Introduction to On-Orbit Thermal Environments

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Introduction

This lesson provides an introduction to On-Orbit Thermal Environments for those unfamiliar with this subject and will also serve as a refresher for practitioners of thermal analysis.

Overview of Natural Thermal Environments

Main focus will be on naturally occurring heating sources driven by solar heating, reflected solar heating and infrared sources;

Some focus will be given to free molecular heating;

Charged particle heating will be briefly discussed.

Scope of this Lesson

Radiation heat transfer;

Basic orbit mechanics;

Derivation of the solar, albedo and planetary infrared heating components;

Beta angle and eclipse effects;

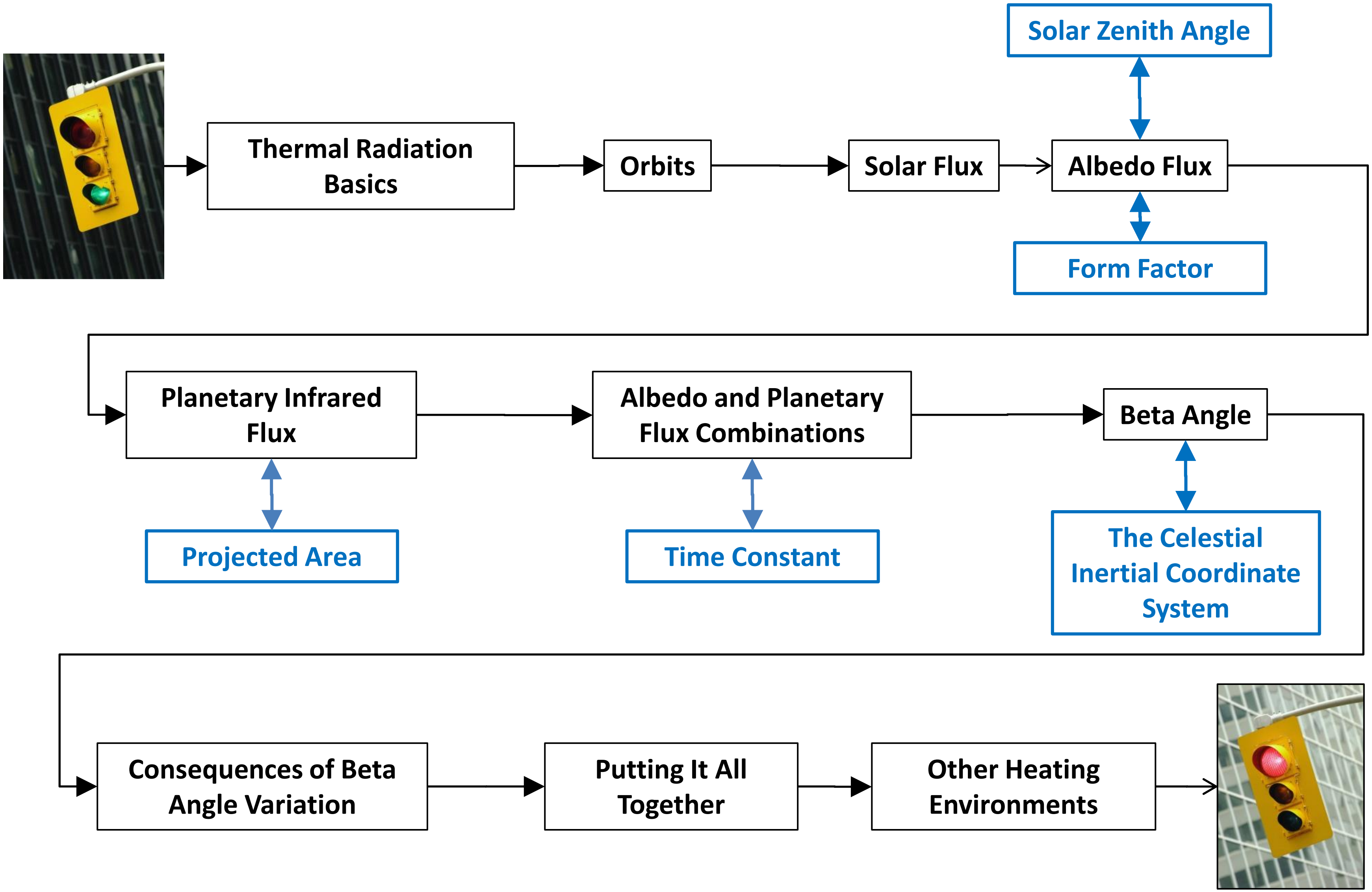
An example problem will be presented.

Scope of this Lesson

Free molecular heating;

Charged particle heating.

Lesson Roadmap



Thermal Radiation Basics

Thermal Radiation Basics

Unlike conduction and convection, thermal radiation is a highly non-linear phenomenon and is proportional to the fourth power of an object's absolute temperature.

Conduction and Convection $\dot{Q} \propto \Delta T$

Radiation $\dot{Q} \propto \Delta(T^4)$

The Rayleigh-Jeans Law and the Ultraviolet Catastrophe (Ref. 1)

Classical theory used the Rayleigh-Jeans law to predict spectral distribution:

$$u(\lambda) = 8\pi kT\lambda^{-4}$$

where u(λ) is the spectral radiance, λ is the wavelength, k is the Boltzmann constant, and T is temperature.

For large λ, there was good agreement with experiment but as λ approached zero, u(λ) approached infinity -- the "ultraviolet catastrophe."

Blackbody Radiation and Planck's Law (Ref. 1)

By assuming that electromagnetic radiation is quantized, Max Planck showed that for a blackbody at a temperature, T:

$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/_{\lambda kT}} - 1}$$

where u(λ) is the spectral radiance, λ is the wavelength, h is Planck's constant, c is the speed of light, and k is the Boltzmann constant.

This is known as Planck's Law.

Blackbody Radiation and Planck's Law (Ref. 1)

The total energy density is given by:

$$U = \int_0^\infty u(\lambda)d\lambda = \int_0^\infty \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}d\lambda$$

Let x = hc/λkT. The integral becomes:

$$U = -\int_\infty^0 \frac{8\pi hc\lambda^{-3}}{e^x - 1}\left(\frac{kT}{hc}\right)dx = 8\pi hc\left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1}dx$$

Blackbody Radiation and Planck's Law (Ref. 1)

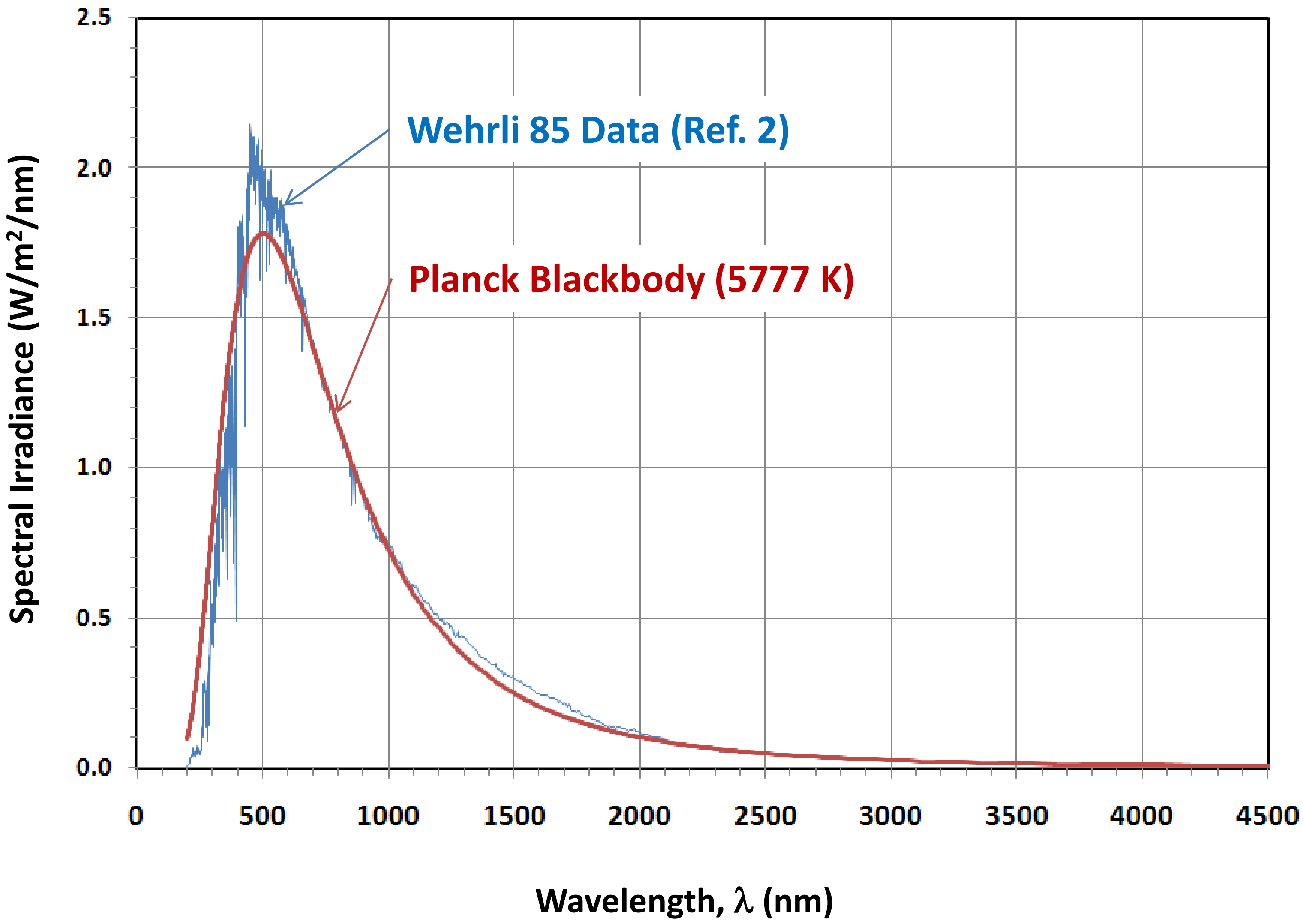
We see that:

$$U = 8\pi hc\left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1}\,dx$$

and observe the quantity in the integral is dimensionless;

We conclude that the spectral radiance is proportional to T^4 - the Stefan-Boltzmann Law.

