Thermal Model Development for an X-Ray Mirror Assembly

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Overview

• Purpose: Present innovations and techniques developed to overcome challenges in the thermal modeling of the mirror assembly for a future space-based x-ray telescope.

• Outline
  – Introduction: X-Ray Optics And Their Thermal Challenges
  – Topic 1: Rapidly Create and Edit Large Number Of Similar Thermal Desktop Solids
  – Topic 2: Creating Complex Geometry From Merged Thermal Desktop Solids
  – Topic 3: Use SINDA Solver To Determine Optimal Heater Set Points
  – Conclusion: Initial STOP Analysis Results
Brief Introduction To X-Ray Optics

- X-Rays Focused Using Grazing-Incidence Optics
  - Grazing Reflection Efficiency Depends on X-Ray Energy
  - High Incidence Soft X-Rays (~1 kev) will be Absorbed by Most Materials
- Future Optics Use Hundreds of Thin, Concentric Shells to Increase Effective Area
  - Each Shell Includes Primary and Secondary Mirror Pair
  - 160 m² Actual Mirror Area Required for 1 m² Effective Collecting Area @ 1keV
- Focusing Goal, Point Spread Function (PSF): 5 arc-second half-power diameter
- Allocated Thermal Distortion PSF: ~1 arc-second half-power diameter
Future X-Ray Mirror Assembly

- Practical Application: Shells Divided into Segments, Mounted in Modules, and Secured in Structure
  - 135 Shells Grouped In 3 Structural Rings: 59 inner, 40 middle, 36 outer shells
  - Rings Divided Into Modules: 18 inner, 30 middle, 36 outer modules per ring
  - 7,116 Mirror Segments Total (3558 Mirror Segment Pairs)
Thermal Challenges for Future X-Ray Mirror Assembly

- Tightly Control Mirror Segments and Modules Near 20 °C to Minimize Distortion
  - Differential Thermal Expansion Distorts Mirror Segments or Changes Focus
  - Heat Flow via Pins Creates Axial Temperature “Waviness” in Mirror Segments

- Thermal Control of Mirror Assembly Complicated
  - Open to Large View Factor of Cold Space at Front
  - Open to Smaller View Factor of Cold Focal Plane at Back

- Thermal Solutions:
  - Surround Mirror Assembly with Heated Structure While Minimally (or Acceptably) Impacting PSF from Introduced Stray Light Reflections
    - Heated Stray-Light Baffles and Pre-Collimators in Front (Stray Light Impact)
    - Heated Structure on Sides (No Stray Light Impact)
  - High Thermal Conductivity for Mirror Segments and Module Structure
  - Low Thermal Conductivity for Pins
  - Best Match Thermal Expansion Coefficient for Pins, Mirrors, and Module Structure
  - Low IR Emissivity for Mirror Segments, High IR Emissivity for Structure

- Detailed Thermal Modeling Together With Structural-Optical-Thermal (STOP) Analysis Required To Verify Design Choices.
Topic 1: Method to Rapidly Build Mirror Segments for Thermal Model

- Thermal Desktop Model of Mirror Assembly with Sufficient Detail for STOP Analysis Requires Building Large Number of Mirror Segments.
- Issue: How to Build Thousands of Mirror Segments and Modify Them as Needed with Minimum Man-Hours?
- Solution: Parameterize with Thermal Desktop Symbols and Develop Tools to Create and Import the Thousands of Symbols Required.

Model for Mirror Segment Pair: 60-Degree Wedge of Mirror Segments in Modules:

- Small Nodal Area Models Contact Area of Clip for Mounting Mirror Segments (4 Places Per Segment)
Step #1: Create and Import Thermal Desktop Symbols that Parameterize Mirrors
- Prescription Data: Height, Top Radius, Base Radius, Axial Position, (Thickness)
- Additional Parameters: Azimuthal Angle, Density Factor
- Used 2160 Symbols (8 each for Primary and Secondary Segments in 135 Shells)
- Use 4-Character Suffix to Identify Mirror Segment: \{P/S\}{nnn}
Topic 1: Building Mirror Segments, Edit Symbol Names in TD Objects

