Bayesian-based Simulation Model Validation for Spacecraft Thermal Systems

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Presentation Overview

• Introduction
  – Background and Motivation
  – Literature Review
  – Research Goal

• Bayesian-based Model Validation (BMV) Methodology
  – Methodology Overview
  – REXIS Solar X-ray Monitor (SXM) Case Study

• Conclusion
  – Primary Contributions
  – Recommendations for Future Work
  – Acknowledgements
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• **Bayesian-based Model Validation (BMV) Methodology**
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Motivation

How effective are current model validation practices?

- Literature review of flight temperatures vs. model predictions
- Thermal systems are successful but:
  - Overdesign w.r.t. stacked worst case scenarios
  - Occasional model inaccuracies

Improve thermal model validation process to reduce *form*-related and *process*-related costs long term
Bayesian-based Model Validation (BMV) Motivation

- **Potential to increase knowledge of the system earlier in the project lifecycle when important design decisions are made.**

- **Graphical representation:**
  - **Knowledge of System**
  - **Conventional**
  - **BMV**
  - **Design Freedom**

- **Key points:**
  - Identify and reduce uncertainty in critical system parameter(s) earlier through BMV.
  - Large increase in system knowledge at the time of an important design decision.

- **Project Lifecycle Time**

- **Legend:**
  - $\star$ = design decision
A Conventional Model Validation Approach

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Cold Case Value</th>
<th>Hot Case Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_N$</td>
<td>4.4</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Thermal Vacuum Chamber

Test/verification Standards

Thermal Balance

Test Data

$T(t)$

Requirement

$T(t)$

Requirement

Correlate to threshold value!

Standardized design margins!
Literature Review Summary and Research Goal

<table>
<thead>
<tr>
<th>Area</th>
<th>State of the Art</th>
<th>Thermal Convention</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uncertainty Propagation (UP)</strong></td>
<td>Probabilistic uncertainty characterization; UA/GSA [13,14]</td>
<td>Convex uncertainty characterization; margin “downstream” of model [15-17]</td>
</tr>
<tr>
<td><strong>Model Calibration</strong></td>
<td>Bayesian [24-27] (K-O approach [22,23])</td>
<td>Manual model correlation [1]</td>
</tr>
</tbody>
</table>

**Research Gap**
In practice, the state of the art methods are used rarely and in limited capacity
- No existing framework to combine state of the art methods for thermal systems

**Research Goal**
Improve the thermal model validation process by developing a tailored methodology that combines the state of the art validation methods of Uncertainty Quantification (UQ) and Design of Experiments (DOE).
Presentation Overview

- **Introduction**
  - Background and Motivation
  - Literature Review
  - Research Goal and Thesis Objectives

- **Bayesian-based Model Validation (BMV) Methodology**
  - Methodology Overview
  - REXIS Solar X-ray Monitor (SXM) Case Study

- **Conclusion**
  - Primary Contributions
  - Recommendations for Future Work
  - Acknowledgements
Visualization of Proposed Methodology

Methodology rigorously quantifies and manages model uncertainties throughout model validation process!

Uncertainty and Global Sensitivity Analysis

Goals

- Model (95% PI)
- Experimental

Methods

- $T(t)$
- $p(x_1)$
- $\text{Var}(T)$

$d_1$

$p(x_1)$

$p(x_N)$

$d_3$

$z_1$, $z_2$, $z_3$

$x_n$, $x_1$, $x_2$
**Figure 2-1: BMV methodology overview.**

Blue = analyses
Orange = decision
Red = hardware required

**Bayesian-based Model Validation (BMV)**

1. **Validation Problem Definition**
2. **UP and Parameter Prioritization**
   - Uncertainty analysis
   - Global sensitivity analysis
   - Prioritized parameter list
3. **Experimental Goal Setting**
4. **Design and Implementation of Experiments**
   - Thermal model adjusted for experimental conditions
   - Experimental data
5. **Experimental Model Calibration and Flight Model Updates**
   - Updated flight models
   - Quantified model inadequacy
   - Small design changes
6. **Validation Problem Documentation**

**Design**

Redesign
REXIS Solar X-ray Monitor (SXM) Case Study
REgolith X-ray Imaging Spectrometer (REXIS)

- One of five payload instruments on OSIRIS-REx
- Complements and enhances other science instruments on OSIRIS-REx
  - Characterizes Bennu among known meteorite groups and map surface elemental distribution

Two assemblies: spectrometer and Solar X-ray monitor (SXM). SXM observes time-variant solar X-ray spectrum to provide context to spectrometer measurements.
- Five node lumped parameter model
- 38 total parameters
- 18 uncertain parameters

Model Structure

<table>
<thead>
<tr>
<th>Deep Space</th>
<th>Sun</th>
<th>O-REx</th>
<th>1 – Bracket</th>
<th>2 – SXM Housing</th>
<th>3 – SEB</th>
<th>4 – SDD Housing</th>
<th>5 – Collimator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ y = \eta_{SX M}(x) \]

Lumped parameter model provides \( y(x,t) \), temperatures versus time for each node

What is max allowable \( T_{O-REx} \)?
SXM Thermal Requirements

At least 99% probability that all temperature ranges are satisfied

Three quantities of interest (QoIs) for SXM – all operational component temperature ranges

<table>
<thead>
<tr>
<th>Component</th>
<th>Operational (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
</tr>
<tr>
<td>SDD Housing</td>
<td>-40</td>
</tr>
<tr>
<td>SEB</td>
<td>-40</td>
</tr>
<tr>
<td>SDD</td>
<td>-100</td>
</tr>
</tbody>
</table>

SDD = silicon drift detector
SEB = SXM electronics board
Summary of SXM Case Study

Blue = analyses
Orange = decision
Red = decision warranted

- Three QoIs
- What is max allowable $T_{O-REx}$?