The Solar Spectrum

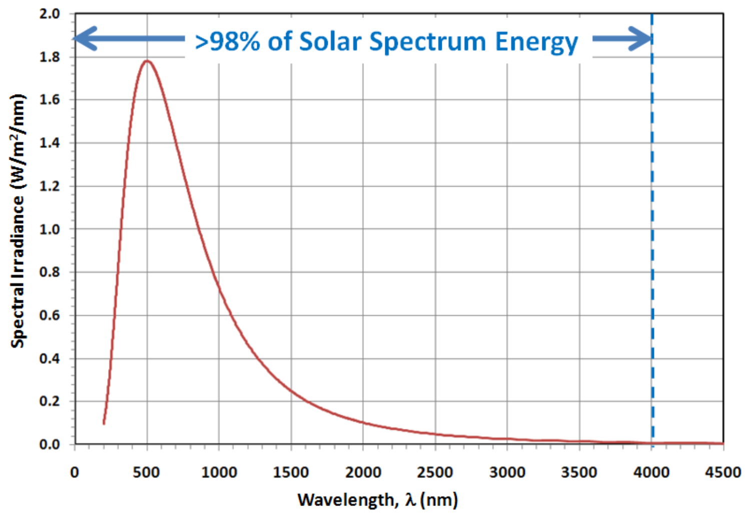


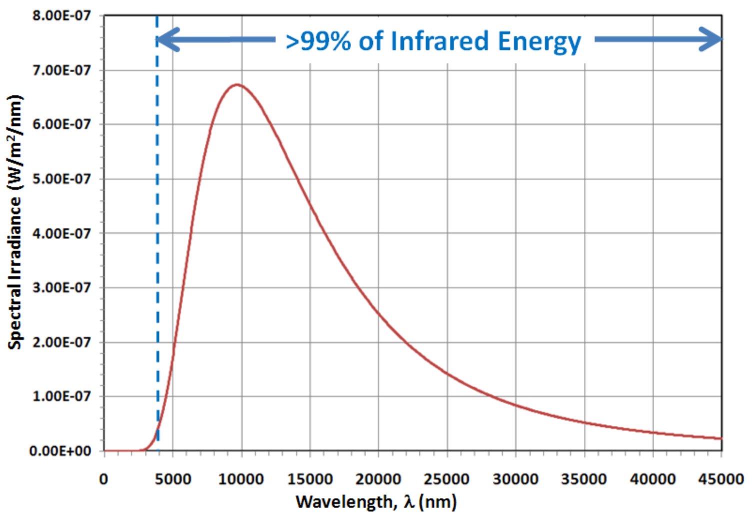
The Solar and Infrared Spectra (Ref. 3)

>98% of the solar spectrum energy lies *below* 4000 nm;

For a blackbody at 300 K, >99% of its energy lies *above* 4000 nm;

Think of these two regions as the **solar** and **infrared** spectra, respectively.





The Blackbody

A blackbody is the perfect absorber and emitter of radiant energy;

The Stefan-Boltzmann law shows that energy radiated from a blackbody is a function only of its absolute temperature, T:

$$\dot{q} = \sigma T^4 \qquad \sigma = \frac{2\pi^5 k^4}{15 h^3 c^2}$$

where $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4.

The Grey Body

Most objects are not perfect blackbody absorbers or emitters -- they are said to be "grey";

To account for imperfect absorption and emission, the Stefan-Boltzmann equation is scaled by an emissivity term, ε;

$$\dot{q} = \varepsilon\sigma T^4$$

ε is a value between 0 and unity.

Solar Absorptance and Infrared Emittance

In the spacecraft thermal lexicon, we differentiate between how well a surface absorbs solar spectrum energy and how well it absorbs and emits infrared energy:

α refers to solar absorptance;
ε refers to infrared emittance;

But ***for a given wavelength***, λ, the absorptance is equivalent to the emittance:

$$\alpha_\lambda = \varepsilon_\lambda$$

Conservation of Energy

Conservation of energy tells us that the energy absorbed by a surface, α_λ plus the energy reflected, ρ_λ plus the energy transmitted, τ_λ must equal the energy incident on that surface:

$$\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$$

For an opaque surface, $\tau_\lambda = 0$ and the equation simplifies to:

$$\alpha_\lambda + \rho_\lambda = 1$$

This will be important when we calculate albedo flux.

Solar Absorptance and Infrared Emittance

When we differentiate between the solar and infrared spectra, ***we drop the λ subscript*** and it is understood that:

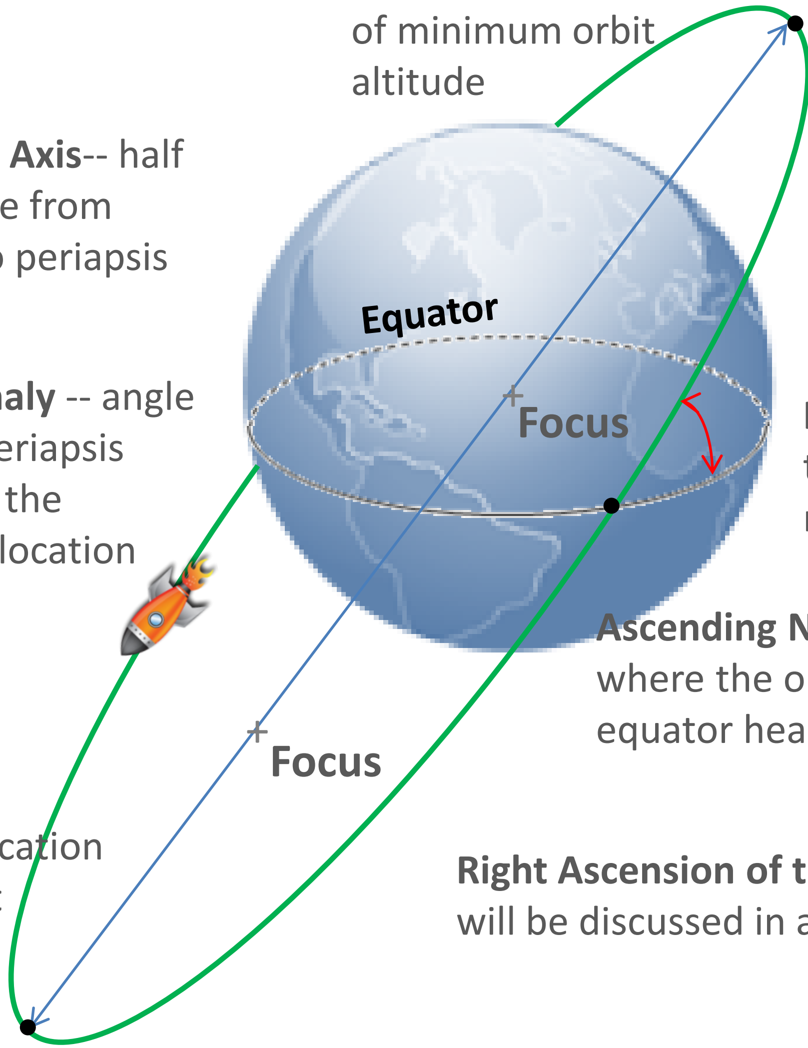
α applies to the 0-4000 nm range;

ρ applies, roughly, to the 0-4000 nm range;

ε applies to the 4000 nm and above range.

Orbits

Anatomy of an Orbit



Periapsis -- the location of minimum orbit altitude

Argument of Periapsis -- the angle, measured in the orbit plane, from the ascending node to the periapsis

Semimajor Axis-- half the distance from apoapsis to periapsis

True Anomaly -- angle from the periapsis location to the spacecraft location

Inclination -- the tilt of the orbit plane with respect to the equator

Ascending Node -- the location where the orbit crosses the equator headed south to north

Apoapsis -- the location of maximum orbit altitude

Right Ascension of the Ascending Node will be discussed in a subsequent section.

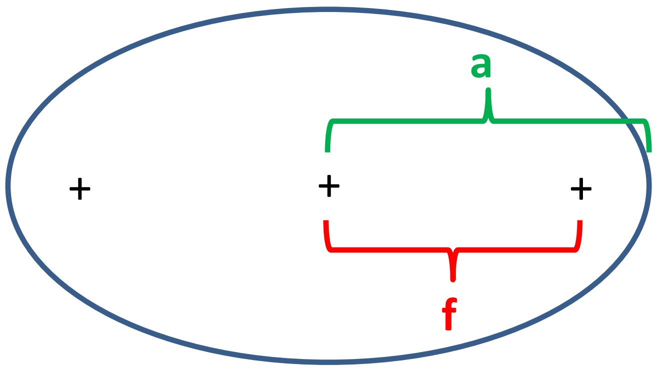
Orbit Eccentricity

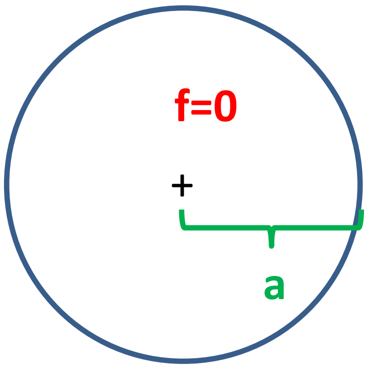
The eccentricity, *e*, of an ellipse is defined as:

$$e = \frac{f}{a}$$

where a is the length of the semimajor axis and f is the distance from the center to one of the foci.

