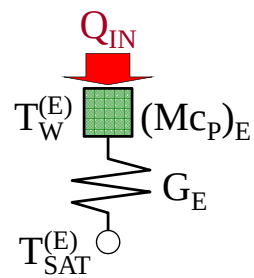

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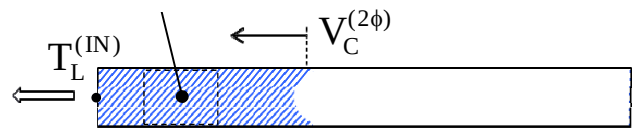

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LHP Nonlinear Dynamics

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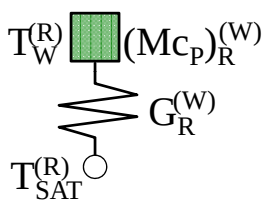
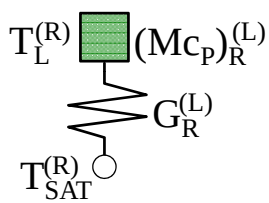
Vapor-Res. Wall Heat Transfer

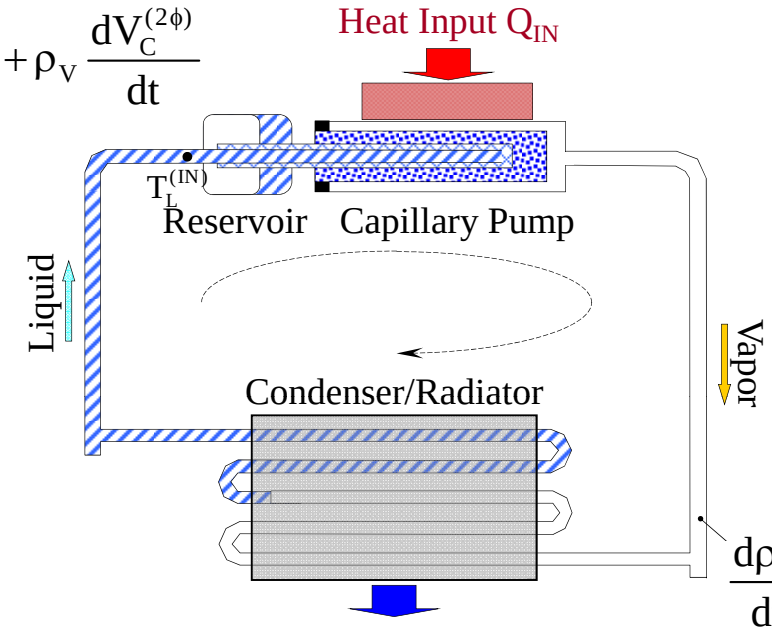


Vapor-Liquid Heat Transfer



$$\frac{dT_W^{(R)}}{dt} = -\frac{\dot{Q}_R^{(W)}}{(Mc_P)_R^{(W)}}$$

$$\frac{dT_L^{(R)}}{dt} = -\frac{\dot{Q}_R^{(L)} + (1-\eta)\dot{Q}_{SC}^{(MAX)}}{(Mc_P)_R^{(L)}}$$



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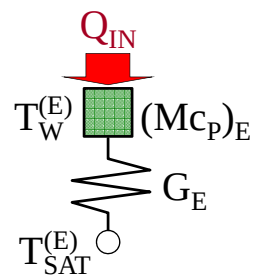

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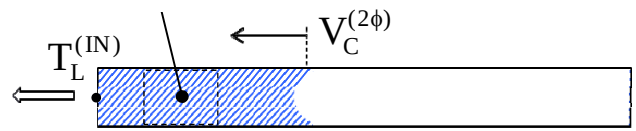

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$$\frac{d\dot{m}_L}{dt} = \frac{(P_{LVI} - P_R) - \left[R_B^{(L)} + R_{LL}^{(L)} + R_{CL}^{(L)}\left(1 - \frac{V_C^{(2\phi)}}{V_C}\right)\right]\dot{m}_L}{\left[\frac{L_B}{A_B} + \frac{L_{LL}}{A_{LL}} + \frac{L_{CL}}{A_{CL}}\left(1 - \frac{V_C^{(2\phi)}}{V_C}\right)\right]}$$