<table>
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<th>Component</th>
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<tr>
<td></td>
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</tr>
<tr>
<td>SDD</td>
<td>-100</td>
</tr>
</tbody>
</table>

Finally, an aluminum collimator fastened to the SXM housing restricts the field of view of the SDD so that the majority of the beam falls on the SDD. The primary heat path from the TEC is through the base of the SDD housing to the SXM housing.

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Uncertainty Analysis

Monte Carlo (MC) Simulation

\[ \bar{\eta}_{SXM,N} = \frac{1}{N} \sum_{i=1}^{N} \eta_{SXM}(x_i) \]

- \( x \) contains all system and environmental parameters
- Parametric uncertainty results in uncertainty in max allowable \( T_{O-Rex} \)

- SDD temperature is driving QoI
- Max allowable \( T_{O-Rex} \) is 63 °C

**Graph:**
- Y-axis: Probability of satisfying temperature limits
- X-axis: Spacecraft Interface Temperature, \( T_{O-Rex} \) (°C)
- Lines for different components:
  - SDD
  - SEB
  - SDD housing
  - Nominal \( T_{O-Rex} \) Value
  - 99% Probability

40 45 50 55 60 65 70 75 80 85 90 95

0.2 0.4 0.6 0.8 1.0

1.0
Global Sensitivity Analysis

Main Effects Sensitivities for $T_{O-REx} = 85 \, ^\circ C$

**Sobol’ Indices**

\[ S_j = \frac{V_j}{V[Q]} = \frac{V[E[Q|x_j]]}{V[Q]} \]  

Main effects

\[ S_{Tj} = 1 - \frac{E[V[Q|x_{\sim j}]]}{V[Q]} \]  

Total effects

**Conductance between SDD housing and SXM housing, $G_h$, is driving uncertain parameter for SDD and SDD housing temperatures**
Summary of SXM Case Study

Bayesian-based Model Validation (BMV)

- Uncertainty analysis
- Global sensitivity analysis
- Prioritized parameter list

1. Validation Problem Definition
2. UP and Parameter Prioritization
3. Experimental Goal Setting
4. Small Redesign
5. Experimental Goal
6. Validation Based on BMV

- Blue = analyses
- Orange = decision
- Red = hardware required

- SDD temperature = driving QoI
- SDD temperature below 99% at $T_{O-REx} = 63 \, ^\circ C$
- GSA indicates $G_h$ parameter as the primary global sensitivity
Experimental Goal Setting

- Uncertainty analysis
- Global sensitivity analysis
- Prioritized parameter list

Two types of experiments will be implemented:
- Parameter inference experiment to reduce uncertainty in $G_h$
- Model validation experiment to validate SXM thermal model
Summary of SXM Case Study

Bayesian-based Model Validation (BMV)

Two types of experiments:
- Parameter inference for $G_h$
- System-level model validation

Experimental Model Calibration and Flight Model Updates

Experimental Goal Setting

Design and Implementation of Experiments

Validation Problem Documentation

Validated Models

Redesign

1. Validation Problem

Blue = analyses
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Two types of experiments:
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Model Validation Experiment

**Parameter inference experiment for G_h**

- **Full-factorial experiment** (classical DOE)
- Small system time constant
- All test phases completed to steady state conditions

Validation experiment designed to span domain of expected TEC voltages and SXM interface temperatures
Parameter Inference Experiment

**Nomenclature**
- \( x \): all model parameters
- \( \theta \): parameter(s) of interest, \( \theta = G_h \)
- \( d \): experimental conditions,
  \[ d = [T_{\text{O-REx}}, V_{\text{TEC}}]^T \]
- \( z \): experimental result/data

**Table of Experimental Design Conditions, \( d \)**

<table>
<thead>
<tr>
<th>Name</th>
<th>Variable</th>
<th>DOE Variable</th>
<th>Units</th>
<th>Nominal Value</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature of O-REx Deck</td>
<td>( T_{\text{O-REx}} )</td>
<td>( d_1 )</td>
<td>°C</td>
<td>40</td>
<td>-100</td>
<td>75</td>
</tr>
<tr>
<td>TEC Voltage</td>
<td>( V_{\text{TEC}} )</td>
<td>( d_2 )</td>
<td>( V_{\text{DC}} )</td>
<td>3.0</td>
<td>0</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Two experimental design conditions varied to create different parameter inference experiments for \( G_h \)
Parameter Inference Experiment

$d^*$ is $T_{O-REx} = -30 \, ^\circ C$, $V_{TEC} = 4.0 \, V$, and $T_w = 23 \, ^\circ C$
Sample Experimental Results

- 43 of 45 test phases completed to steady state
- Final recorded temperature is shown (very small observation error variance)

![Graph showing SDD temperature vs. TEC heat load](image)

- Max expected TEC power is <2.0 W
- Preliminary thermal model has good predictive accuracy for SDD temperature
Summary of SXM Case Study

Bayesian-based Model Validation (BMV)

1. Validation Problem Definition
2. UP and Preparation
3. Experimental Goal Setting
4. Design and Implementation of Experiments
5. Validation
6. Model Resolution

Blue = analyses
Orange = decision
Red = hardware required

- Uncertainty analysis
- Global sensitivity analysis
- Prioritized parameter list

Thermal model adjusted for experimental conditions
Experimental data
Updated flight models
Quantified model inadequacy
Small design changes
Experimental Goal Setting
Experimental Goal
Experimental Model
Calibration and Flight Model Updates
Validated Models
Redesign
Design and Implementation of Experiments

Summary of SXM Case Study
General process for calibration of model parameters and quantifying the model inadequacy
Markov Chain Monte Carlo Results

Posterior parameter distributions yield acceptable fit to all data.