For a circle, *e* = 0.



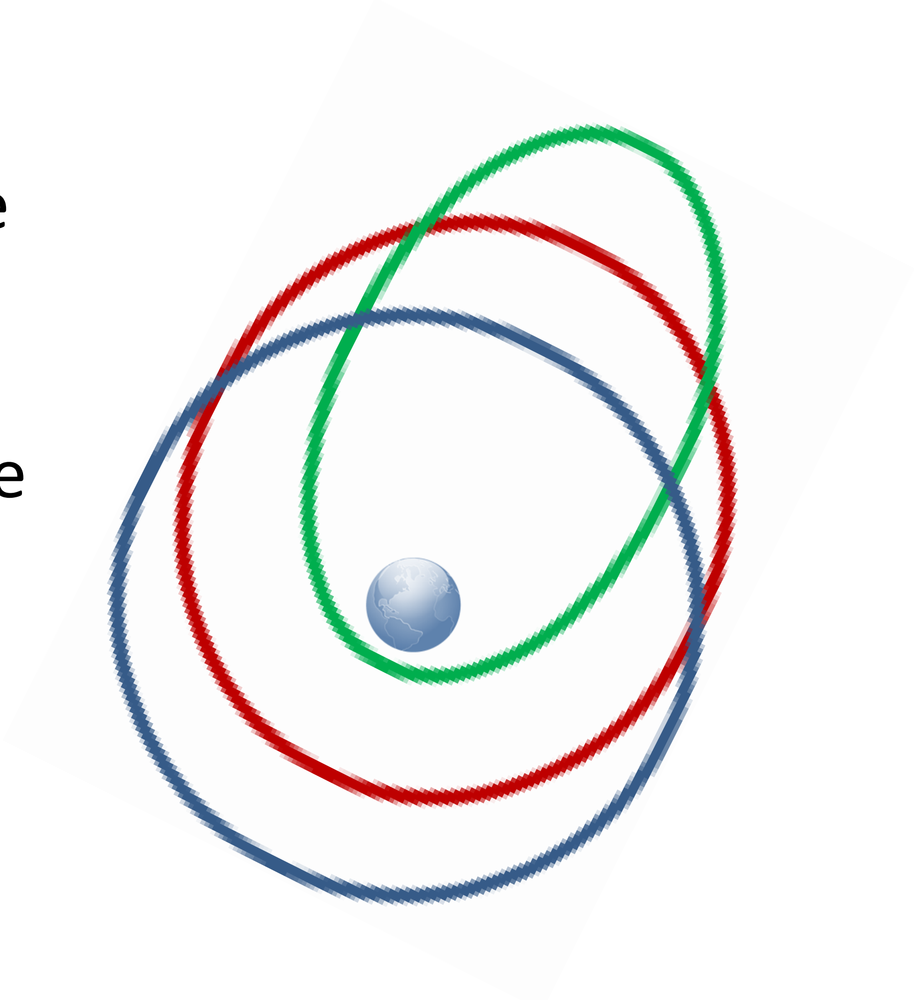


Orbit Eccentricity

Three orbits with the same semimajor axis are shown here;

The only difference is the orbit eccentricity:

***e* = 0.0 (circular)**
***e* = 0.4**
***e* = 0.8**



Circular Orbits

Circular orbits are easier to describe because they are a degenerate case of the ellipse;

For a circular orbit, the semimajor and semiminor axes are equal in length and, because of this, the argument of periapsis is undefined;

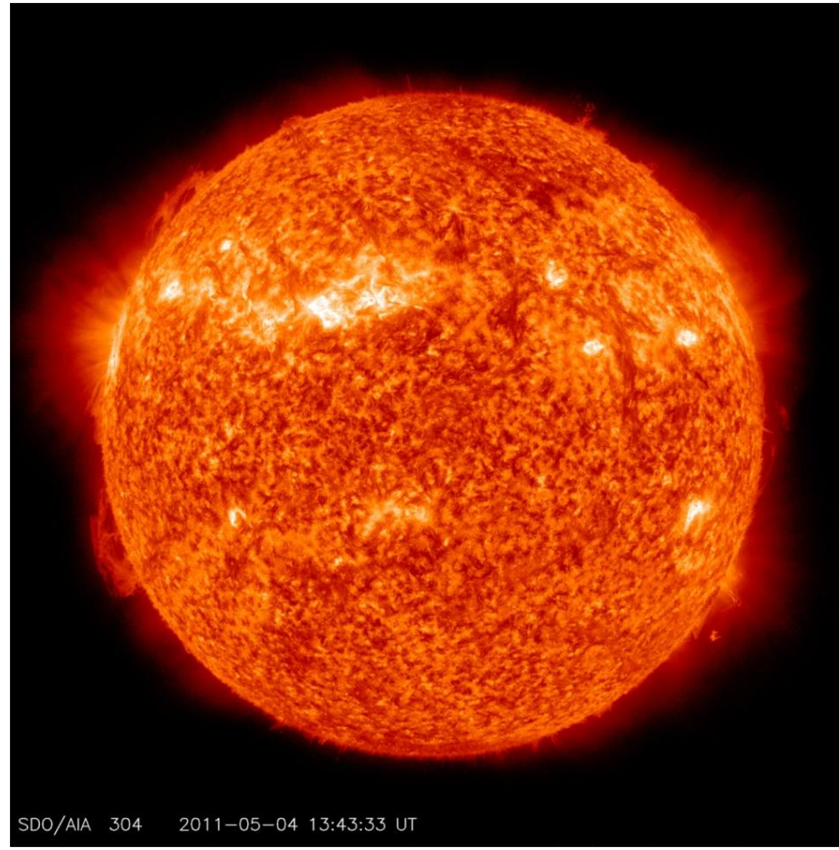
Hence, the shape and orientation of a circular orbit may be described by its radius, inclination and right ascension of the ascending node, only.

Solar Flux

Solar Flux

At a distance of 1 a.u., the intensity of the incoming solar flux is 1367 W/m^2 (Ref. 4);

We seek an expression that allows us to calculate the intensity of the solar flux at any distance.



The Sun as seen by the Solar Dynamics Observatory on 04 May 2011

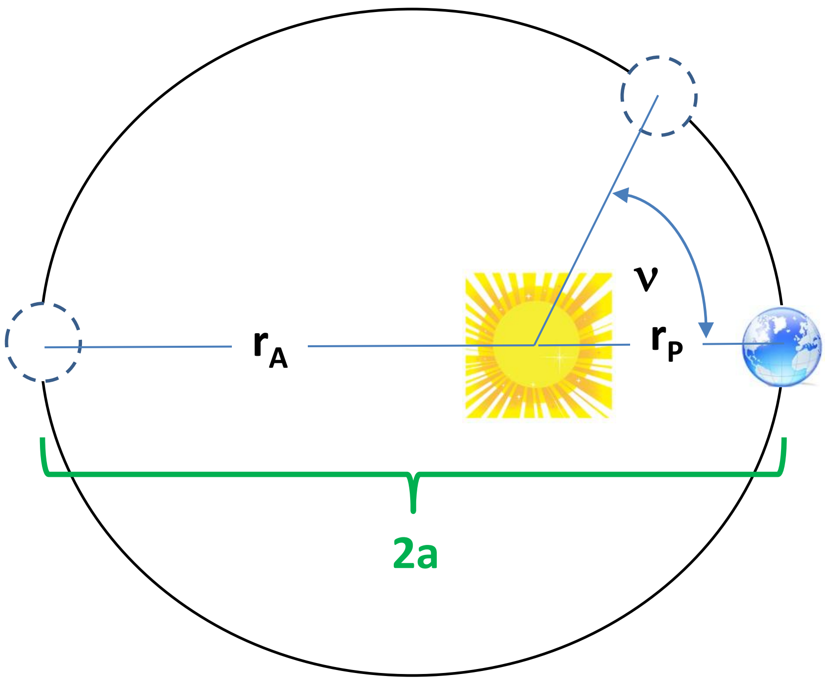
NASA Photo

Solar System Geometry

Earth's orbit around the sun is slightly elliptical (e = 0.0167);

Earth closer to the sun during part of the year;

Earth's average distance from the sun, a, is one half of the ellipse's major axis.