LHP Governing Equations

Fluid Dynamics:

$$\frac{dV_C^{(2\phi)}}{dt} = \frac{1}{\rho_L}\left(-\frac{\dot{Q}_C^{(2\phi)}}{\lambda} + \dot{m}_L\right)$$

$$\frac{d\dot{m}_L}{dt} = \frac{(P_{LVI} - P_R) - \left[R_B^{(L)} + R_{LL}^{(L)} + R_{CL}^{(L)}\left(1 - \frac{V_C^{(2\phi)}}{V_C}\right)\right]\dot{m}_L}{\left[\frac{L_B}{A_B} + \frac{L_{LL}}{A_{LL}} + \frac{L_{CL}}{A_{CL}}\left(1 - \frac{V_C^{(2\phi)}}{V_C}\right)\right]}$$

Thermodynamics:

$$\frac{dT_{SAT}^{(E)}}{dt} = \frac{1}{\left(\frac{\partial \rho_V}{\partial T}\right)_{SAT}\left(V_{VL} + V_C^{(2\phi)}\right)}\left(\frac{\dot{Q}_1 - \dot{Q}_C^{(2\phi)}}{\lambda} - \rho_E^{(V)}\frac{dV_C^{(2\phi)}}{dt}\right)$$

$$\frac{dT_{SAT}^{(R)}}{dt} = \frac{1}{\left(\frac{\partial \rho_V}{\partial T}\right)_{SAT}\left(V_{LHP}^{(V)} - V_{VL} - V_C^{(2\phi)}\right)}\left(\frac{-\eta\dot{Q}_{SC}^{(MAX)} + \dot{Q}_2 + \dot{Q}_R^{(W)} + \dot{Q}_R^{(L)}}{\lambda} + \rho_R^{(V)}\frac{dV_C^{(2\phi)}}{dt}\right)$$

Heat Transfer:

$$(Mc_P)_E\frac{dT_W^{(E)}}{dt} = \dot{Q}_{IN} - \dot{Q}_E \qquad (Mc_P)_R^{(W)}\frac{dT_W^{(R)}}{dt} = -\dot{Q}_R^{(W)} \qquad (Mc_P)_R^{(L)}\frac{dT_L^{(R)}}{dt} = -\dot{Q}_R^{(L)} - (1-\eta)\dot{Q}_{SC}^{(MAX)}$$

LHP Linear Stability Theory

Governing Equations: $$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x})$$

where, $$\mathbf{x} \equiv \begin{pmatrix} V_C^{(2\phi)} & \dot{m}_L & T_{SAT}^{(E)} & T_{SAT}^{(R)} & T_W^{(E)} & T_W^{(R)} & T_L^{(R)} \end{pmatrix}^T$$

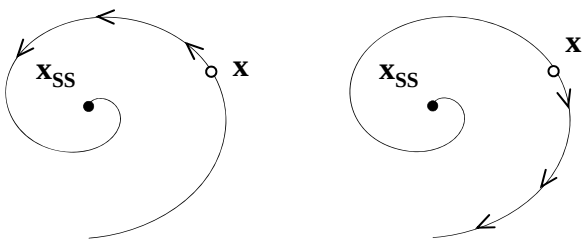
$$\mathbf{f} \equiv \begin{pmatrix} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 \end{pmatrix}^T$$

Is LHP state x capable of reaching steady state x_{SS} ?

Let x_{SS} *be the equilibrium (steady) state and* $|x - x_{SS}| < \varepsilon$

Linearized Equations: $$\frac{d}{dt}(\mathbf{x} - \mathbf{x}_{ss}) = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)_{SS} (\mathbf{x} - \mathbf{x}_{ss}) \equiv \mathbf{J}_{ss}(\mathbf{x} - \mathbf{x}_{ss})$$

$$\therefore \quad \boxed{\mathbf{x}(t) - \mathbf{x}_{ss} = \sum_k \mathbf{A}_k e^{\lambda_k t}}$$



where λ_k*'s are eigenvalues of* J_{SS}.

LHP Linear Stability Theory

Characteristic Equation: $\det(\lambda \mathbf{I} - \mathbf{J}_{ss}) = 0$

$$\therefore \quad \left\{\lambda^2 + b\lambda + c\right\} \lambda \left\{\lambda - \Omega\right\} \lambda \left\{\lambda - \Theta\right\} = 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4c}}{2} \qquad \lambda_3 = 0 \qquad \lambda_4 = \Omega \qquad \lambda_5 = 0 \qquad \lambda_6 = \Theta$$

- ***Real parts of all λ_k's are negative $\Rightarrow$ solution decays to zero $\Rightarrow$ <u>steady state</u>***
- ***At least one real part of λ_k's is positive $\Rightarrow$ solution grows exponentially $\Rightarrow$ <u>unstable operation</u>***
- ***At least one real part is zero and the rest are negative $\Rightarrow$ <u>inconclusive</u>***