- Step #2a: Create 135 blank TD Solid Cones. Use Multiple Edit Mode to populate all with default symbols and name (suffix “_P000”). Also populate common inputs (subdivisions, optical and thermophysical properties, radiation active sides).
- Step #2b: Edit each only to update the following.
  - Increment suffix of symbols and name in 9 locations to _Pnnn, nnn is shell ID.
  - Input Node Numbering Start ID as 1nnn01 (leading 1 or 2 signifies P versus S).
- Step #2c: Repeat Steps #2a and #2b for 135 Secondary Mirror Segments.
Step #3a: Create a TD Assembly and a SINDA submodel for each module in each ring.
Step #3b: Copy modules, attach to TD Assembly, and rotate in place.
Step #3c: Using Multiple Edit Mode, assign each complete module to independent SINDA submodel to prevent duplicate node numbers without requiring renumbering of nodes.
Topic 2: Develop Tool To Calculate Large Nodal Boundary Lists

- Creating Stray-Light Baffles From Separate Finite-Difference Objects Using Edge Nodes Requires Merging As Many As 59 Vanes (TD Solid Cones/Cylinders) To Radial Sides (TD Solid Bricks).
- Issue: Calculate Large Subdivision Nodal Boundary List.
- **Solution:** Develop Excel Tool To Calculate Nodal Boundary Lists Given Node Location Inputs (Shown Using Edge Nodes)

Ring 1 Stray-Light Baffles Require 60 Nodal Boundaries in List
• Thermal Desktop Places Nodes at Mid-Point of Each Pair of Interior Boundaries (plus at the edges of the object when using edge nodes).
• Inputs: Node Locations along One Dimension of a Thermal Desktop Finite-Difference Object.
• Outputs/Unknowns: Non-Dimensional Nodal Boundary Locations.
• Matrix Formulation
  o  \( N = \) total number of nodes (including the edge nodes)
  o  \( A = \) column vector of inputs, \( N - 2 \) non-dimensional node locations
  o  \( X = \) column vector of unknowns, \( N - 1 \) non-dimensional boundary locations
  o  Matrix is simple and sparse with \( N - 2 \) rows x \( N - 1 \) columns

\[
\begin{bmatrix}
0.5 & 0.5 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0.5 & 0.5
\end{bmatrix} X = A
\]
Topic 2: Nodal Boundary List Tool, Refined Problem Statement

• One Additional Input Required To Make Problem Determinate. Must Specify One Non-Dimensional Nodal Boundary Location.
• Solve By Gaussian Elimination
  – Caution: Although Unique Algebraic Solution Always Exists, Not All Sets of Node Location Inputs Yield Physically Valid Solution.
  – Validity Check: If Resulting Nodal Boundary List is Monotonic, Solution Is Physically Valid.

\[
x_i = 2a_i - x_{i-1} = a_i + (a_i - x_{i-1}) \quad \text{for } i > k
\]

\[
x_i = 2a_i+1 - x_{i+1} = a_{i+1} - (x_{i+1} - a_{i+1}) \quad \text{for } i < k
\]
### Topic 2: Nodal Boundary List Tool, Excel Tool

- Tool Builds by VBA Macro for Specified Number of Edge Nodes.
- Tool Runs with Excel Formulas and Conditional Formatting. Can be Cut and Pasted.
- Includes Prompts and Validity Checks.
  - Relative Gap Between Nodes. (Typically Specify Boundary in Smallest Gap.)
  - Position of Solved Boundaries in Node Gaps. (Indicates Quality of Solution.)
  - Complete and Valid Solutions Output In Column on Right for Easy Cut and Paste into Thermal Desktop.
- Tool Allows for Edge or Centered Nodes and Boundaries Location Inputs substituting for some Node Location Inputs