On average, difference between model prediction and data is less than 1 °C.

$$|\Delta T_{avg}| = \frac{1}{P} \sum_{i=1}^{P} |E[\eta_{SXM}(x, d_i)] - z_i|$$

Posterior Predictive Check for all 43 Test Phases

"Bracket" = SXM Housing
"SDD Housing" = Collimator
"SDD" = SDD Component
Summary of SXM Case Study

- MCMC to calibrate parameters
- Persisting model discrepancy <1 °C, on average
Uncertainty Analysis

Monte Carlo (MC) Simulation

\[ \overline{\eta}_{SXM,N} = \frac{1}{N} \sum_{i=1}^{N} \eta_{SXM}(x_i) \]

- \( x \) contains all system and environmental parameters
- Parametric uncertainty results in uncertainty in max allowable \( T_{O-REx} \)

- SDD temperature is driving QoI
- Max allowable \( T_{O-REx} \) is 63 °C

![Graph showing the relationship between probability of satisfying component temperature limits and spacecraft interface temperature. The graph includes lines for SDD, SEB, SDD housing, and nominal \( T_{O-REx} \) value, with 99% probability indicated.]
Updated Uncertainty Analysis

\[
\zeta_{SXM}(x) = \eta_{SXM}(x) + \delta(d) \quad \text{True physical process, } \zeta_{SXM}
\]

\[
\bar{\zeta}_{SXM,N} = \frac{1}{N} \sum_{i=1}^{N} \zeta_{SXM}(x_i) \quad \text{Monte Carlo simulation of true physical process}
\]

- SDD temperature is still driving QoI
- Max allowable \(T_{O-REx}\) is 60.8 °C

Low uncertainty in max allowable \(T_{O-REx}\)
Summary of SXM Case Study

Bayesian-based Model Validation (BMV)

1. Validation Problem Definition
2. UP and Parameter Prioritization
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4. Experimental Model Calibration and Flight Model Updates
5. Small Redesign
6. Validated Models

• Max allowable $T_{O-REx}$ is 60.8 °C
• Low uncertainty in max allowable $T_{O-REx}$

Blue = analyses
Orange = decision
Red = hardware required
BMV Motivation – SXM Case Study

- G_h = critical system parameter
- Post-calibration: reduced uncertainty in G_h, G_s,b, G_b
- Quantified model discrepancy

Knowledge of System

- Conservative, conventional approach set T_O-REx to 50 °C
- BMV: max temperature could have been up to 10 °C warmer

Design Freedom

- All requirements met for T_O-REx ≤ 60.8 °C

Potential to increase knowledge of the system earlier in the project lifecycle when important design decisions are made
Conclusion

• Application of state of the art model uncertainty methods for thermal systems
• Created BMV methodology using state of the art UQ and DOE
• Implemented BMV on REXIS hardware
  – System level form and validation process improvements
• Future work:
  – Demonstrate BMV on larger, more complex thermal systems
  – Improve BMV interface with Thermal Desktop
  – Create databases of parameter uncertainty distributions


References – Literature Review


Backup Slides
Posterior Sampling Formulation

**Calibration Parameters**

\[ \gamma = [G_h, G_{s,b}, G_b]^T \]

**Bayes’ Theorem**

\[ p(\gamma|z, x) = \frac{p(z|\gamma, x)p(\gamma)}{p(z|x)} \]

**Metropolis-Hastings Algorithm [28,29], method for Markov Chain Monte Carlo (MCMC)**

1. Draw proposal, \( \gamma_{new} \), from \( q(\gamma_{new}|\gamma_{old}) \)

2. Calculate acceptance ratio:

\[ \alpha(\gamma_{old}, \gamma_{new}) = \left[ 1, \frac{\pi(\gamma_{new})q(\gamma_{old}|\gamma_{new})}{\pi(\gamma_{old})q(\gamma_{new}|\gamma_{old})} \right] \]

3. Set the next value in the chain:

\[ \gamma_{n+1} = \begin{cases} 
\gamma_{new} \text{ with probability } \alpha(\gamma_{old}, \gamma_{new}) \\
\gamma_{old} \text{ with probability } 1 - \alpha(\gamma_{old}, \gamma_{new}) 
\end{cases} \]

where \( \pi(\gamma) = p(\gamma|z, d) \propto p(z|\gamma, d)p(\gamma) \)

**Bayesian inference**: given the test data, MCMC is used to sample the posterior distributions of the calibration parameters
• **Calibration parameters:** $\gamma = [G_h \ G_{s,b} \ G_b]^T$
• All other parameters in $\mathbf{x}$ are fixed
Posterior Predictive Check for T36

Bracket

SXM Housing

SDD Housing

Collimator

Model discrepancy function improves model accuracy (all data plausible under model output)
Model Discrepancy Formulation

- Kennedy-O’Hagan formulation [22], additive model discrepancy
- Gaussian Process (GP) models
- Squared Exponential ARD covariance kernel

\[ \delta(d) \sim \mathcal{GP}(m(d), k(d, d')) = \mathcal{GP}(0, k(d, d')) \]