LHP Instability Criteria

$$\lambda_4 = \Omega = \chi_V \frac{\partial \dot{Q}_E}{\partial T_{SAT}^{(E)}} \left\{ 1 + \left(\eta \frac{\rho_L}{\rho_V} - 2 \right) \left(\frac{\dot{Q}_2}{\dot{Q}_E} \right)_{SS} \right\} - \chi_V G_C \left\{ 1 + \eta \left(\frac{\rho_L}{\rho_V} - 1 \right) \left(\frac{\dot{Q}_2}{\dot{Q}_E} \right)_{SS} \left[1 + \left(\frac{\dot{Q}_2}{\dot{Q}_E} \right)_{SS} \right] \right\} - \chi_R G_R^{(L)} \left(1 - \frac{\partial T_L^{(R)}}{\partial T_{SAT}^{(R)}} \right)$$

$$\lambda_6 = \Theta = -\frac{1}{(Mc_P)_E} \left(\frac{\partial \dot{Q}_E}{\partial T_W^{(E)}} \right)_{SS} - \frac{1}{(Mc_P)_R^{(W)} + (Mc_P)_R^{(L)}} \frac{c_P \dot{Q}_{IN}}{\lambda}$$

$\therefore$

$$\boxed{\left(\frac{\partial \dot{Q}_E}{\partial T_{SAT}} \right)_{SS} > \frac{G_C^{(MAX)} \frac{V_C^{(2\phi)}}{V_C} \left\{ 1 + \eta \left(\frac{\rho_L}{\rho_V} - 1 \right) \left(\frac{\dot{Q}_2}{\dot{Q}_E} \right)_{SS} \left[1 + \left(\frac{\dot{Q}_2}{\dot{Q}_E} \right)_{SS} \right] \right\} + \frac{\chi_R}{\chi_V} G_R^{(L)} \left(1 - \frac{\partial T_L^{(R)}}{\partial T_{SAT}^{(R)}} \right)}{1 + \left(\eta \frac{\rho_L}{\rho_V} - 2 \right) \left(\frac{\dot{Q}_2}{\dot{Q}_E} \right)_{SS}} \equiv G_{HFLA}^{(CRIT)}}$$

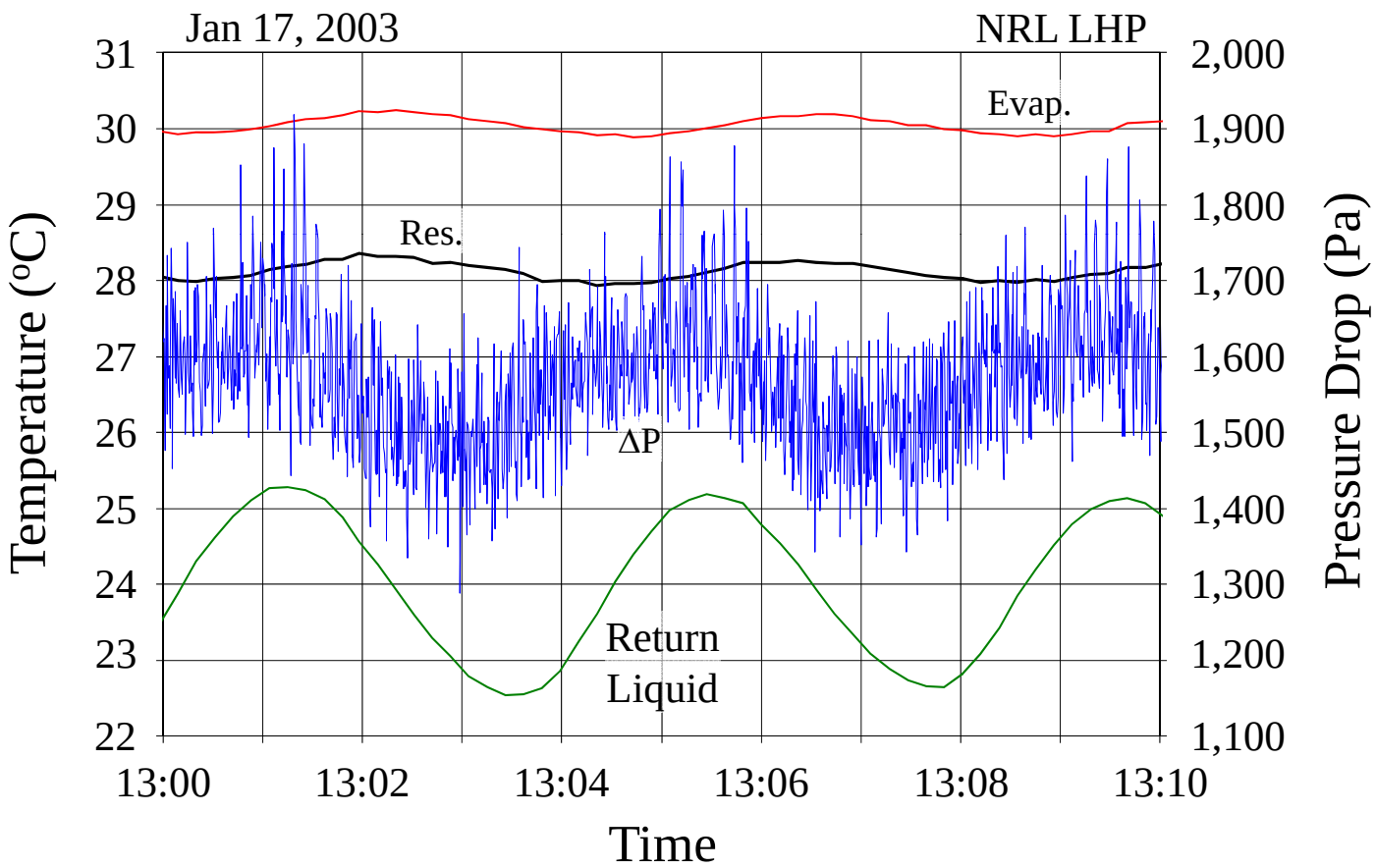
$$\boxed{\left(\frac{\partial T_W^{(E)}}{\partial \dot{Q}_E} \right)_{SS} < -\frac{(Mc_P)_R^{(W)} + (Mc_P)_R^{(L)}}{(Mc_P)_E} \frac{\lambda}{c_P} \frac{1}{\dot{Q}_{IN}} \equiv -\frac{1}{G_{LFHA}^{(CRIT)}}}$$