<table>
<thead>
<tr>
<th>Ring 3 Pre-Collimators: Bndry List</th>
<th>Local Information</th>
<th>Input Node Locations</th>
<th>Specified Boundary</th>
<th>Gaps: Min to Max</th>
<th>Solution (Non-Dimensional)</th>
<th>Check Node Locations</th>
<th>Bndry List</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCL_Ring3InnerHoopRatCL</td>
<td>1</td>
<td>491.299</td>
<td>0</td>
<td>44.46025</td>
<td>0.160885</td>
<td>0.160885 59% TRUE</td>
<td>1</td>
</tr>
<tr>
<td>PCL_Ring3HoopVane1RatCL</td>
<td>2</td>
<td>535.759</td>
<td>0.274201</td>
<td>554.1335</td>
<td>0.274201</td>
<td>0.387518 50% TRUE</td>
<td>2</td>
</tr>
<tr>
<td>PCL_Ring3HoopVane2RatCL</td>
<td>3</td>
<td>572.507</td>
<td>0.500835</td>
<td>38.4361</td>
<td>0.500835</td>
<td>0.614152 48% TRUE</td>
<td>3</td>
</tr>
<tr>
<td>PCL_Ring3HoopVane3RatCL</td>
<td>4</td>
<td>610.943</td>
<td>0.737883</td>
<td>42.5008</td>
<td>0.737883</td>
<td>0.861615 47% TRUE</td>
<td>4</td>
</tr>
<tr>
<td>PCL_Ring3OuterHoopRatCL</td>
<td>5</td>
<td>653.444</td>
<td>1</td>
<td>42.5008</td>
<td>0.861615</td>
<td>0.861615 47% TRUE</td>
<td>5</td>
</tr>
</tbody>
</table>
• Mirror Assembly Temperature Controlled By Heaters on Stray-Light Baffles, Thermal Pre-Collimators, and Interface Rings.
• Issue: Determine Optimal Set Point Temperatures For Heaters.
• Solution: Use SINDA Solver.

Because Geometry Or Radiation Exchange Factors Unchanged, Dynamic SINDA Not Required.
Topic 3: Optimal Set Point Temperatures, Objective Function

- Developed Logic Object Calculating Various Statistics
  - First Order Statistics: e.g. Temperature Range, Temperature Average
  - Second Order Statistics: e.g. Temperature Variance
- Report Various Measures of Merit. Pick One as the Solver Objective Function.
  - Max Value of Statistic in any Segment, Pair, Shell, Module, or Ring and in Mirror Segment Overall
  - Max RMS Value of Statistic over all Segments, Pairs, Shells, Modules, or Rings and over Mirror Segment Overall
- Most Indicative of STOP Analysis: Second Order Statistic of RMS Difference From Least Squares Fit Of Axial Temperature Profile in Mirror Segment Pair, Reported as RMS Over All Pairs

RMS Difference From Least Squares Fit

\[ y = 0.0002x + 22.694 \]

RMS Diff = 0.0203 °C
Conclusion:
Initial STOP Analysis Results

- Initial Results From STOP Analysis Using Optimization of Heater Set Point Temperatures Is Encouraging: HPD of 0.9 arc-seconds From Representative Module in Middle Ring
BACK UP
Additional Material

Topic #1
Topic 1: Building Mirror Segments, Build One Module Per Ring

- Step A: Complete one module per ring by adding module structure and conductors modeling pin connections.
- Step B: Add unique Subdivision Boundary Lists to each mirror segment, creating small nodal regions for each pin contact. (Also redefined small nodes as zero mass “arithmetic nodes” and used unique Density Factor for each mirror segment to equate mass.)

Separate Model Calculating Effective Conductance Between Module and Mirror Segments via Pin/Clip
Additional Material

Topic #2
• Step #0: Reuse Existing Module Structure and Mirror Segments.
• Step #1: Create New Symbols with Different Prefix (Change “Rx_” to “SLB_”)
  – Set Base Radii to Top Radii (Baffle Vanes are Cylinders rather than Cones)
  – Geometry Changes: Extend Azimuthal Angles To Center of Structure, Etc.
• Step #2: Copy DWG File. Highlight All “Rx_” Symbols in Symbol Manager, and Select Rename. Replace Prefix “Rx_” with “SLB_”. This Changes Symbol Names In TD Objects.
• Step #3: Import “SLB_” Symbols. Geometry updates to Stray Light Baffle.
• Step #4: Use BLOCK to Copy Stray-Light Baffles into Model DWG File.
• Step #5: Update Subdivision Nodal Boundary List in Structure Sides and Merge Nodes.