Zero-mean Gaussian Process, each discrepancy term is an independent function

\[ k(d, d') = \sigma_0^2 \exp \left\{ -\frac{1}{2} \left( \frac{V_{TEC} - V_{TEC}'}{\lambda_1} \right)^2 + -\frac{1}{2} \left( \frac{T_{O-REx} - T_{O-REx}'}{\lambda_2} \right)^2 \right\} \]

\[ \delta(d) = \begin{pmatrix} \delta_b \\ \delta_{sxm,h} \\ \delta_{sdd,h} \\ \delta_{coll} \\ \delta_{sdd} \end{pmatrix} \]

5x1 vector corresponding to measurements on 5 SXM components

\[ \delta(x) = z - \eta_{SXM}(x, \gamma) - \epsilon_m \]

discrepancy experimental observations calibrate d model observation error

function of d only

GP models used to quantify the calibrated model discrepancy for all 43 test phases
Calibration Parameter Selection

Prior Predictive Check Sequence

- **G_h Only**
  - SDD & SDD housing plausible

- **G_h and G_s,b**
  - SDD & SDD housing plausible

- **G_h and G_s,b, Relaxed Lower Bound**
  - SDD, SDD housing, SXM housing, & collimator plausible

- **G_h, G_b, and G_s,b, Relaxed Lower Bound**
  - Data for all components plausible

**Calibration parameters:** \( \gamma = [G_h \ G_{s,b} \ G_b]^T \)

All other parameters in \( \mathbf{x} \) are fixed
GP Model Regression Results

By inspection, regressed GP model mean is a good approximation of the mean of the discrepancy samples.
Simulation Model Validation

Process of confirming a model is an adequate representation of the system and is capable of predicting the system’s behavior accurately with respect to requirements over the domain of the intended application of the model [3,4]
**Background – Thermal Simulation Models**

### Parameters

- System component geometry, connectivity, and material properties
- Power dissipation of spacecraft components

### General Spacecraft Thermal Environment

- **Sun**
- **Solar Radiation**
- **Albedo**
- **Radiation to space**
- **Planet**

### Model

#### General Heat Transfer Equation

\[ c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q(T, t) \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Density</td>
</tr>
<tr>
<td>( c_p )</td>
<td>Specific heat</td>
</tr>
<tr>
<td>( k )</td>
<td>Conductivity</td>
</tr>
<tr>
<td>( Q(T,t) )</td>
<td>Source heat</td>
</tr>
</tbody>
</table>

#### Thermal Analysis Tools

- Analytical models
- Lumped parameter models
- Commercially available software packages
  - TSS/SINDA and Thermal Desktop (Finite Difference)

#### Increasing fidelity

### Output

Predictions for spacecraft component temperatures for a given operational mode and thermal environment.
What is max allowable $T_{O-REx}$?

- Thermoelectric cooler (TEC) to cool SDD
- Conduction dominates
- Thermally coupled to OSIRIS-REx interface, $T_{O-REx}$
- Nominally, $T_{O-REx} = 50$ °C
Example model modifications
- Include sensor/observation error, $\varepsilon_m$
- Thermal vacuum wall temperature is external radiation sink
- No sunlight
- TEC not software-controlled

Nomenclature
- $\mathbf{x}$: all model parameters
- $\theta$: parameter(s) of interest, $\theta = G_h$
- $\mathbf{d}$: experimental conditions,
  - $\mathbf{d} = [T_{O-REx}, V_{TEC}, T_w]^T$
- $\mathbf{z}$: experimental result/data

SSL Thermal Vacuum Chamber

SSL chamber used for both parameter inference and model validation experiments
The Kullback-Leibler (KL) divergence utility function:

\[ u(d, z, \theta) = D_{KL}(p(\theta | z, d) || p(\theta)) \]

inserted into Lindley’s expected experimental utility form [21]:

\[ U(d) = E[D_{KL}(p(\theta | z, d) || p(\theta))] \]

\[ U(d) \approx \frac{1}{n_{out}} \sum_{i=1}^{n_{out}} \left( ln[p(z_i | \theta_i, d)] - ln[p(z_i | d)] \right) \]

\[ p(z_i | d) \approx \frac{1}{n_{in}} \sum_{j=1}^{n_{in}} p(z_i | \theta_{i,j}, d) \]

\[ d^* = \max_{d \in D} U(d) \]
Background – Bayesian Probability

- **Interpretation of probability**: instead of quantifying “frequency” or “propensity,” a Bayesian probability is a quantity defining a state of knowledge

- **Bayesian inference**
  - Given new information, the probability is updated via Bayes’ Theorem

- **Broadly applicable to many engineering disciplines**
  - “Natural” fit to many engineering problems
  - “Common sense” interpretation of statistical conclusions

---

**Bayes’ Theorem**

\[
p(\gamma | z, x) = \frac{p(z | \gamma, x)p(\gamma)}{p(z | x)}
\]

---

**Comparison of Probability Interpretations**

<table>
<thead>
<tr>
<th>Frequentists</th>
<th>Bayesians</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probabilities represent long term frequencies of repeatable random experiments</td>
<td>Probabilities describe the incomplete knowledge of a fixed parameter or quantity</td>
</tr>
<tr>
<td>Data are repeatable, random sample</td>
<td>Data observed from realized sample</td>
</tr>
<tr>
<td>Unknown parameters are constant</td>
<td>Parameters are unknown and described probabilistically</td>
</tr>
</tbody>
</table>