Multi Time Scale Dynamical System

- **Fluid Dynamic Subsystem λ_1 and λ_2**
  - **time constant τ_1 = 0.1–1.0 seconds**
  - **perturbations decay with time, i.e. $Re(\lambda_{1,2}) < 0$**
- **Thermodynamic Subsystem λ_3 and λ_4**
  - **time constant τ_2 = 1–10 minutes**
  - **$\lambda_3 = 0$ and $\lambda_4 = \Omega$**
  - **condensation of reservoir vapor causes instability**
- **Heat Transfer Subsystem λ_5 and λ_6**
  - **time constant τ_3 = 1–10 hours**
  - **$\lambda_3 = 0$ and $\lambda_4 = \Theta$**
  - **large attached thermal mass causes instability**

Very High Frequency Oscillations

Pressure Oscillation Frequency f > 1Hz



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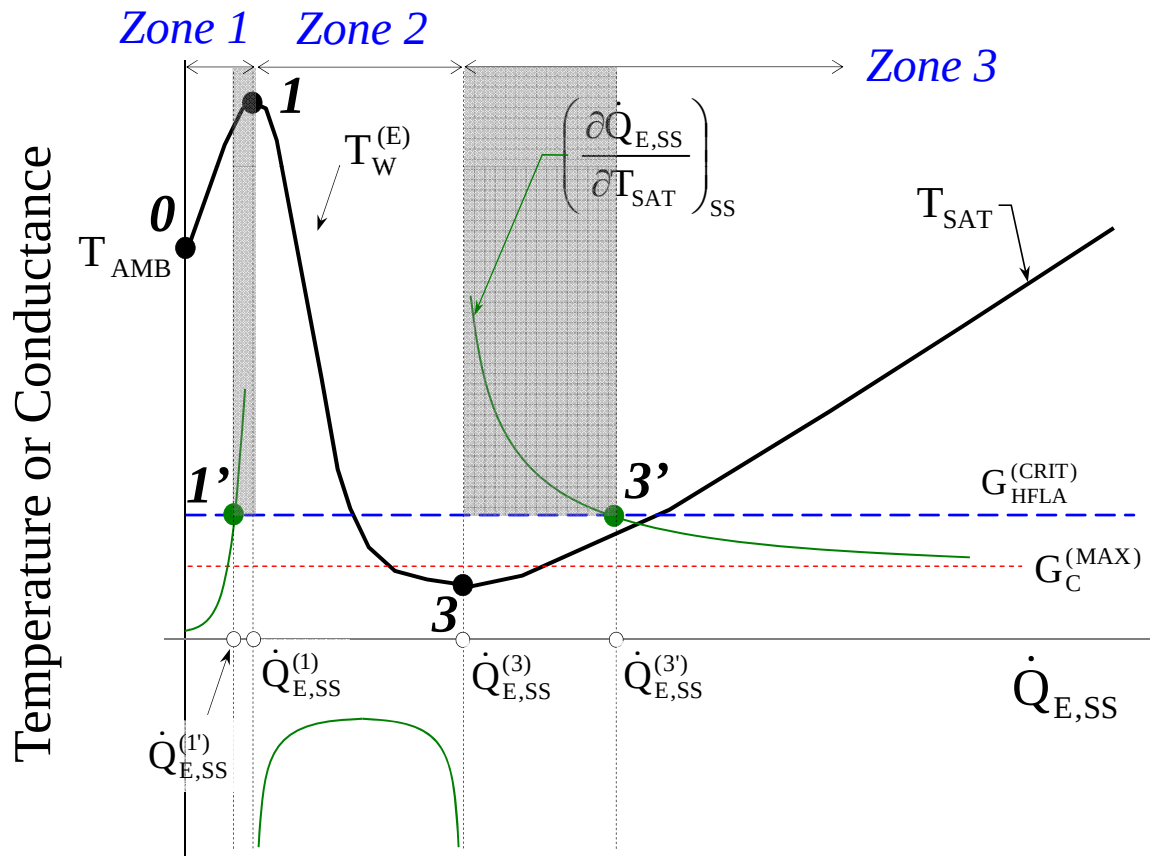
High Frequency Oscillations

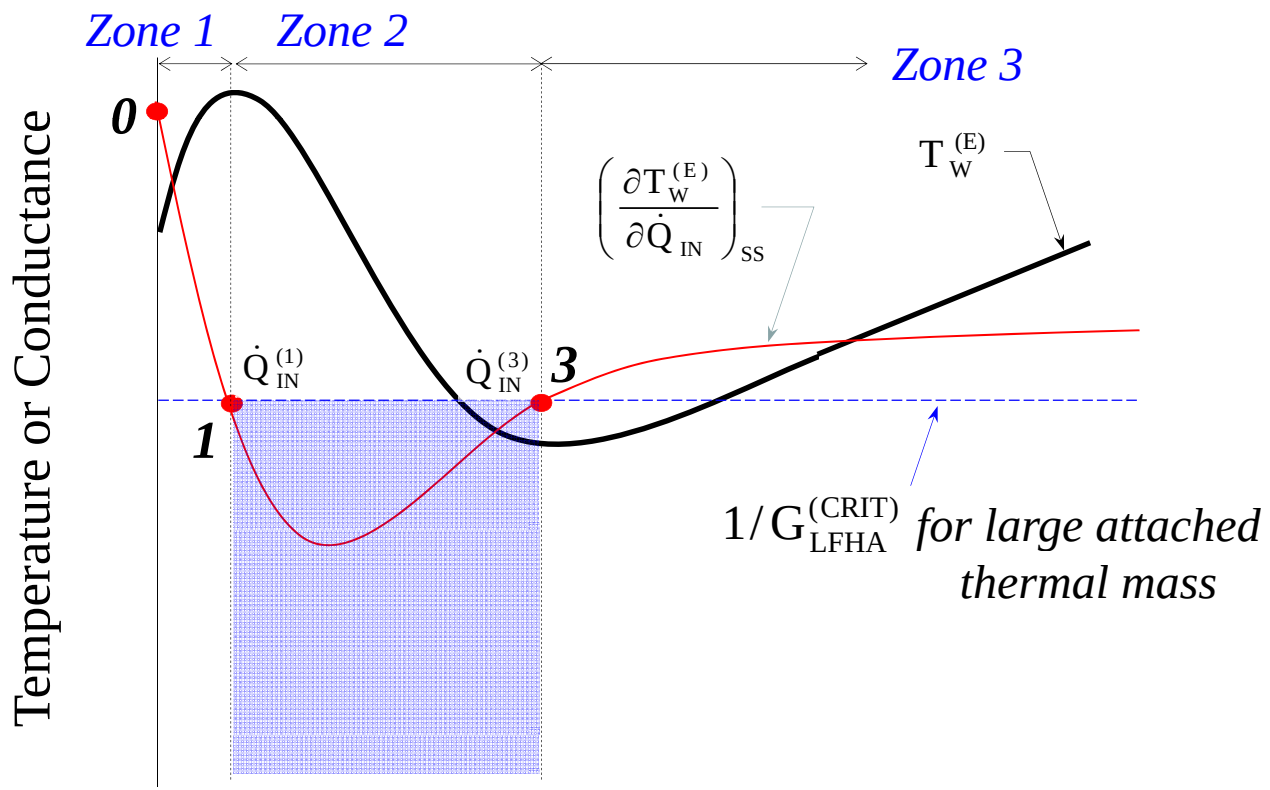
For Unstable LHP Operation: $\left(\frac{\partial \dot{Q}_E}{\partial T_{SAT}}\right)_{SS} > G_{HFLA}^{(CRIT)}$



