Verification of LHP Stability Theory Part I

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Outline

• Loop Heat Pipe (LHP) Stability Theory
• Instability Criterion for Low-Frequency Oscillations
• Theory Verification Process
• Model Predictions vs. Test Data
• Summary / Conclusion
Loop Heat Pipe

LHP is a capillary-pumped device having no mechanical moving part.
Is LHP state $x$ capable of reaching steady state $x_{SS}$?

Instability Criterion for Low-Frequency/High-Amplitude Oscillations

$$\frac{\partial T_W^{(E)}}{\partial \dot{Q}_E}_{SS} < - \frac{(M_c_p)_R^{(W)}}{(M_c_p)_E} + \frac{(M_c_p)_R^{(L)}}{(M_c_p)_E} \frac{\lambda}{c_p} \dot{Q}_{IN} \equiv - \frac{1}{G_{LFHA}^{(CRIT)}}$$
**LHP Linear Stability Theory**

Is LHP state \( x \) capable of reaching steady state \( x_{SS} \)?

\[
x(t) - x_{SS} = \sum_k A_k e^{\lambda_k t}
\]

**Instability Criterion for Low-Frequency/High-Amplitude Oscillations**

\[
\left( \frac{\partial T_W^{(E)}}{\partial \dot{Q}_E} \right)_{SS} < - \left( \frac{(Mc_p)_R^{(W)}}{(Mc_p)_E} \right) R \frac{\lambda}{c_p \dot{Q}_{IN}} = - \frac{1}{G_{LFHA}^{(CRIT)}}
\]
For Unstable LHP Operation:

\[
\left( \frac{\partial T_W^{(E)}}{\partial \dot{Q}_E} \right)_{SS} < - \left( \frac{(Mc_p)_R^{(W)}}{(Mc_p)_E} + \frac{(Mc_p)_R^{(L)}}{(Mc_p)_E} \right) \frac{\lambda}{c_p} \frac{1}{\dot{Q}_{IN}} \equiv - \frac{1}{G_{LFHA}^{(CRIT)}}
\]

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**Criterion for Low Frequency Oscillations**

For large attached thermal mass
Criterion for Low Frequency Oscillations

For Unstable LHP Operation:

\[
\left( \frac{\partial T_W^{(E)}}{\partial Q_E} \right)_{SS} < - \frac{(Mc_p)_R^{(W)}}{(Mc_p)_E} + \frac{(Mc_p)_R^{(L)}}{(Mc_p)_E} \frac{\lambda}{c_p} \frac{1}{\dot{Q}_{IN}} \equiv - \frac{1}{G_{LFHA}^{(CRIT)}}
\]

Zone 1 Zone 2 Zone 3

Temperature or Conductance

\[\dot{Q}_{IN}^{(1)} \quad \dot{Q}_{IN}^{(3)}\]

\[T_W^{(E)} \quad \left( \frac{\partial T_W^{(E)}}{\partial Q_{IN}} \right)_{SS}\]

\[-1/G_{LFHA}^{(CRIT)} \text{ for decreasing thermal mass}\]
Criterion for Low Frequency Oscillations

For Unstable LHP Operation:

\[
\frac{\partial T_W^{(E)}}{\partial \dot{Q}_E} \bigg|_{SS} < - \frac{(Mc_p)_R^{(W)} + (Mc_p)_R^{(L)}}{(Mc_p)_E} \lambda \frac{1}{c_p \dot{Q}_I} \equiv - \frac{1}{G_{LFHA}^{(CRIT)}}
\]

Temperature or Conductance

Zone 1

Zone 2

Zone 3

\[\dot{Q}_I^{(1)} \quad \dot{Q}_I^{(3)} \quad 1 \quad \dot{Q}_I^{(3)} \quad \dot{Q}_I^{(1)}\]

\[-1/G_{LFHA}^{(CRIT)} \text{ for decreasing thermal mass}\]
Criterion for Low Frequency Oscillations

For Unstable LHP Operation:

$$\left( \frac{\partial T_W^{(E)}}{\partial \dot{Q}_E} \right)_{SS} < \left( \frac{(Mc_p)_R^{(W)}}{(Mc_p)_E} + \frac{(Mc_p)_R^{(L)}}{(Mc_p)_E} \right) \frac{\lambda}{c_p} \frac{1}{\dot{Q}_{IN}} \equiv -\frac{1}{G_{LFHA}^{(CRIT)}}$$