Motivation – Evidence* (Welch 2006)

- Revisited military standards for uncertainty margin
- Examined variety of programs, e.g. military, NASA, and ESA programs

<table>
<thead>
<tr>
<th>Flight Program</th>
<th>Model vs. Flight Temperature Difference $\mu \pm 2\sigma$ (°C)</th>
<th>Derived Thermal Uncertainty Margin (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOD Program A</td>
<td>$+5.9 \pm 10.0$</td>
<td>15.9</td>
</tr>
<tr>
<td>DOD Program B</td>
<td>$+1.3 \pm 8.4$</td>
<td>9.7</td>
</tr>
<tr>
<td>Iridium</td>
<td>$-3.3 \pm 11.9$</td>
<td>15.2</td>
</tr>
<tr>
<td>NASA TIMED</td>
<td>$+4.3 \pm 11.2$ (cold) – $-13.5 \pm 15.6$ (hot)</td>
<td>15.5 – 29.1</td>
</tr>
<tr>
<td>DOD Program C</td>
<td>$+6.6 \pm 9.0$</td>
<td>15.6</td>
</tr>
<tr>
<td>DOD Program D</td>
<td>$+0.5 \pm 10.0$</td>
<td>10.5</td>
</tr>
<tr>
<td>ESA Italsat-1</td>
<td>$+2.2 \pm 7.8$</td>
<td>10.0</td>
</tr>
<tr>
<td>ESA Italsat-2</td>
<td>$-1.5 \pm 7.7$</td>
<td>9.2</td>
</tr>
<tr>
<td>ESA SAX</td>
<td>$-3.1 \pm 6.6$</td>
<td>9.7</td>
</tr>
</tbody>
</table>

Red = hot cases, Blue = cold cases

(a) NASA TIMED
- Very biased thermal model
- Intermediate environments significantly more benign than worst-case hot scenario

(b) DOD Program D
- Very little model bias, i.e. mean near zero
- Large variance about hot/cold case mean

*All data, tables, and figures from Welch [11]
**Motivation – Evidence** (Karpati et al. 2012)

**Flight temperatures vs. model predictions for seven recent GSFC missions**

- Nearly all worst hot case predicted temperatures greater than those observed
  - Results agree with Welch [11] and Peabody, et al. [12]
  - Evidence that stacked worst case scenarios have low likelihood/frequency

- Estimated that the 5 °C NASA uncertainty margin [17] will result in radiator mass growth between 0.3-0.7 kg per 100 W heat load
  - Radiator growth leads to power draw increase of 4-6 W per 100 W heat load for survival heaters

---

*All data from Karpati, et al. [5]

**Daily/orbit max temperatures polled for 209 sensors for entire life of missions.

---
Uncertainty Propagation (UP)

**State of the Art**

**Uncertainty Propagation Process [13]**
- Goal setting
- Model selection and documentation
  - Surrogate modeling
- Uncertainty classification
- Uncertainty characterization
- Uncertainty Analysis (UA)
- Sensitivity Analysis (SA)

**Thermal Convention**

Most programs follow the philosophy in NASA GOLD Rules [17]:

**Rule**: Use model to show adequate margin between component temperature limits and stacked worst case temperature predictions.

**Rationale**: Positive margins account for uncertainties in power dissipations, environments, and thermal system parameters.

**Global SA concept**

*Stacked worse case scenarios [1]*
- Heat loads
- Coating degradations
- Power dissipations
- Beta angles
- Critical conductances
- MLI e*
## State of the Art

### Classical DOE
- Ronald Fisher [18,19]
  - Est. null hypothesis
- Principles of DOE
  - Randomization, blocking, replication, orthogonality
- No unified strategy and predefined experiments for general system

### Optimal Bayesian Experimental Design (OBED)
**Culminate to Huan and Marzouk** [21]:
- Update prior parameter distributions to reduce uncertainty
- Framework allows for *different experimental goals*, e.g. parameter inference
- Measure utility based on experimental result
  - Utility function based on predictive variance or parameter of interest, e.g. Kullback-Leibler divergence

Bayesian statistics offer inference from noisy, indirect, and incomplete data.

## Design of Experiments (DOE)

### Thermal Convention
- Models validated through thermal balance testing
- Classical DOE approach (same testing philosophy):
  - NASA – GEVS [17]
  - Other, e.g. universities

**What cases and how** a system should be tested to achieve model validation
- Test levels
- Environmental conditions
- Duration

**Min Requirement:** Two test conditions shall be imposed: one each at mission hot and cold case. NASA engineers shall select one additional case, per GEVS.

**Primary Objective:** Validate the design/model, which will be used to make predictions for the entire range of modes/mission environments.
**Model Calibration**

**State of the Art**

**Parameter optimizations**
- Cullimore [#]
- Masterson [#]

**Bayesian Calibration**

Seminal Paper – Kennedy and O’Hagan [22]
- General Bayesian calibration framework
- Non-linear, black box models
- Captures all parametric and non-parametric uncertainties
- Model inadequacy quantified after experiment

**K-O Approach Enhancements**
- Brynjarsdottir and O’Hagan [#]
  - Model the model inadequacy
- Higdon et al. [#]
  - High dimensional output
- Bayarri et al. [#]
  - Model validation framework

**Thermal Convention**

Correlation process outlined by Gilmore [1] followed for most space-based thermal systems:

1. Configure model based on environment and power modes tested

2. For a single test phase, adjust model to match data. Common adjustments to the model include:
   - Physical model omissions, i.e. model inadequacy
   - View factor geometries
   - Conductances
   - Power dissipations