Low Frequency Oscillations

For Unstable LHP Operation:

$$\left(\frac{\partial T_W^{(E)}}{\partial \dot{Q}_E}\right)_{SS} < -\frac{(Mc_P)_R^{(W)} + (Mc_P)_R^{(L)}}{(Mc_P)_E}\frac{\lambda}{c_P}\frac{1}{\dot{Q}_{IN}} \equiv -\frac{1}{G_{LFHA}^{(CRIT)}}$$



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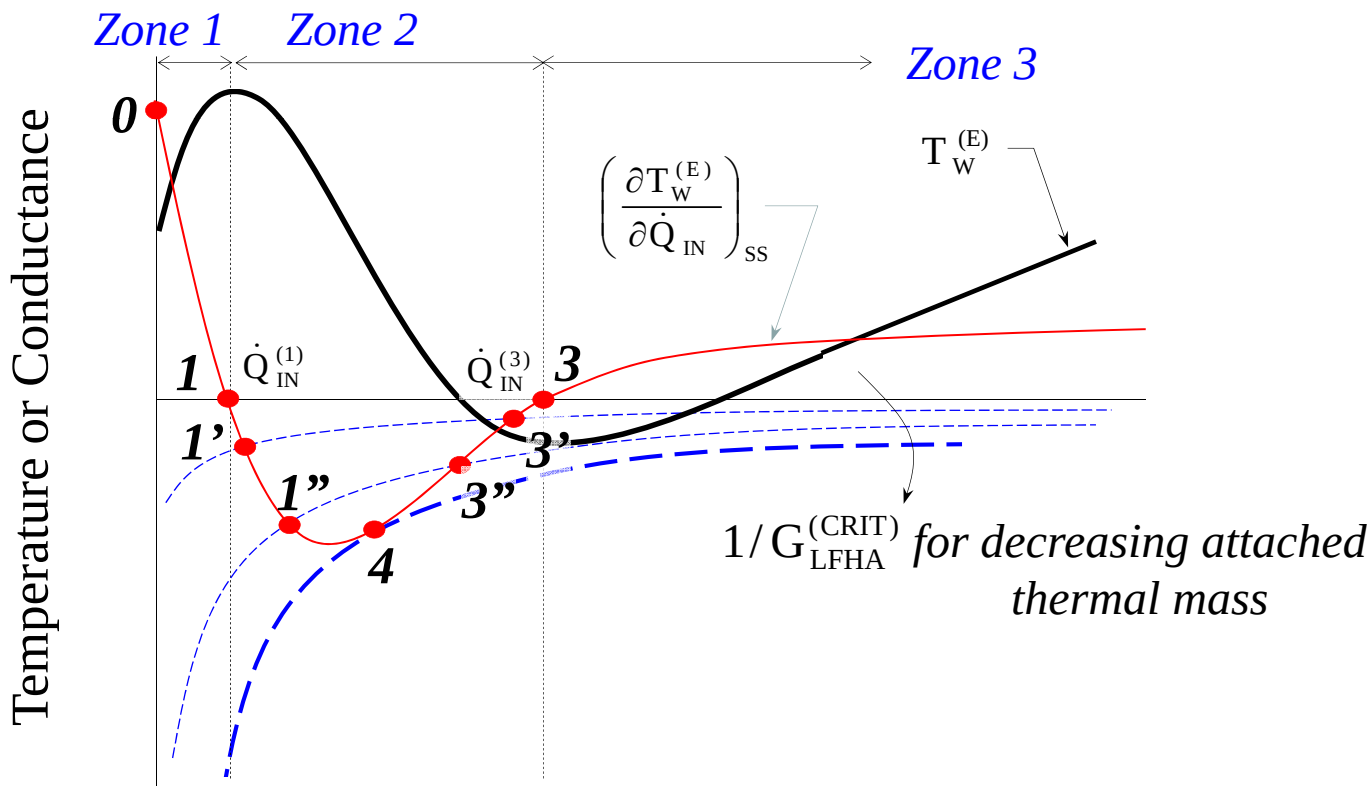
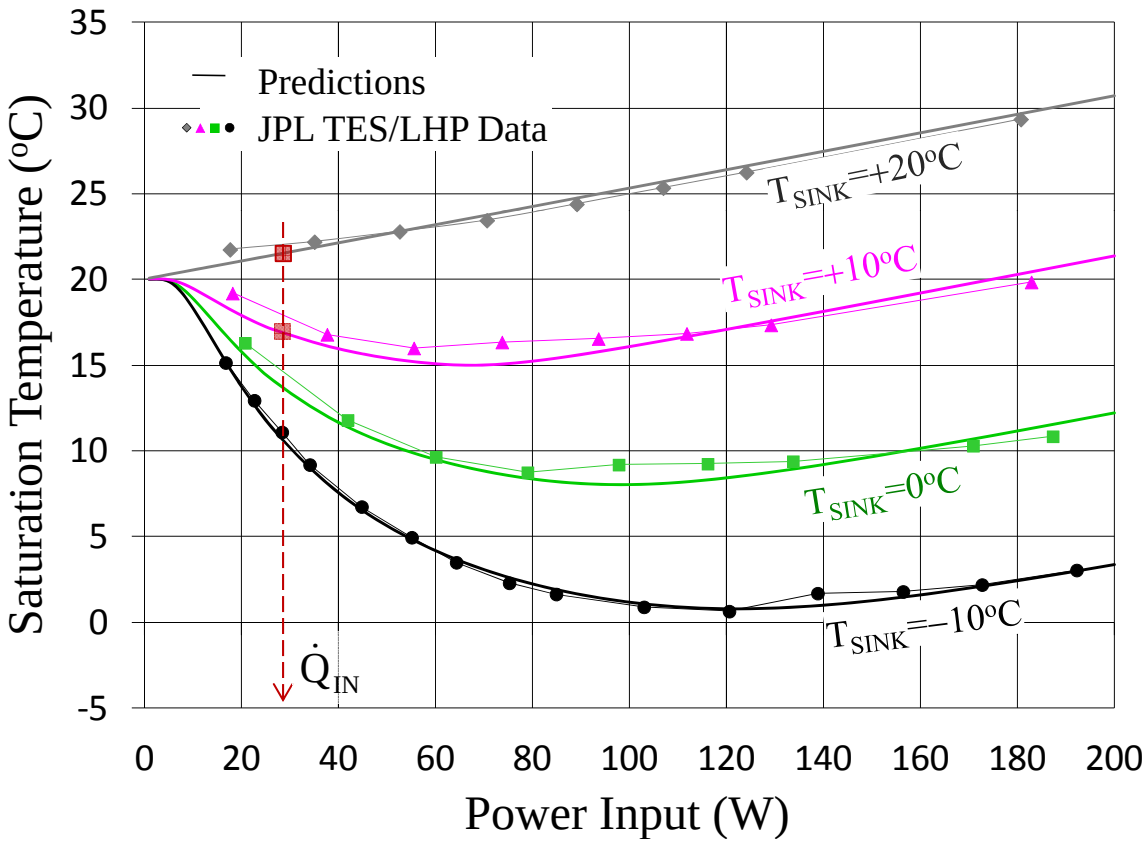


Discussion

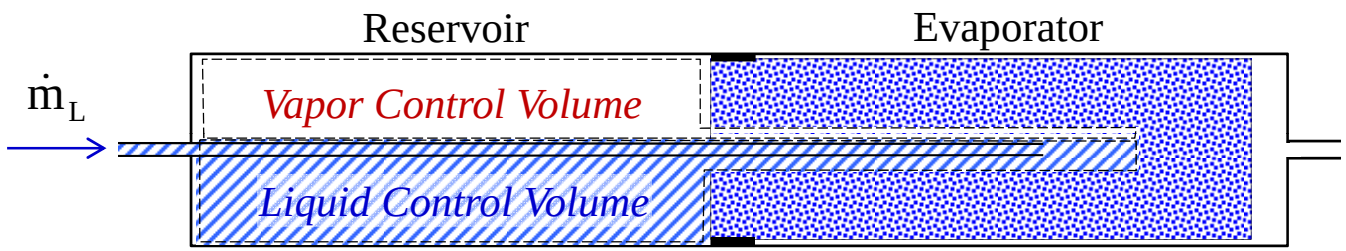
Effects of Sink Temperature on LHP Stability