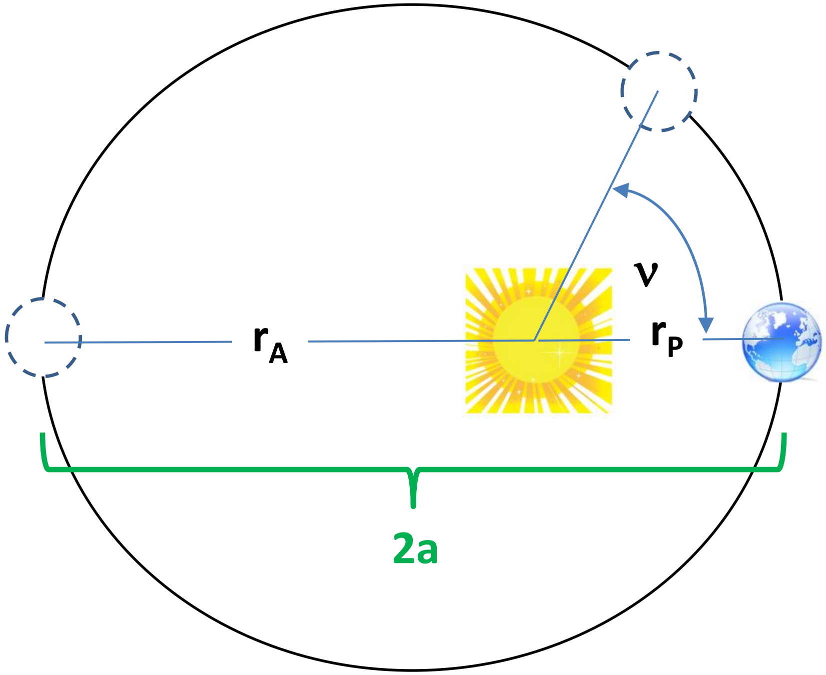


Solar System Geometry

Earth's closest approach to the sun is called **perihelion** (r_P);

Earth's most distant point from the sun is called **aphelion** (r_A);

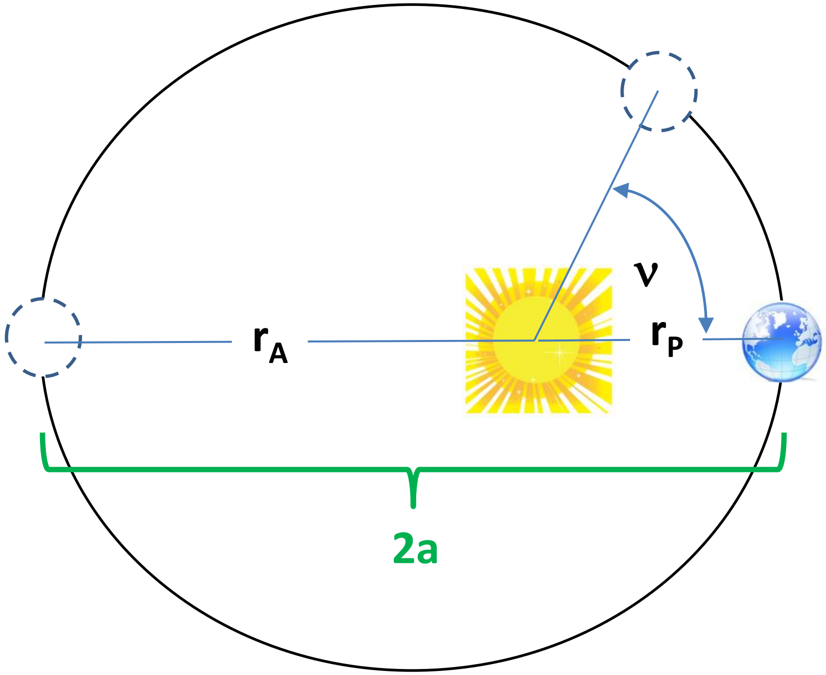
Earth's angular position from perihelion is called **true anomaly** (ν).



Solar System Geometry

Earth's distance from the sun may be determined for any value of true anomaly, ν by using the formula:

$$r(\nu) = \frac{a(1-e^2)}{1+e\cos(\nu)}$$



Solar System Geometry

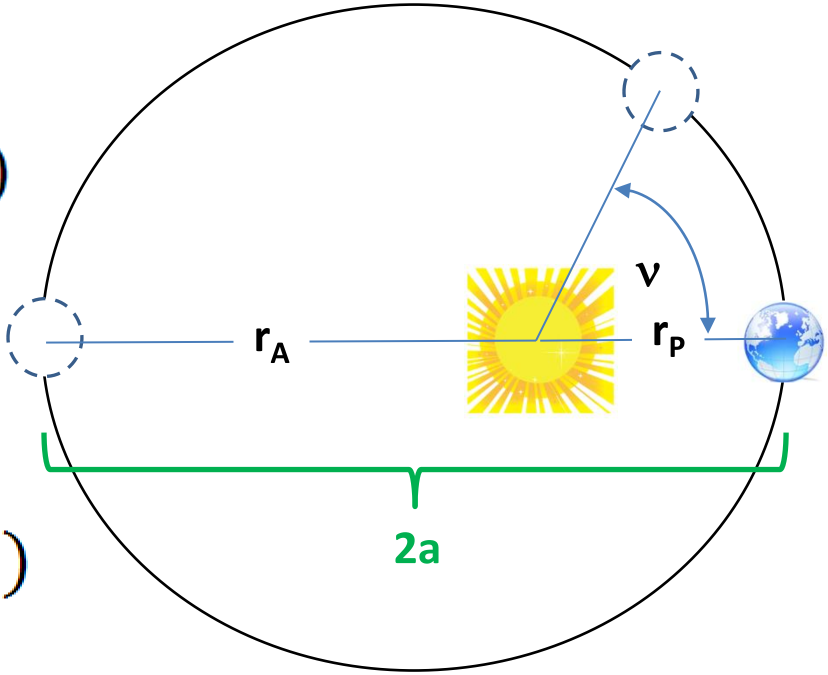
Perihelion, then, is determined as:

$$r(0) = \frac{a(1-e^2)}{1+ecos(0)} = a(1-e)$$

And, similarly, aphelion is:

$$r(\pi) = \frac{a(1-e^2)}{1+ecos(\pi)} = a(1+e)$$

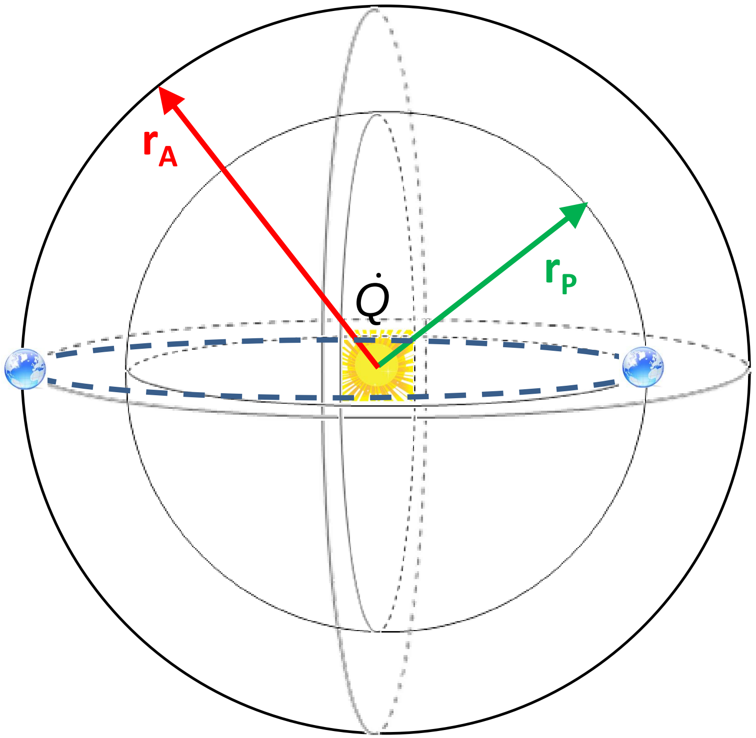
For Earth, *a* = 1 a.u.



Calculating Solar Flux

The sun broadcasts its energy in all directions;

If we construct imaginary spherical surfaces about the sun, we know that the total energy crossing each surface must be the same.

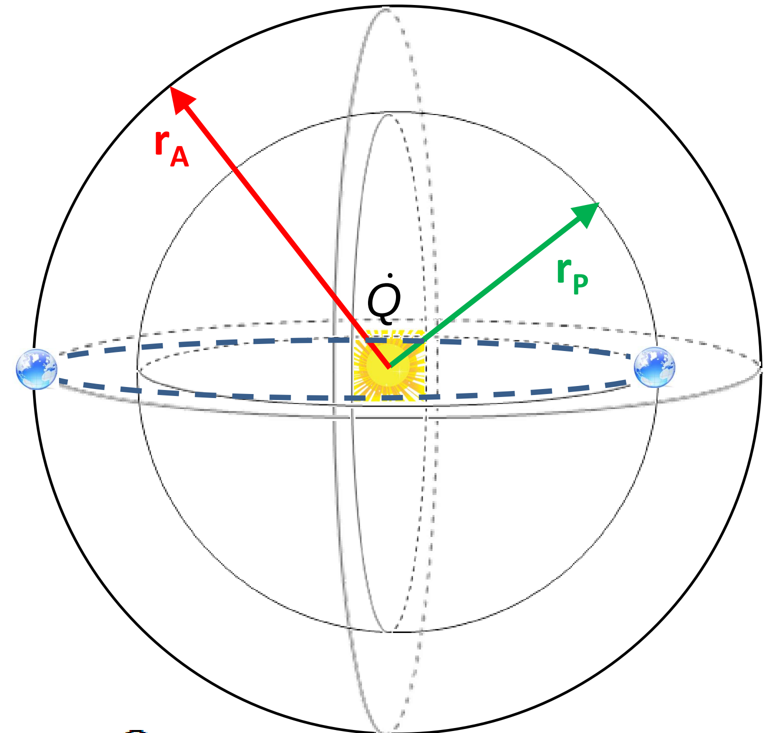


Calculating Solar Flux

We can set up an energy balance as follows:

$$(\dot{q}_{sol})_{r=1} = \frac{\dot{Q}}{4\pi(1)^2}$$

But since the same amount of energy must cross each sphere:



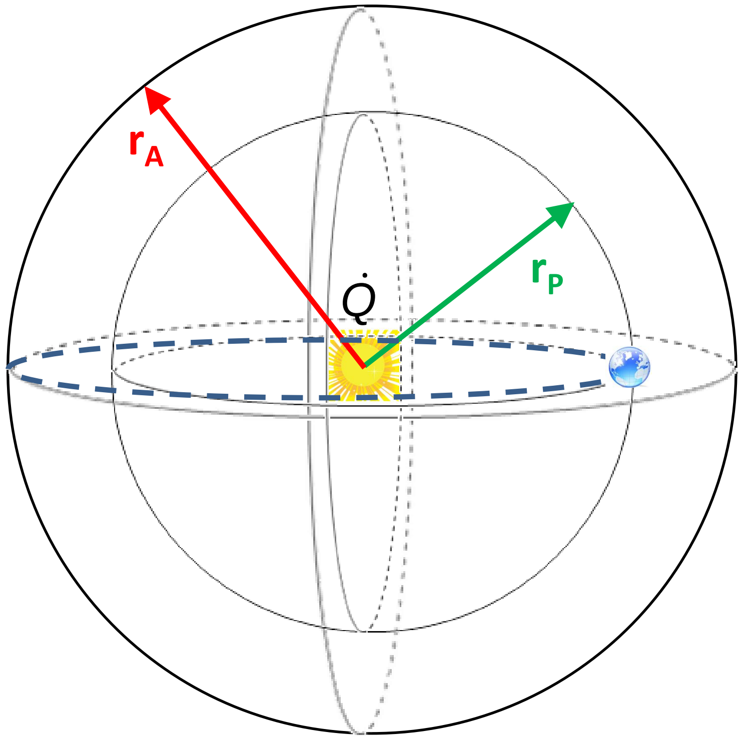
$$\dot{Q} = (\dot{q}_{sol})_{r=1} 4\pi 1^2 = (\dot{q}_{sol})_{r_P} 4\pi r_P^2$$
$$= (\dot{q}_{sol})_{r_A} 4\pi r_A^2$$

Calculating Solar Flux

The flux at Earth perihelion and aphelion is, respectively:

$$(\dot{q}_{sol})_{r_P} = \frac{(\dot{q}_{sol})_{r=1}}{r_P^2} = \frac{(\dot{q}_{sol})_{r=1}}{(1-e)^2}$$

$$(\dot{q}_{sol})_{r_A} = \frac{(\dot{q}_{sol})_{r=1}}{r_A^2} = \frac{(\dot{q}_{sol})_{r=1}}{(1+e)^2}$$



Calculating Solar Flux

For a = 1 a.u., e = 0.0167 and a solar flux of 1367 W/m^2 at 1 a.u., the following values are obtained:

$$(\dot{q}_{sol})_{r_P} = 1414 \quad W/m^2$$
$$(\dot{q}_{sol})_{r_A} = 1322 \quad W/m^2$$

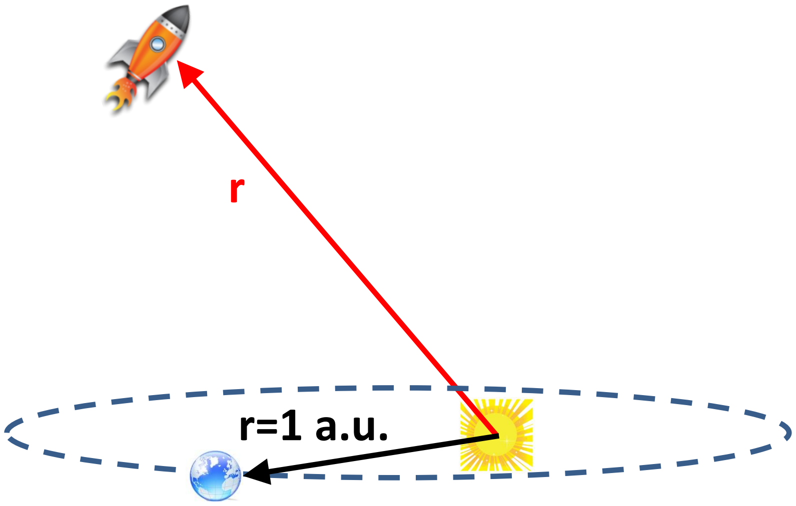
These typically have a ±5 W/m^2 accuracy.

Note: A value of 1423 W/m^2 is a typical hot case solar flux design parameter and depends on the mean value of solar constant assumed.

Calculating Solar Flux

Using this same formulation, we can calculate the solar flux at any distance, *r*, from the sun:

$$\dot{q}_{sol}(r) = \frac{(\dot{q}_{sol})_{r=1}}{r^2}$$



For the remainder of this lesson, solar flux will be referred to as, simply $\dot{q}_{sol}$.

Albedo Flux

Albedo Heating

Albedo heating is solar energy reflected from the planet and its atmosphere;

It is not a point source -- it subtends a solid angle and has non-uniform area intensity.

Partially Illuminated Earth as Seen from Apollo 12 (NASA Photo)

A Simplified Albedo Model

Analysis of albedo is complicated by its non-uniform area intensity;

We can, however, gain an understanding of the contribution of albedo heating by making some simplifying assumptions;

Corrections to this will be discussed.

Assumptions for the Simplified Albedo Model

Simplifying assumptions:

Low altitude, circular orbit -- restricts planet viewing to local regions with similar illumination conditions;

Constant albedo factor planet-wide -- uniform albedo factor, with diffuse reflection, is the easiest to model.

Simplified Albedo Flux Calculation

The expression for the simplified albedo flux is formed by considering the following:

intensity of the sunlight striking the planet, $\dot{q}_{sol}$;

fraction of sunlight reflected, ρ (i.e., the **albedo factor** is just 1 - α);

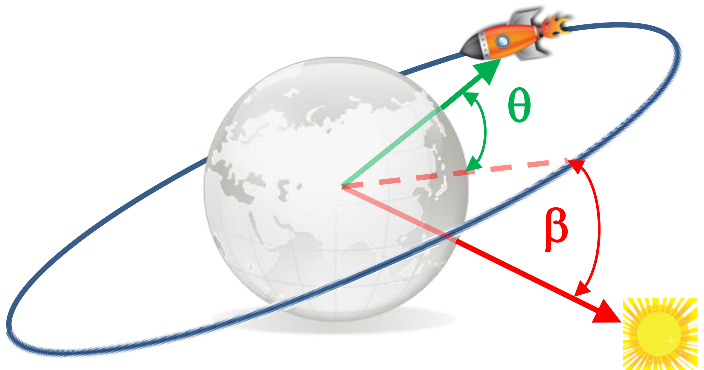
scaling of intensity from orbit noon (i.e., **solar zenith angle**, ξ);

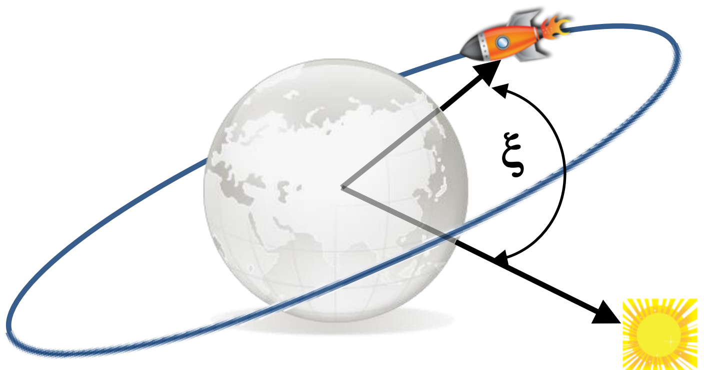
local **form factor** to the planet, FF.

Aside: Solar Zenith Angle, ξ

The **solar zenith angle**, ξ, is a measure of angular distance from orbit noon;

If orbit angle, θ is measured from orbit noon and β* is measured from a plane containing the sun, an increase in either parameter results in an increase in ξ.





* β will be discussed in detail later on

Aside: Solar Zenith Angle, ξ

Mathematically, ξ is expressed as:

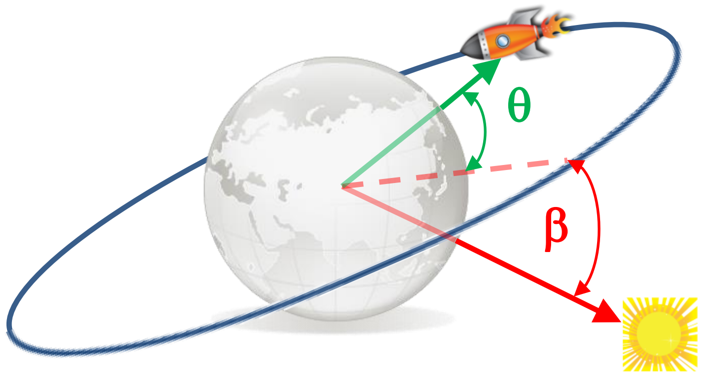
$$\cos\xi = \cos\theta\cos\beta$$

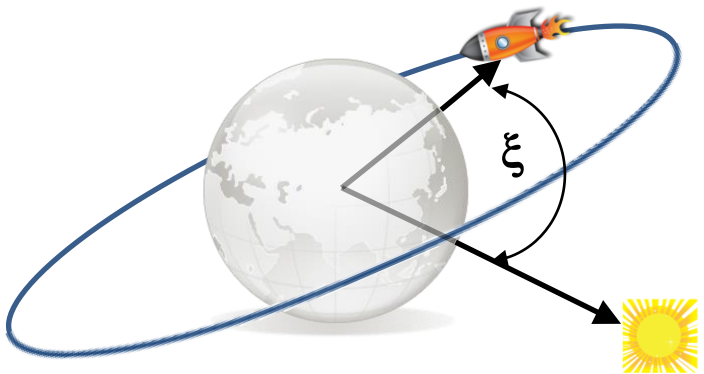
for:

$$-90^\circ \le \theta \le +90^\circ$$

$$-90^\circ \le \beta \le +90^\circ$$

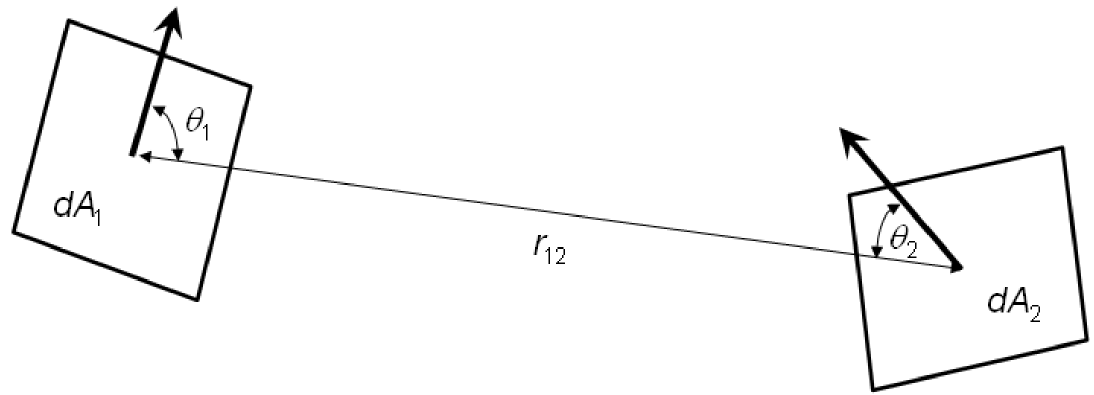
We'll explore β in detail in a subsequent section.





Aside: The Form Factor

A **form factor** describes how well one object can "see" another object;



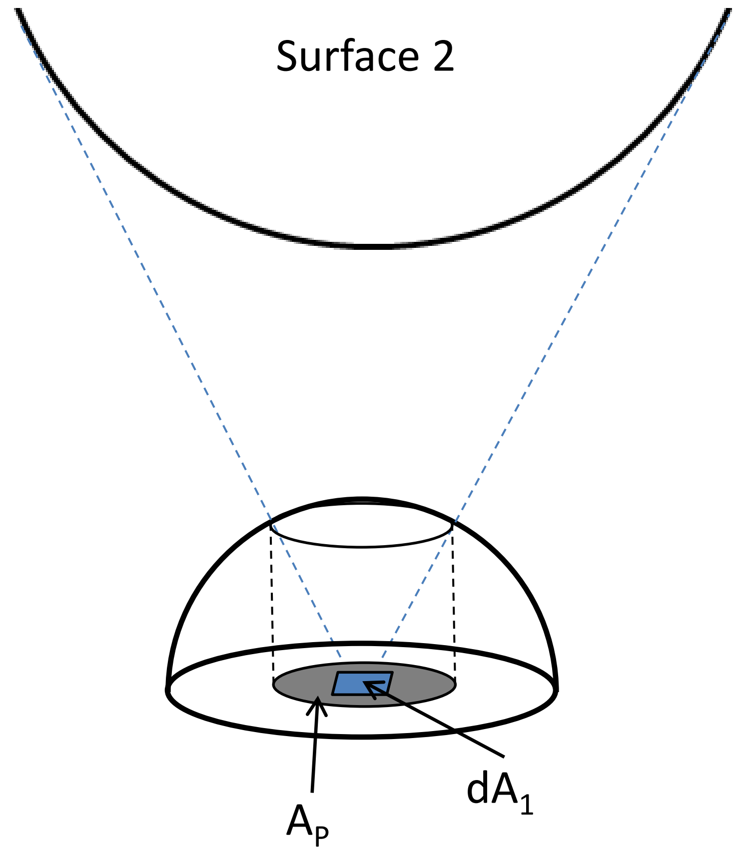
The form factor may take on a value from zero to unity.

$$FF_{12} = \frac{1}{A_1} \iint \frac{\cos\theta_1 \cos\theta_2}{\pi r_{12}^2} dA_1 dA_2$$

Aside: Form Factor

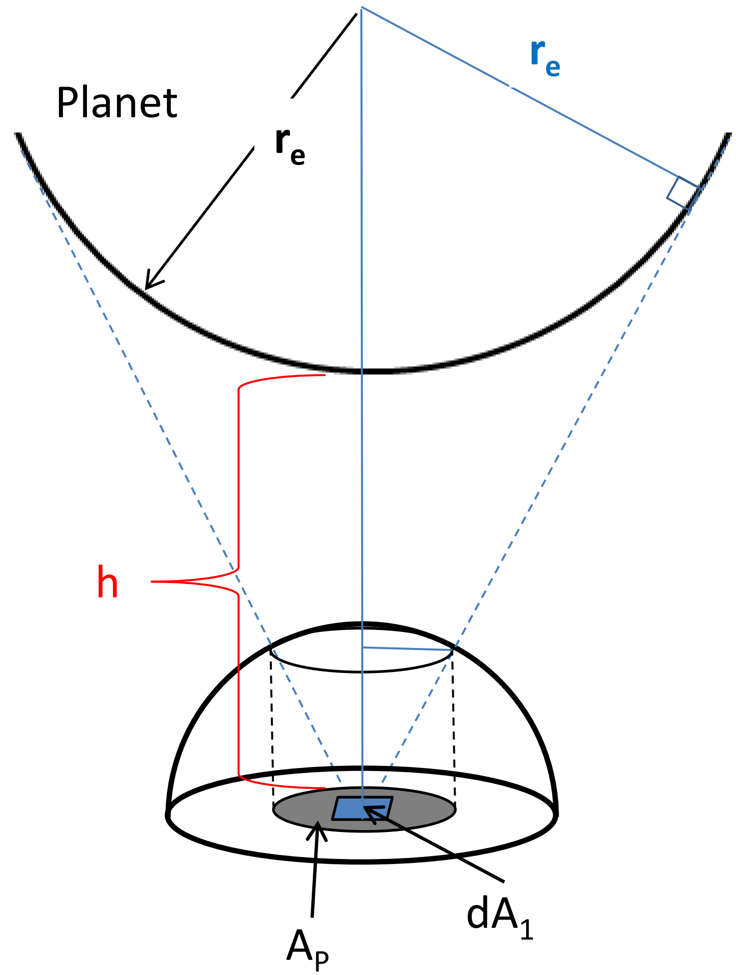
The Nusselt Sphere technique is one of many ways to calculate form factors;

The form factor from dA_1 to Surface 2 is calculated as the projected area, A_P, divided by the area of the hemisphere's circular base.



Aside: Form Factor

Let's use the Nusselt Sphere technique for calculating the form factor to the planet from an orbiting plate, at altitude h above the planet, whose surface normal faces the nadir direction.

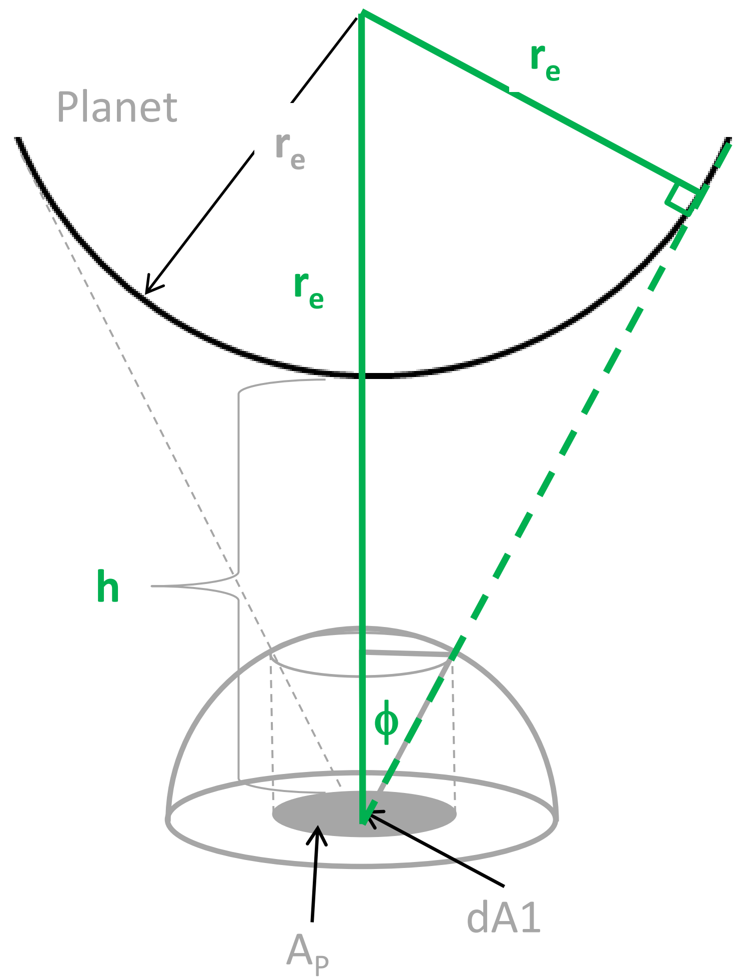


Aside: Form Factor

We see that we can construct a right triangle (in green) with a short side measuring r_e and a hypotenuse measuring r_e+h;