3. Correlate all temperature differences between model and test data to less than some threshold value, e.g. ±3°C per MIL-HDBK-340 [16]

4. Repeat 2-3 for the remaining test phases, ensuring that changes made in each remaining phase do not undo the correlation from a previous phase

**Ad hoc search for best fitting model parameters: relies heavily on engineering experience and intuition.**
Model Formulation

Lumped Parameter Concept [1]

![Lumped Parameter Diagram]

Model Formulation

\[ y = \eta_{SXM}(x) \]

where the three QoIs are identified in the output \( Q \subset y \) and \( Q = [T_h, T_{pa}, T_{sdd}]^T \)

\[
\frac{dT}{dt} = f(T, t) \quad \text{where the nodal temperatures are} \quad T = [T_1, T_2, \ldots, T_n]^T
\]

Simplifying Assumptions:

- Heterogeneous material globally, but the material assigned to each node is homogeneous
- All SXM material is isotropic
- All material within a nodal region is isothermal

\[
f(T, t) = C^{-1}[GT + Q(T, t)]
\]

\[
T_j(t_{i+1}) = \Delta t \left. \frac{dT}{dt} \right|_{T_j(t_i)} + T_j(t_i)
\]

Select \( \Delta t \) such that solver is stable and has acceptable error

\[
C = \begin{pmatrix}
m_{1,1}c_{p1,1} & 0 & \cdots & 0 \\
0 & m_{2,2}c_{p2,2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & m_{n,n}c_{p3,3}
\end{pmatrix}
\]

\[
G = \begin{pmatrix}
G_{1,1} & G_{1,2} & \cdots & G_{1,n} \\
G_{2,1} & G_{2,2} & \cdots & G_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
G_{n,1} & G_{n,2} & \cdots & G_{n,n}
\end{pmatrix}
\]
Thermoelectric Cooler (TEC)

\[ T_h \xrightarrow{T_{TEC}} \text{TEC Model} \xrightarrow{T_{SDD}} i_{TEC} \]

Proportional Control to Setpoint, \( T_s \)

\[ V(t_{i+1}) = K_p e(t) = K_p (T_{sdd} - T_s) \]

- Performance estimates provided by Amptek, Inc. used to predict SDD temperature
- Parameters of polynomial curves are fixed values
- In flight, \( V_{TEC} \) will be controlled by flight software
- As the hot side temperature, \( T_h \), increases, more power is required

Thermal analysis uses TEC model and controller to focus on the ability of the TEC to achieve \( T_s = -30^\circ C \).
SXM Model Nominal Parameters

Results for Nominal Case

- 38 total parameters
  - 18 are uncertain or naturally exhibit variation
- What is meant by nominal?
  - Default design value
  - Current best estimate
  - Median parameter value

Effect of TEC proportional controller

Low thermal inertia, thus small time constant
Structure relatively isothermal

Requirements

<table>
<thead>
<tr>
<th>Component</th>
<th>Operational (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
</tr>
<tr>
<td>SDD Housing</td>
<td>-40</td>
</tr>
<tr>
<td>SEB</td>
<td>-40</td>
</tr>
<tr>
<td>SDD</td>
<td>-100</td>
</tr>
</tbody>
</table>

Nominally, all three steady-state temperature requirements are satisfied
# SXM Model Uncertain Parameters