For Unstable LHP Operation:

$$\left( \frac{\partial T_W^{(E)}}{\partial \dot{Q}_E} \right)_{SS} < \left( \frac{(Mc_p)_R^{(W)}}{(Mc_p)_E} + \frac{(Mc_p)_R^{(L)}}{(Mc_p)_E} \right) \frac{\lambda}{c_p} \frac{1}{\dot{Q}_{IN}} \equiv -\frac{1}{G_{LFHA}^{(CRIT)}}$$

Temperature or Conductance

Zone 1

Zone 2

Zone 3

0

1

1

$\dot{Q}_1$ (IN)

$\dot{Q}_3$ (IN)

$-1/G_{LFHA}^{(CRIT)}$ for decreasing thermal mass
Tests with Heater Block

With Large Attached Thermal Mass

## Dimensions and Properties of LHP Components

### Evaporator

**Primary Wick**
- **Material:** Sintered Powder Nickel
- **Outer Diameter:** 24.21mm (0.950”)
- **Inner Diameter:** 9.525mm (0.375”)
- **Active Length:** 0.1524m (6”)
- **Max. Pore Radius:** 1.2µm
- **Permeability:** $4.0 \times 10^{-14} m^2$
- **Effective Conductivity:** $7.8W/m-K$

### Casing/Saddle, 1st Wick, and Attached Thermal Mass
- **Attached Thermal Mass:** 9,080J/K
- **Thermal Mass-to-Vapor Conductance $G_{E}$:** 8.16 W/K
- **Saddle:** 7.62cm x 15.24cm x 1.91cm Al 6061

### Vapor Grooves
- **Number of Channels:** 4
- **Hydraulic Diameter:** 0.05”

### Transport Lines

**Vapor Line**
- **Outer Diameter:** 5.54mm
- **Wall Thickness:** 0.508mm
- **Length:** 1.0m

**Liquid Line**
- **Outer Diameter:** 5.54mm
- **Wall Thickness:** 0.508mm
- **Length:** 1.2264m (incl. bayonet)

### Condenser

**Number of Parallel Passes:** 1

**Heat Exchanger Tubing**
- **Inner Diameter:** 3.99mm
- **Length:** 3.81m (200”)
- **Conductance $G_{C}^{(MAX)}$:** 25W/K

### Reservoir
- **Outer Diameter:** 43.94mm
- **Wall Thickness:** 2.20mm
- **Active Length:** 0.08023m
- **Thermal Mass ($M_{C_P}$):** 190J/K
- **Conductance $G_{R}$:** 22W/K
JPL TES LHP Steady State (w/o Large Thermal Mass)

- Predictions
- JPL TES/LHP Data

Power Input (W)

Saturation Temperature (°C)

$T_{\text{SINK}} = -10^\circ \text{C}$
LHP Saturation Temperature

**JPL TES LHP Steady State (w/o Large Thermal Mass)**

- **Predictions**
- **JPL TES/LHP Data**

Saturation Temperature (°C) vs. Power Input (W)

- $T_{SINK}=+20^\circ C$
- $T_{SINK}=+10^\circ C$
- $T_{SINK}=0^\circ C$
- $T_{SINK}=-10^\circ C$
- $T_{SINK}=-20^\circ C$
- $T_{SINK}=-30^\circ C$
Thermal Mass Temperature

JPL TES LHP Steady State (w/o Large Thermal Mass)

Power Input (W)

Thermal Mass Temperature (°C)

T_{SINK} = +20°C
T_{SINK} = +12.5°C
T_{SINK} = +10°C
T_{SINK} = 0°C
T_{SINK} = -10°C
T_{SINK} = -20°C
T_{SINK} = -30°C
Predicted Instability Map

JPL TES LHP

Predicted Region of Instability

Power Input (W)

Sink Temperature (°C)
Predictions vs. Test Data

JPL TES LHP

Power Input (W)

Sink Temperature (°C)

Test Without Oscillations

Predicted Region of Instability
Predictions vs. Test Data

**JPL TES LHP**

- **Test Without Oscillations**
- **Test With Oscillations**

**Predicted Region of Instability**

- **Power Input (W)**
- **Sink Temperature (°C)**

Graph showing the relationship between power input and sink temperature for JPL TES LHP tests with and without oscillations.
Transient Model