We define the angle ϕ by noting:

$$\sin\phi = \frac{r_e}{r_e + h}$$

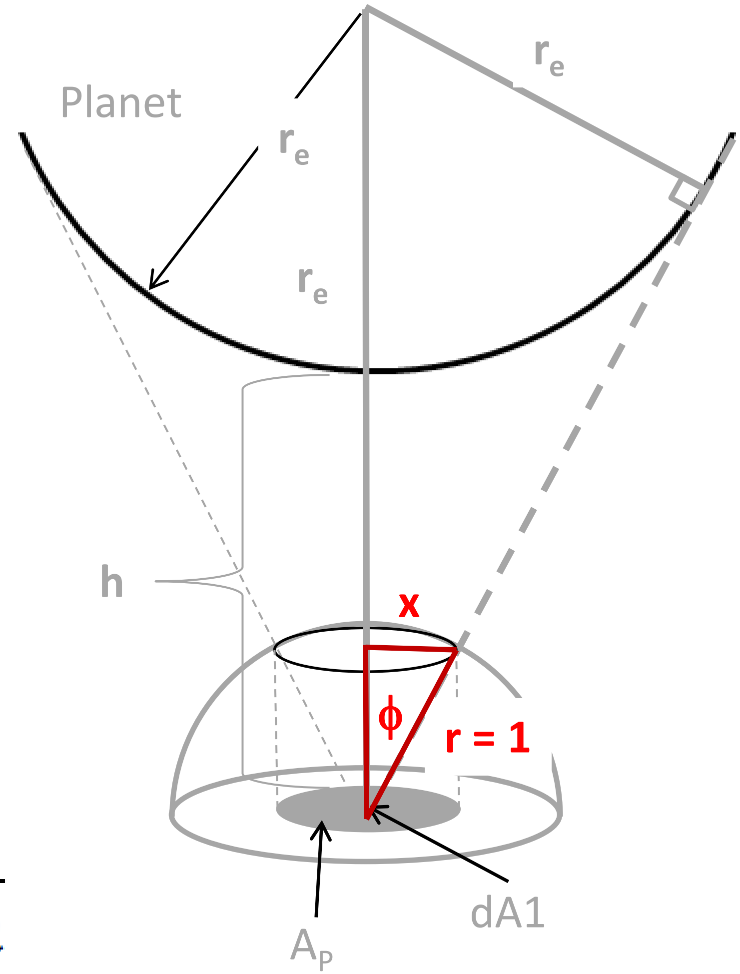


Aside: Form Factor

Similarly, we can construct a right triangle (in red) with a hypotenuse measuring unity and the angle ϕ, already defined;

We define the distance x by noting:

$$\sin\phi = \frac{x}{r} = \frac{x}{1} = x = \frac{r_e}{r_e + h}$$

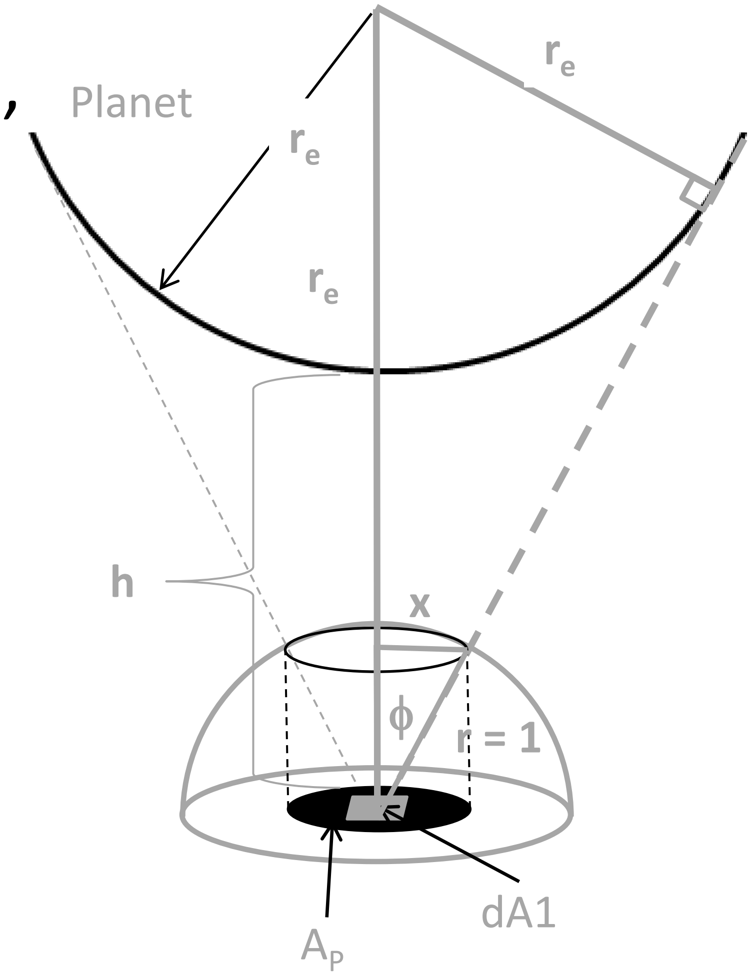


Aside: Form Factor

Projecting x down to the base, we see that the ratio of the projected circular area to the total area of the base is:

$$FF = \frac{\pi x^2}{\pi 1^2} = \left(\frac{r_e}{r_e + h}\right)^2$$

We'll come back to this result in our heating calculations.



Aside: Form Factor

The forward (east)-, aft (west)-, north- and south-facing surfaces have a different view to the planet due to their orientation.

For these "perpendicular" surfaces, the form factor is presented in Ref. 5 and shown here, without derivation.

$$FF_{\perp} = \left(\frac{1}{2\pi}\right)\left[\pi - 2\sin^{-1}\left(\sqrt{1-\left(\frac{r_e}{r_e+h}\right)^2}\right) - \sin\left(2\sin^{-1}\left(\sqrt{1-\left(\frac{r_e}{r_e+h}\right)^2}\right)\right)\right]$$

Simplified Albedo Flux Calculation

The equation for a nadir-facing plate is:

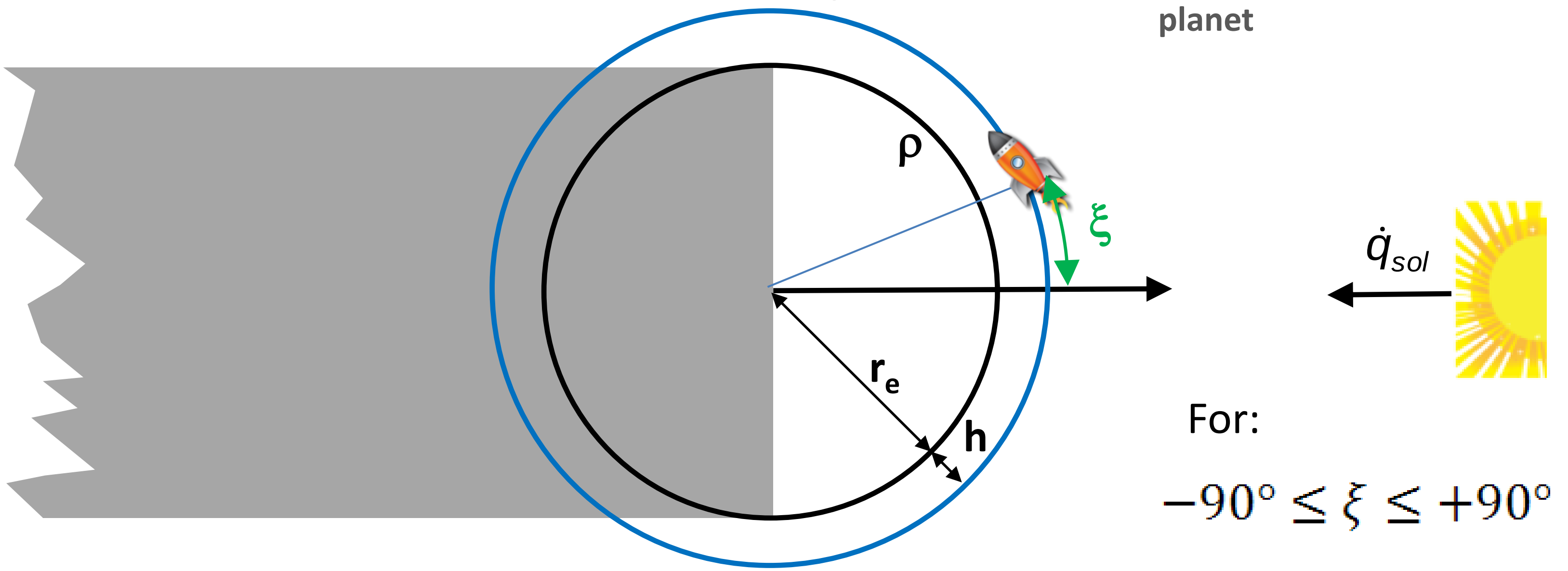
$$\left(\dot{q}_{alb}(\xi)\right)_{nadir} = \dot{q}_{sol}\rho\left(\frac{r_e}{r_e + h}\right)^2 \cos\xi$$

Intensity of sunlight striking the planet

Fraction of the sunlight reflected

Local Form Factor to the planet

Scaling of intensity from orbit noon



For:

$$-90° \leq \xi \leq +90°$$

Solar Zenith Angle Corrections to Albedo (Ref. 6)

