CFD-Thermal Interactions
Short Course, TFAWS 2003

Integrated Fluid-Thermal Analysis from a Thermal-Structures Perspective

Kim S. Bey
Metals and Thermal Structures Branch
NASA Langley Research Center
Hampton, VA
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GWU M.S. Students
James Tomey, Ford Motor Company
Christopher Lang, MTSB LaRC
David Walker, ATK Thiokol

Kim.S.Bey@nasa.gov  8/21/03
Integrated Fluid-Thermal Analysis for High Speed Flight Vehicles

- Coupling at the fluid-thermal interface depends on the type of structure: insulated or non-insulated (hot)

- “Next Generation” Thermal Analysis Methods for Hot Built-up Structures
Thermal-Structural Airframe Concepts for Reusable Launch Vehicles

Cryotank with aeroshell and insulating thermal protection system

Integrated hot structure where thermal protection also carries load
Airframe Thermal Analysis State-of-the-Art

Transient nonlinear problem
- Conduction
- Radiation exchange
- Convection

Full 3D finite element models

Through-the-thickness plug models
CFD-Thermal Interactions for Insulated Structures are (Approximately) Decoupled

Heat conducted into the structure is small

\[ q_s \approx 0 \]

\[ \downarrow \]

\[ q_{aero} \approx q_{rad} \]

Iterate the fluid energy equation at the wall boundary until

\[ q_W = k_f \frac{\partial T_f}{\partial z} \bigg|_w = \varepsilon \sigma (T_W^4 - T_\infty^4) \]

Through-the-Thickness Plug Models are Adequate for Insulated Structures since In-plane Temperature Gradients are Small
Through-the-Thickness Plug Model of Complex Metallic TPS Concept

Armor TPS panel

Cryogenic fuel

Purge cavity

Tank wall

Insulation

Sandwich

Thermally compliant sides & support

Box beam & bolt

TPS support

Bolt

q = 0 or

q = h(T - T_a)
Thermal Response Predicted with TPS-it

Heating Rate (btu/ft²-s)

Time (sec)

TPS top surface, 1

TPS bottom surface, 3

insulation mid-plane, 2

Substructure, 4

$q_w(t)$

$q_w$
Decoupled CFD-Thermal Interactions Simplify the Design Process

TPS Sizing and Material Selection are performed independently
CFD-Thermal Interactions for Hot (non-insulated) Structures are Coupled

Structure absorbs thermal energy

- Heating strongly depends on wall temperature
  \[ q_{\text{aero}} = -k_f \frac{\partial T_f}{\partial z} = q_{\text{aero}}(T_W) \]

- Wall temperature strongly depends on thermal energy absorbed by structure
  \[ T_W = T_W(q_s) \]

At the fluid-solid interface

\[
\begin{aligned}
-k_f \frac{\partial T_f}{\partial z} &= k_s \frac{\partial T_s}{\partial z} = q_w \\
T_f &= T_s = T_W
\end{aligned}
\]

Fluid: \(k_f, T_f\)

Solid: \(k_s, T_s\)
CFD-Thermal Coupling Approaches

Globally Iterative:

\[ T_f \big|_W = T_0 \rightarrow \text{CFD soln} \rightarrow q_W \rightarrow \text{Thermal soln} \rightarrow T_W = T_f \big|_W \]

Locally Iterative:

\[ T_f \big|_W = T_0 \rightarrow \text{CFD soln} \rightarrow q_W \rightarrow \text{Thermal soln} \rightarrow T_W = T_f \big|_W \]

Fully Coupled:

• Solid is a fluid with \( u=v=w=0 \)

• Cast thermal problem in conservation form, use same CFD algorithm, coupled energy equation at interface

\[
\begin{bmatrix}
u
\end{bmatrix} = \begin{bmatrix}
R
\end{bmatrix}
\]
Integrated Fluid-Thermal-Structural Analysis using Unstructured Meshes

Fully coupled analysis using Taylor-Galerkin finite element formulation

- artificial viscosity for high speed flow
- flux-based formulation for heat transfer

Structural analysis of built-up structures rarely use meshes of the “continuum” (3D elasticity equations).