<table>
<thead>
<tr>
<th>Parameter Number</th>
<th>Name</th>
<th>Variable</th>
<th>Units</th>
<th>Nominal Value</th>
<th>Distribution Type</th>
<th>Parameter 1 (minimum)</th>
<th>Parameter 2 (maximum)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Node Specific Heats</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1 - Bracket</td>
<td>$c_{p,1}$</td>
<td>J/kg-K</td>
<td>961</td>
<td>Uniform</td>
<td>921</td>
<td>972</td>
</tr>
<tr>
<td>2</td>
<td>2 - SXM Housing</td>
<td>$c_{p,2}$</td>
<td>J/kg-K</td>
<td>961</td>
<td>Uniform</td>
<td>921</td>
<td>972</td>
</tr>
<tr>
<td>3</td>
<td>3 - Pre-amp board</td>
<td>$c_{p,3}$</td>
<td>J/kg-K</td>
<td>800</td>
<td>Uniform</td>
<td>378</td>
<td>880</td>
</tr>
<tr>
<td>4</td>
<td>4 - SDD Housing</td>
<td>$c_{p,4}$</td>
<td>J/kg-K</td>
<td>461</td>
<td>Uniform</td>
<td>378</td>
<td>461</td>
</tr>
<tr>
<td>5</td>
<td>5 - Collimator</td>
<td>$c_{p,5}$</td>
<td>J/kg-K</td>
<td>961</td>
<td>Uniform</td>
<td>921</td>
<td>972</td>
</tr>
<tr>
<td><strong>Power Dissipations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>SDD</td>
<td>$Q_{SDD}$</td>
<td>W</td>
<td>0.01</td>
<td>Uniform</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>7</td>
<td>Pre-amp board</td>
<td>$Q_{pA}$</td>
<td>W</td>
<td>0.20</td>
<td>Uniform</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Conduction Parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Temperature of O-REx Deck</td>
<td>$T_{O-REx}$</td>
<td>°C</td>
<td>40</td>
<td>Uniform</td>
<td>-30</td>
<td>50</td>
</tr>
<tr>
<td>9</td>
<td>Conductance between O-REx and bracket</td>
<td>$G_b$</td>
<td>W/m²·°C</td>
<td>2,000</td>
<td>Uniform</td>
<td>100</td>
<td>4,000</td>
</tr>
<tr>
<td>10</td>
<td>Conductance per screw between bracket and SXM housing</td>
<td>$G_{s,b}$</td>
<td>W/C</td>
<td>0.42</td>
<td>Uniform</td>
<td>0.11</td>
<td>1.32</td>
</tr>
<tr>
<td>11</td>
<td>Conductance per screw between pre-amp and SXM housing</td>
<td>$G_{s,pa}$</td>
<td>W/C</td>
<td>0.26</td>
<td>Uniform</td>
<td>0.07</td>
<td>0.80</td>
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<tr>
<td>12</td>
<td>Conductivity of pins on SDD package</td>
<td>$k_{pins}$</td>
<td>W/m·°C</td>
<td>400</td>
<td>Uniform</td>
<td>350</td>
<td>405</td>
</tr>
<tr>
<td>13</td>
<td>Conductance between SDD housing and SXM housing</td>
<td>$G_s$</td>
<td>W/m²·°C</td>
<td>2,000</td>
<td>Uniform</td>
<td>100</td>
<td>4,000</td>
</tr>
<tr>
<td>14</td>
<td>Conductance per screw between collimator and SXM housing</td>
<td>$G_s,roll$</td>
<td>W/C</td>
<td>0.21</td>
<td>Uniform</td>
<td>0.03</td>
<td>0.42</td>
</tr>
<tr>
<td><strong>Radiation Parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Solar Flux</td>
<td>$\phi_s$</td>
<td>W/m²</td>
<td>1,367</td>
<td>Uniform</td>
<td>700</td>
<td>1,752</td>
</tr>
<tr>
<td>16</td>
<td>Collimator Absorptivity</td>
<td>$\alpha_c$</td>
<td>--</td>
<td>0.50</td>
<td>Uniform</td>
<td>0.31</td>
<td>0.60</td>
</tr>
<tr>
<td>17</td>
<td>Collimator Emissivity</td>
<td>$\varepsilon_c$</td>
<td>--</td>
<td>0.80</td>
<td>Uniform</td>
<td>0.78</td>
<td>0.82</td>
</tr>
<tr>
<td>18</td>
<td>SDD Housing Absorptivity</td>
<td>$\alpha_b$</td>
<td>--</td>
<td>0.50</td>
<td>Uniform</td>
<td>0.30</td>
<td>0.52</td>
</tr>
</tbody>
</table>
Fourier Amplitude Sensitivity Testing (FAST)

- Variance-based global sensitivity analysis method
- Can be more efficient to evaluate “main” or “total” effect sensitivity indices over other methods

\[ X_i = G_i \sin(\omega_i s) \]

Explore N-dimensional space of model parameters via search curve defined by parametric equations

\[ s \sim \text{scalar from } [-\infty, +\infty] \]
\[ G_i \sim \text{transfer function} \]
\[ \omega_i \sim \text{frequencies} \]

In classic FAST, main effects sensitivities are approximated via Fourier coefficients
Sensor Importance Study

- **Objective:** identify through analysis which temperature sensors are most important w.r.t. experimental utility

- **Procedure** can be used to answer:
  - Where to measure?
  - How accurately to measure?

- **Value in knowing sensor importance:**
  - Sensor could fail during test
  - Addition of redundant sensors for critical locations
  - Testbed may have sensor quantity restrictions
  - Planned sensor may not be possible to install on system

---

**Pearson’s Correlation Coefficient**

\[ R_{A,B} = \frac{\text{cov}(A, B)}{\sigma_A \sigma_B} \]

Will indicate whether the presence of a sensor is, on average, correlated to high utility

**Matrix of Sensor Permutations (64x6)**

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & \\
0 & 0 & 0 & 1 & 1 & \\
& & & & & & \\
1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}
\]

---

Vector of observations, \( z \), is now 5x1.

Occurred on SXM – not possible to place RTD on SXM electronics board.

SDD and SDD housing measurements are most important, on average, for realizing high experimental utility.
Experimental Results: Sample for T9

System time constant is small, allowing for many different tests

Very small $\Delta T$ between bracket and interface

Chamber baseplate: 25 °C
TEC Voltage: 4.0 V

SXM thermal time constant is approximately 10 min due to small thermal capacitance.
Prior Predictive Check (PPC): $G_h$ Only

- Propagate prior uncertainty through SXM thermal model
- All parameters have fixed values except for $G_h$
- PPC for only test phase T36 (4.0 V, -30°C)

Location of discrepancy and previous GSA suggests to repeat PPC including the uncertainty in conductance between SXM housing and bracket, $G_{s,b}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Min Value</th>
<th>Max Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_h$</td>
<td>W/m²/C</td>
<td>100</td>
<td>4,000</td>
</tr>
</tbody>
</table>
PPC: \( G_h \) and \( G_{s,b} \) Only

- Propagate prior uncertainty through SXM thermal model
- All parameters have fixed values except for \( G_h \) and \( G_{s,b} \)
- PPC for only test phase T36 (4.0 V, -30 °C)

**Current Parametric Model Uncertainty**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Min Value</th>
<th>Max Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_h )</td>
<td>W/m^2/C</td>
<td>100</td>
<td>4,000</td>
</tr>
<tr>
<td>( G_{s,b} )</td>
<td>W/C</td>
<td>0.11</td>
<td>1.32</td>
</tr>
</tbody>
</table>
- Propagate prior uncertainty through SXM thermal model
- All parameters have fixed values except for $G_h$ and $G_{s,b}$
- PPC for only test phase T36 (4.0 V, -30 °C)

