A Comparison of Geometric Discretization Methods

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Background

• Thermal analyses often require a system-level model
  – Quick evaluation of the overall system
  – Interactions between components
  – Boundary conditions for component-level models

• System-level models should
  – Adequately represent components
    • Accurate mass drives transient solution accuracy
    • Accurate area drives convection and radiation accuracy
  – Run quickly for evaluating design space or design changes
  – Correlate to test data

• This presentation will focus on discretization methods appropriate for system-level models
  – Compare models created with various discretization methods
  – Evaluate the strengths and weaknesses of each method
Discretization Methods

• Finite Difference
  – Geometry defined using geometric primitive shapes

• Flat Finite Elements
  – Structured or unstructured meshes define geometry shape
  – Curved geometry is faceted, requiring many elements

• Curved Elements
  – Curved geometry is accurately represented using few elements
  – Tessellated and exact options for radiation calculations
    • Tessellated subdivides curved surface elements using facets with area correction factors
    • Exact uses precise geometric representation
Conduction and Radiation Model

- Reaction wheel with thermal strap
- Conduction and radiation boundary conditions
- Radiation*
  - Minimum 10k rays per node
  - 1% statistical error
  - Maximum 1M rays per node
- Transient thermal solution

* Not typical values; purposefully over-resolved
Reaction Wheel Models with ~500 Nodes

- Finite Difference: 477 nodes
- Flat Elements: 533 nodes
- Curved Elements: 533 nodes

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Flat elements underestimate model mass

- **Node Count**: The number of nodes in the model.
- **Error**: The percentage difference between the model's mass and the actual mass.

**Legend**:
- Red: Finite Difference
- Blue: Flat Elements
- Green: Curved Element - Tessellated
- Black: Curved Element - Exact
Minimizing node count is important to solution speed.
Reaction Wheel Radk Calculation Time

Calculation time (s)

- Finite Difference
- Flat Elements
- Curved Element - Tessellated

<1% mass error
Reaction Wheel Solution Time

Solution time (s)

- Finite Difference
- Flat Elements
- Curved Element - Tessellated

<1% mass error
Reaction Wheel Discussion

• Geometry accuracy
  – Finite difference and curved elements provide accurate mass and surface area at all model sizes
  – Flat elements require more nodes for mass and surface area accuracy

• Calculation time
  – Flat element model must be increased in size to improve mass accuracy
    • Decreases efficiency of the model
  – Solution times are dependent on node count
    • Solutions may be repeated many times
    • Smaller models are better
  – The exact method for curved elements is not shown
    • It is computationally more expensive but only needed for special situations (discussed later)

• Conclusion
  – Finite difference and curved elements are the better options
    • Curved elements allow arbitrary geometry
• Parabolic trough
  – Source surface emitting parallel rays
  – Black-body collector tube at trough focus
  – 1 million rays from source

• Reflection must be precise
  – All radiation should be absorbed by collector
    • $Bij_{space}$ represents poor reflection of rays
    • Special case that requires precise reflections
Parabolic Trough with 10 Nodes

Finite Difference

Flat Elements

Curved Element - Tessellated

Curved Elements - Exact
Precision Radiation Model Discussion

• Curved elements with exact radiation and finite difference are intrinsically accurate regardless of model size

• Flat elements and tessellated curved elements can get the correct answer, however…
  – Flat elements require more nodes
  – Tessellated curved elements require more nodes and/or tessellations
  – Trial and error required to find the model that gives the “correct” answer
    • Multiple runs for trial and error increase the cost
    • Not all models have a predetermined answer: what is “correct”? 
  – Increased node count will increase solution time

• Not all geometries can be represented by finite difference objects
Compound Paraboloid

- Otherwise known as Winston cone
  - Radiator enhancer and shade
  - Solar concentrator
- Accurate representation requires curved elements or *many* flat elements
Odd-shaped Mirrors
## Discretization Method Comparison

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<th>Method</th>
<th>Strengths</th>
<th>Weaknesses</th>
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<td>Finite Difference</td>
<td>• Extremely low node count possible</td>
<td>• Limited shapes</td>
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<tr>
<td></td>
<td>• Accurate geometry</td>
<td></td>
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<td></td>
<td>• Precise radiation with few nodes</td>
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<tr>
<td></td>
<td>• Fast radiation calculations</td>
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<tr>
<td>Finite Element</td>
<td>• Arbitrary shapes</td>
<td>• Requires many nodes to represent curvature</td>
</tr>
<tr>
<td>Curved Element</td>
<td>• Arbitrary shapes</td>
<td>• Requires many nodes count or tessellations for precise reflections from curved surfaces</td>
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<td>• Tessellated radiation</td>
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Conclusions

• **Use finite difference objects**
  – For system-level models when geometry can be represented with provided geometric primitives
  – Early in design process when CAD geometry or access to a direct modeler (such as SpaceClaim) is not available

• **Use curved elements**
  – For system-level models with arbitrary geometry
  – Early in the design process along with a direct modeler for concept designs
  – With tessellation option when precise radiation is not required
  – With exact option for optics or concentrators

• **Use flat finite elements**
  – For arbitrary geometry
    • Without curvature
    • When high node count is required for temperature gradients