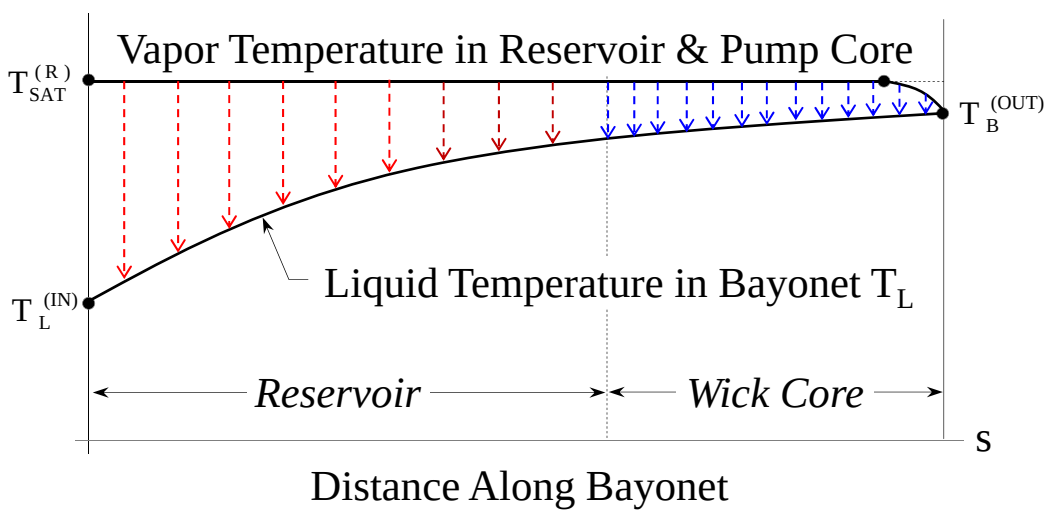


Ku, J., "Low Frequency High Amplitude Temperature Oscillations in Loop Heat Pipe Operation," Paper No. 2003-01-2388, SAE 33rd International Conference on Environmental Sciences (ICES), Vancouver, British Columbia, Canada, July 7-10, 2003.

Discussion

Effects of Bayonet Heat Exchanger on High-Frequency/Low-Amplitude Oscillations





$$\dot{Q}_{SC}^{(MAX)} \equiv \dot{m}_L c_P \left(T_{SAT} - T_L^{(IN)}\right)$$

$$\eta \equiv \frac{\dot{m} c_P \left(T_B^{(OUT)} - T_L^{(IN)}\right)}{\dot{Q}_{SC}^{(MAX)}}$$

Summary

- LHP as Dynamical System
  - three (3) synergistically interacting subsystems: fluid dynamics, thermodynamics, heat transfer processes
  - three (3) corresponding time scales: τ_1 (0.1–1 seconds), τ_2 (1–10 minutes), τ_3 (1–10 hours)
- LHP Linear Stability Theory
  - *Is LHP state* **x** *capable of reaching steady state* $\mathbf{x}_{SS}$ *?*
- Instability Criteria

$$\left(\frac{\partial \dot{Q}_E}{\partial T_{SAT}}\right)_{SS} > \frac{G_C^{(MAX)} \frac{V_C^{(2\phi)}}{V_C}\left\{1+\eta\left(\frac{\rho_L}{\rho_V}-1\right)\left(\frac{\dot{Q}_2}{\dot{Q}_E}\right)_{SS}\left[1+\left(\frac{\dot{Q}_2}{\dot{Q}_E}\right)_{SS}\right]\right\}+\frac{\chi_R}{\chi_V}G_R^{(L)}\left(1-\frac{\partial T_L^{(R)}}{\partial T_{SAT}^{(R)}}\right)}{1+\left(\eta\frac{\rho_L}{\rho_V}-2\right)\left(\frac{\dot{Q}_2}{\dot{Q}_E}\right)_{SS}} \equiv G_{HFLA}^{(CRIT)}$$

$$\left(\frac{\partial T_W^{(E)}}{\partial \dot{Q}_E}\right)_{SS} < -\frac{(Mc_P)_R^{(W)}+(Mc_P)_R^{(L)}}{(Mc_P)_E}\frac{\lambda}{c_P}\frac{1}{\dot{Q}_{IN}} \equiv -\frac{1}{G_{LFHA}^{(CRIT)}}$$

Path Forward

Verify LHP stability theory against available test data