Persisting *small* discrepancy in bracket suggests to repeat PPC including uncertainty in conductance between bracket and interface, $G_b$
PPC: $G_h$, $G_{s,b}$ and $G_b$ Only, Relaxed $G_{s,b}$ Lower Bound

- Propagate prior uncertainty through SXM thermal model
- All parameters have fixed values except for $G_h$, $G_{s,b}$ and $G_b$
- PPC for only test phase T36 (4.0 V, -30 °C)

### Current Parametric Model Uncertainty

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Min Value</th>
<th>Max Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_h$</td>
<td>W/m²°C</td>
<td>100</td>
<td>4,000</td>
</tr>
<tr>
<td>$G_{s,b}$</td>
<td>W/°C</td>
<td>0</td>
<td>1.32</td>
</tr>
<tr>
<td>$G_b$</td>
<td>W/m²°C</td>
<td>100</td>
<td>4,000</td>
</tr>
</tbody>
</table>

Current parametric uncertainty can explain all data for T36. Now, update parameter distributions and calibrate to all test phases.
Quantify Calibrated Model Discrepancy for SDD

- No obvious functional relationship between SDD temperature and $V_{\text{TEC}}$, $T_{O-\text{REx}}$
- Updated, empirical TEC thermal model under-predicts/over-predicts SDD temperature
  - If more accuracy were required, additional refinements to TEC model would increase predictive accuracy
- Histogram of all discrepancy samples for all 43 test cases reveals that discrepancy can be conservatively captured via Gaussian distribution

SDD discrepancy function will be stationary Gaussian distribution → conservative approach because maximum possible discrepancy variance is considered for all possible $V_{\text{TEC}}$, $T_{O-\text{REx}}$
GP model section shows variance reduction and trends in discrepancy samples well-matched with discrepancy model.

Variable voltage and $T_{O-REx}$ constant at 25 °C.

GP mean
- GP 95% confidence interval
- Discrepancy samples
- 95% confidence interval of samples
### Comparison of BMV to a Conventional Approach

<table>
<thead>
<tr>
<th>Validation Step</th>
<th>Analogous BMV Step</th>
<th>BMV</th>
<th>A Conventional Approach</th>
</tr>
</thead>
</table>
| Analysis        | 2: UP and Parameter Prioritization | • All system and environmental parameters probabilistically characterized and propagated through model for many thousands of bounding and intermediate thermal cases; all requirements satisfied for $T_{O-REx}$ up to 50 °C  
• Global sensitivity analysis uses information within model to rigorously, systematically identify critical system sensitivities; SXM conductance $G_h$ is critical sensitivity | • Likely only two analysis cases, corresponding to worst-case hot and cold operational scenarios  
• Identification of critical system sensitivity up to individual engineer; often manual local sensitivity analysis; heavy reliance on experience/intuition |
| Test            | 4: Design and Implementation of Experiments | • Parameter inference experiment to maximize information gain in $G_h$ at $V_{TEC} = 4.0$ V and $T_{O-REx} = -30$ °C  
• Full factorial model validation experiment with focus on bounding important parameters of domain of intended application of SXM | • System-level thermal balance test at worst hot case, cold case, and possibly a few intermediate cases |
| Model Update    | 5: Experimental Model Calibration and Flight Model Updates | • SXM thermal model parameters were *updated* (not replaced) via systematic, Bayesian calibration approach  
• Remaining model inadequacy was quantified via Gaussian Process Models to predict inadequacy for any SXM power mode or spacecraft interface temperature | • Manual correlation or parameter optimization model update procedure  
• Differences between model predictions and experimental data are less than a threshold value (e.g., ±3 °C) |

For SXM case study, BMV led to additional information being available to the engineer at each major step of the validation process. BMV focused validation efforts to critical areas of SXM thermal system and provided a more rigorous quantification of model uncertainties before and after testing.
Importance of $T_{O-REx}$ as System Design Parameter

- Cooling the SXM interface is driving thermal system accommodation for REXIS SXM
  - SXM is nominally facing the sun
  - Need to cool the SXM interface to 50 °C with the GEVS [17] standard thermal design margin of 5 °C

- Due to the 50 °C spacecraft interface upper limit, design changes to OSIRIS-REx included:
  - Heat spreader and RTV added to interface to decrease thermal resistance across interface
  - Changes in surface coatings near the SXM to help cool the mounting structure
  - Redesign of MLI blankets near the interface to increase heat rejection from structure to cooler parts of spacecraft

- Power cycling of REXIS could be necessary if temperatures are slightly warmer than expected
  - Operational mission plan has changed since the 50 °C upper limit was set
  - Power cycling introduces risk to spectrometer detector array that would require major rework to spectrometer electronics so that detectors could remain on if SXM were power cycled

The spacecraft-SXM interface temperature, $T_{O-REx}$, is an important system design parameter. If the upper limit had been higher, some or all of the design changes and potential operational constraints would not have been necessary.
MCMC Results

**Glasgow and Kittredge** [30]: Cho-Therm 1671 (applied to $G_h$ interface) tested near its vendor-specified value of 6,700 W/m$^2$/C

Good mixing, but MCMC hitting “wall” at 4,000 W/m$^2$/C

Correlated posterior distributions: $G_{s,b}$ and $G_b$ affected by $G_h$ “wall”

Increase upper bound of $G_h$ distribution and update MCMC results