Built-up structures are “modeled” with plates, shells, beams, and 3D elements.

Need thermal analogues

Dechaumphai et. al. LaRC, Circa 1990
"Next Generation" Thermal Analysis Methods for Hot Built-up Structures

Hierarchical through-the-thickness modeling

Parameter: $p_z$

p-version finite elements with discontinuous Galerkin time marching

Parameters: $h$, $p_{ip}$, $\Delta t$, $p_t$

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Why p-Version Finite Elements?

\[ T(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_p x^p \]

FV, FD, h-FE methods (algebraic convergence)

spectral, p-FE methods without pollution (exponential convergence)

hp-FE methods on good meshes

Higher accuracy for fixed number of unknowns
Fewer elements for fixed accuracy
Element shapes that reflect actual geometry
Finite Element Options for Multi-layered Plates

Conventional elements (p=1)

- Piecewise linear in (x,y)
- Piecewise linear in z

Improve accuracy by adding more elements

p-Version elements

- Polynomial in (x,y)
- Piecewise polynomial in z

Improve accuracy by increasing polynomial degree
Homogenized Through-the-Thickness Modeling of Conduction in Multi-layered Plates

**p-element**
- Polynomial in (x,y)
- Mathematically equivalent to single polynomial in z

**Hierarchical model**
- Geometrically collapsed
- Thermal “higher-order plate theory”
- Structurally compatible

+ Fewer degrees of freedom than multiple layers of p-elements
+ Good for single-layer
- Bad for multiple layers
  - Lacks convergence with increasing model order
  - Jump in the flux across material interfaces
Optimal Through-the-Thickness Modeling of Conduction in Multi-layered Plates

Basis functions are single polynomials defined piecewise by scaling the homogenized basis functions by the thermal conductivity of each layer.

Homogenized basis functions

Optimal basis functions

Same number of DOF's as homogenized hierarchical model

Converges with model order and plate thickness

Ref: Volgelius & Babuska

Kim.S.Bey@nasa.gov
Steady-State Conduction in a Two-Layer Plate

\[ q(x) = \frac{-40x(L-x)}{L^2} \]

**Exact Solution**

\[
T = \sum_{n=1}^{\infty} \sin \left( \frac{n \pi x}{L} \right) \left\{ A_n \sinh \left( \frac{n \pi z}{L} \right) + B_n \cosh \left( \frac{n \pi z}{L} \right) \right\}
\]

\[
C_n \sinh \left( \frac{n \pi z}{L} \right) + D_n \cosh \left( \frac{n \pi z}{L} \right)
\]
Convergence of Through-the-Thickness Hierarchical Models of Conduction in Two-Layer Plate
A Posteriori Error Estimation

- Error in finite element solution \( e = u - \hat{u} \)
- Global error equation

\[
- \nabla \cdot (\kappa \nabla u) = Q \\
+ \nabla \cdot (\kappa \nabla \hat{u}) - \nabla \cdot (\kappa \nabla u) = Q + \nabla \cdot (\kappa \nabla \hat{u}) \\
- \nabla \cdot (\kappa \nabla e) = Q + \nabla \cdot (\kappa \nabla \hat{u})
\]

- Can solve this global problem using the same FE approach, but this would be as computationally expensive as obtaining the solution
- Instead, solve a local problem on each element

\[
- \int_K \nabla \cdot (\kappa \nabla e) v \, d\Omega = \int_K Qv \, d\Omega + \int_K \nabla \cdot (\kappa \nabla \hat{u}) v \, d\Omega
\]
Local Problem for Element Error

- Weak formulation of element error problem
  \[ \int_K (\kappa \nabla e) \cdot \nabla v d\Omega = \int_K Qv d\Omega - \int_K (\kappa \nabla \hat{u}) \cdot \nabla v d\Omega + \int_{\partial K} (\kappa \nabla u) \cdot \vec{n} v ds \]

- Approximate boundary flux \( \hat{q} = \bar{q} + \tilde{q} \) where \( \bar{q} \) is unknown