Governing Equations for Large Attached Thermal Mass

\[
\dot{Q}_E = \dot{Q}_1 + \dot{Q}_2 \quad \text{and} \quad \dot{Q}_1 - \dot{Q}_c^{(2\phi)} = 0 \quad \text{and} \quad -\dot{Q}_{sc} + \dot{Q}_2 = 0
\]  

\[
\frac{dT_W^{(E)}}{dt} = \frac{\dot{Q}_{in} - \dot{Q}_E}{(Mc_p)_E} \quad \frac{dT_{sat}}{dt} = \frac{\dot{Q}_2 - \dot{Q}_1 \left( \frac{c_p}{\lambda} \right) (T_{sat} - T_L^{(IN)})}{\left( (Mc_p)_R^{(W)} + (Mc_p)_R^{(L)} \right)}
\]  

Solution of Eq. (1):

\[
T_{sat} = f_1(\dot{Q}_E) \quad \dot{Q}_1 = f_2(\dot{Q}_E) \quad \dot{Q}_2 = f_3(\dot{Q}_E) \quad T_L^{(IN)} = f_4(\dot{Q}_E)
\]

Integrate Eq. (2) with Runge-Kutta-Fehlberg (RKF45) method to obtain \( T_W^{(E)} \) and \( \dot{Q}_E \) as functions of time \( t \).
Predictions vs. Test Data

01/24/2000

$T_{\text{SINK}} = -10^\circ\text{C}$

Elapsed Time (hours)

Temperature ($^\circ\text{C}$) or Power (W)

Evaporator (Data)

Reservoir (Data)

Reservoir (Model)

Evaporator (Model)

Power Input
**Predictions vs. Test Data**

**Graph 1:**
- **Evaporator (Model):** 15W/20W/30W Power Input
- **Heat Input:** T\(_{\text{SINK}} = -10^\circ\text{C}\)
- **Temperature (°C):**
  - Elapsed Time (hours): 0, 5, 10, 15, 20, 25, 30
  - Power Input (W): 0, 10, 20, 30, 40, 50, 60, 70, 80

**Graph 2:**
- **02/14/2000**
- **Evaporator (Data)**
- **Reservoir (Data)**
- **Heat Input:** T\(_{\text{SINK}} = -10^\circ\text{C}\)
- **Temperature (°C):**
  - Elapsed Time (hours): 0, 4, 8, 12, 16, 20, 24, 28
  - Power Input (W): 0, 10, 20, 30, 40, 50, 60, 70, 80

**Graph 3:**
- **Tesla/EDU Data for T\(_{\text{SINK}} = -10^\circ\text{C}\)**
- **Peak-to-Peak Amplitude (°C):**
  - Temperature (°C): 0, 10, 20, 30
  - Power Input (W): 0, 10, 20, 30, 40, 50, 60, 70, 80

**Graph 4:**
- **Tesla/EDU Data for T\(_{\text{SINK}} = -20^\circ\text{C}\)**
- **Peak-to-Peak Amplitude (°C):**
  - Temperature (°C): 0, 10, 20, 30
  - Power Input (W): 0, 10, 20, 30, 40, 50, 60, 70, 80

**Graph 5:**
- **Tesla/EDU Data for T\(_{\text{SINK}} = -30^\circ\text{C}\)**
- **Peak-to-Peak Amplitude (°C):**
  - Temperature (°C): 0, 10, 20, 30
  - Power Input (W): 0, 10, 20, 30, 40, 50, 60, 70, 80
Conclusion / Summary

• LHP Linear Stability Theory
  – low-frequency/high-amplitude oscillations are caused by large thermal mass ratio between evaporator and reservoir
  – unstable range of power input is between those of two extrema (Points 1 & 2) of $T_W^{(E)}$ vs. $Q_{IN}$ curve, i.e. dependent on environmental heating of liquid line and reservoir
  – theory is limited to single-pass condenser LHPs

• Theory Verification
  – model predictions agreed very well with JPL TES test data
  – unfortunately, test data from only one LHP are available

• Path Forward
  – continue with verification when additional data available
  – add multiple parallel evaporators/condensers to theory