Multi-String Rename Form:
One Additional Input Required To Make Problem Determinate. Must Specify One Non-Dimensional Nodal Boundary Location.

Subscript Key
- \( N \) = total number of nodes
- \( a_i \) = elements of vector A, with i from 2 to \( N - 1 \)
- \( x_i \) = elements of vector X, with i from 1 to \((N-1)\) (where \( x_i < a_{i+1} < x_{i+1} \))
- \( x_k \) = single, specified non-dimensional boundary location

\[
\begin{bmatrix}
0.5 & 0.5 & 0 & \cdots & 0 & 0.0 \\
\vdots & \ddots & \vdots & & \vdots & \vdots \\
0.0 & 0.0 & 0 & \cdots & 0 & 0.5 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
\vdots \\
x_{(k-1)} \\
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
\vdots \\
0.5x_k \\
\end{bmatrix}
= 
\begin{bmatrix}
a_2 \\
\vdots \\
a_k \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.5 & 0 & \cdots & 0 & 0.0 & 0.0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0.0 & 0 & \cdots & 0 & 0.5 & 0.5 \\
\end{bmatrix}
\begin{bmatrix}
x_{(k+1)} \\
\vdots \\
x_{(N-1)} \\
\end{bmatrix}
+ 
\begin{bmatrix}
0.5x_k \\
\vdots \\
0 \\
\end{bmatrix}
= 
\begin{bmatrix}
a_{k+1} \\
\vdots \\
a_{N-1} \\
\end{bmatrix}
\]
Topic 2: Nodal Boundary List Tool, Algebraic Solution

• Solve By Gaussian Elimination
• Caution: Unique Algebraic Solution Exists for Any Set Of Node Location Inputs. However, Not All Sets of Node Location Inputs Provide A Physically Valid Solution.
• Validity Check: Solution Is Valid If Resulting Nodal Boundary List is Monotonic.

\[ x_i = 2a_{i+1} - x_{(i+1)} = a_{i+1} - (x_{(i+1)} - a_{i+1}) \quad \text{for } i < k \]

\[ x_i = 2a_i - x_{(i-1)} = a_i + (a_i - x_{(i-1)}) \quad \text{for } i > k \]
Additional Material
Topic #3
Topic 3: Optimal Temperature Control of Mirror Assembly

- Mirror Assembly Temperature Controlled By Heaters on Stray-Light Baffles, Thermal Pre-Collimators, and the Interface Rings at the Mirror
Topic 3: Optimal Set Point Temperatures For Heaters

- Radiant Heating from Temperature Controlled Stray-Light Baffles, Thermal Pre-Collimators, and Interface Rings Can Offset Heat Loss To Space And Control Mirror Assembly Temperature.
- However, Optimal Heater Set Point Temperatures Not Obvious.
- Issue: Determine Optimal Set Point Temperatures For Heaters.
- Solution: Use SINDA Solver.

Because Changing Set Point Temperatures Does Not Change Geometry Or Radiation Exchange Factors, Dynamic SINDA Calls To Thermal Desktop Are Not Required.

The Solver Can Efficiently Run Many Steady-State Solutions, Even For This Large Geometry.
Topic 3: Optimal Set Point Temperatures, Design Variables

- Define TD Symbols/SINDA Registers For Each Set Point
- Constrain Set Points Between 10 °C – 50 °C.
Topic 3: Optimal Set Point Temperatures, Average Temperature Constraint

- Desire Solution Providing Average Mirror Segment Temperature of 20 °C.
- Solver Struggled With This Tight Restraint. Solution Involved OPERATIONS Logic And Two Calls To Solver.
  - 1st Solver Call: Loosely Constrain Average Temperature Between 19.5 °C – 20.5 °C
  - Interim Logic: Adjust All Set Points To Equating Average Temperature To 20 °C
  - 2nd Solver Call: Re-Optimize With Tight Constraint of 20 °C In Vicinity Of 1st Solution