  \( \bar{q} = \) Average flux

  \( \tilde{q} = \) Correction to equilibrate \( \hat{q} \)

Find \( \tilde{q}_K \) = polynomial of degree \( p_z \) such that

\[ \int_{\partial K} \tilde{q}_K \theta_n \, ds = \int_K (\kappa \nabla \hat{u}) \cdot \nabla \theta_n \, d\Omega - \int_K Q\theta_n \, d\Omega - \int_{\partial K} \bar{q}_K \theta_n \, ds \]

\( \theta_n = \) nodal basis functions, \( n = 1, \ldots, p_z \)

\[ [M] \{\tilde{q}\} = \{R\} \]

Ref: Ainsworth & Oden
On Each Element, Solve the Local Problem for the Estimated Error

- Approximate element error

\[
\hat{e} = \sum_{i=0}^{p_x+1} \sum_{j=0}^{p_z+1} \varphi_i(x) \psi_j(z) \varepsilon_{ij}
\]

such that

\[
\int_K (\kappa \nabla \hat{e}) \cdot \nabla v d\Omega = \int_K Qv d\Omega - \int_K (\kappa \nabla \hat{u}) \cdot \nabla v d\Omega + \int_{\partial K} \hat{q} v ds
\]

for all admissible \( v \)

\[
[K]\{e\} = \{F\}
\]

- Element error indicator

\[
\| \hat{e} \|_K = \sqrt{\int_K (\kappa \nabla \hat{e}) \cdot \nabla \hat{e}}
\]

- Global error estimate

\[
\| \hat{e} \|_\Omega = \sqrt{\sum_K \| \hat{e} \|_K^2}
\]
Performance of the Error Estimate on the Two-Layer Example
Performance of Error Estimate
Steady Conduction with Internal Heat Generation

Rough Exact Solution

\[ q \]

\[ T = 0 \quad T = 0 \]

\[ Q \]

\[ \text{insulated} \]

\[ \|e\| \]

\[ 1/h \quad \text{(number of elements)} \]

\[ p_z = 1 \]

\[ p_z = 2 \]

\[ p_z = 3 \]

\[ p_z = 4 \]

\[ p_z = 5 \]

\[ \text{actual} \]

\[ \text{estimated} \]

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Performance of Element Error Indicator
Steady Conduction with Internal Heat Generation

32 elements
$p_x=p_z=2$

actual error
Max. error
Max. gradient

$\|\|e\|\|_K$

$\|\|e\|\|=2.980$
$\|\|\dot{e}\|\|=3.248$

actual
estimate
Estimating Contributions of Hierarchical Modeling and Finite Element Error

- Hierarchical modeling error: \( e_{HM} = u - u_{HM} \)
- Finite element error: \( e_{FE} = u_{HM} - \hat{u} \)
- Total error: \( e = u - \hat{u} = e_{HM} + e_{FE} \)

\[
\| e \|^2 = \| e_{HM} \|^2 + \| e_{FE} \|^2
\]

- Solve local problem twice

\[
\hat{e}_{HM} = \sum_{i=0}^{p_x} \sum_{j=0}^{p_z+1} \varphi_i(x) \psi_j(z) c_{ij}
\]

\[
\hat{e}_{FE} = \sum_{i=0}^{p_x+1} \sum_{j=0}^{p_z} \varphi_i(x) \psi_j(z) d_{ij}
\]
Performance of Estimated Error Contributions

\[ \|e\| = \sum_{p=1}^{5} \begin{cases} \text{estimated model error} & \text{for } p_z = 1, 2, 3 \\text{estimated FE error} & \text{for } p_z = 4, 5 \end{cases} \]

1/\(h\) (number of elements)
Error Estimates are Sufficiently Accurate to Drive an Adaptive Strategy
Concluding Remarks

• Examples shown here were constructed to have exact solutions to study behavior of the solution method and error estimates.

• Similar approach has been used with same success for transient 3D linear conduction using 2D elements and steady-state 3D nonlinear conduction.

• Can homogenization using p-version finite elements be used to accurately represent the thermal effects of all the internal structure on the surface?