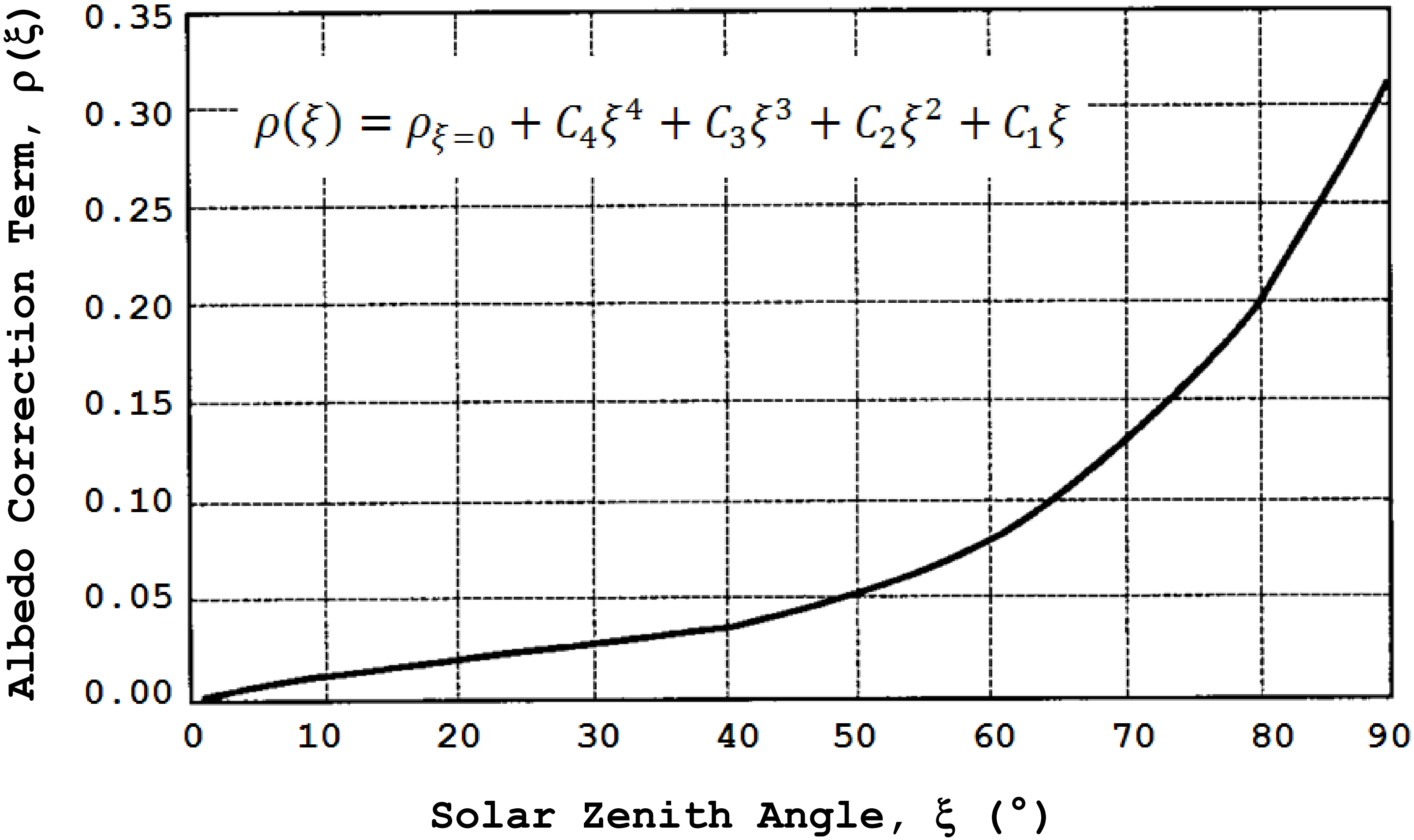
Our simplified model isn't perfect but there's a way to correct for albedo by applying this formula:

$$\rho(\xi) = \rho_{\xi=0} + C_4\xi^4 + C_3\xi^3 + C_2\xi^2 + C_1\xi$$

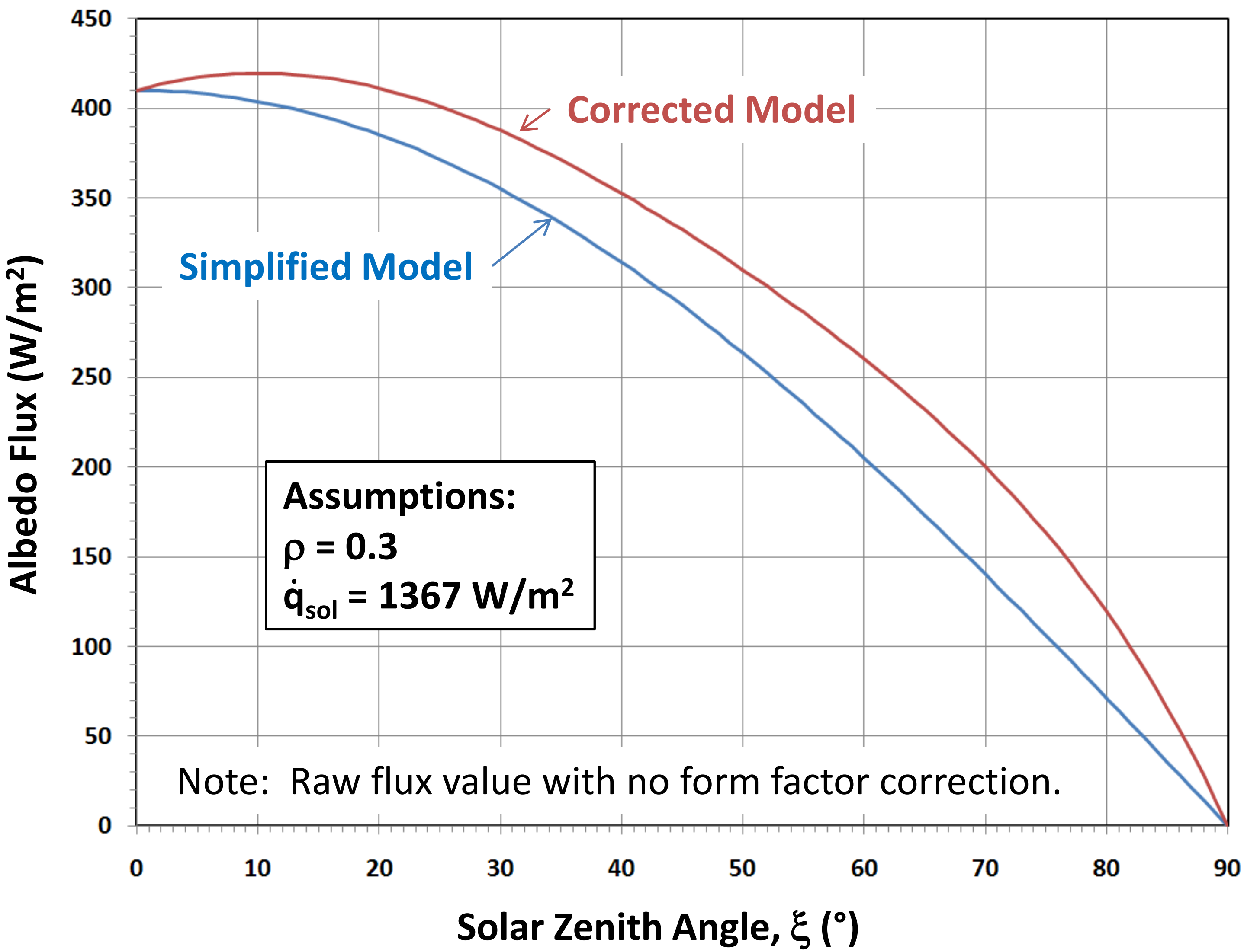
$$C_4 = +4.9115 \times 10^{-9}$$
$$C_3 = +6.0372 \times 10^{-8}$$
$$C_2 = -2.1793 \times 10^{-5}$$
$$C_1 = +1.3798 \times 10^{-3}$$

Note that this correction was derived from data restricted to latitudes of +30° to -30°.

Solar Zenith Angle Corrections to Albedo (Ref. 6)



Solar Zenith Angle Corrections to Albedo



Planetary Infrared Flux

Planetary Infrared Heating

Thus far, we've discussed natural heating components using illumination in the solar spectrum;

But there is another heating source incident on a spacecraft when in proximity to a planet/moon;

And the heating is concentrated in the infrared portion of the spectrum -- it's called **planetary infrared** or **outgoing long-wave radiation** (OLR);

Simplifying Assumptions for Earth Infrared

We can learn much by examining a simplified representation of Earth's heat balance;

For this analysis, we'll assume that Earth's atmosphere and relatively rapid rotation with respect to the sun results in uniform temperatures over the entire globe; Earth is at thermal radiation equilibrium; and, Earth's albedo is constant over the entire surface.

The Overall Planetary Heat Balance

The overall planetary heat balance assumes steady state heat transfer -- the amount of heat absorbed by the planet must equal the amount radiated:

$$\dot{Q}_{in} = \dot{Q}_{out}$$

Let's take a closer look at each of these terms.

The Overall Planetary Heat Balance

The heat absorbed is the amount of incoming solar flux times the area that intercepts the flux times the fraction absorbed:

$$\dot{Q}_{in} = \dot{q}_{sol} A_{proj} (1 - \rho)$$

But what is meant by the projected area (A_{proj})?

Aside: Projected Area

The entire planetary sphere is not illuminated;

Only half is illuminated at any given instant and even that illumination is not uniform;

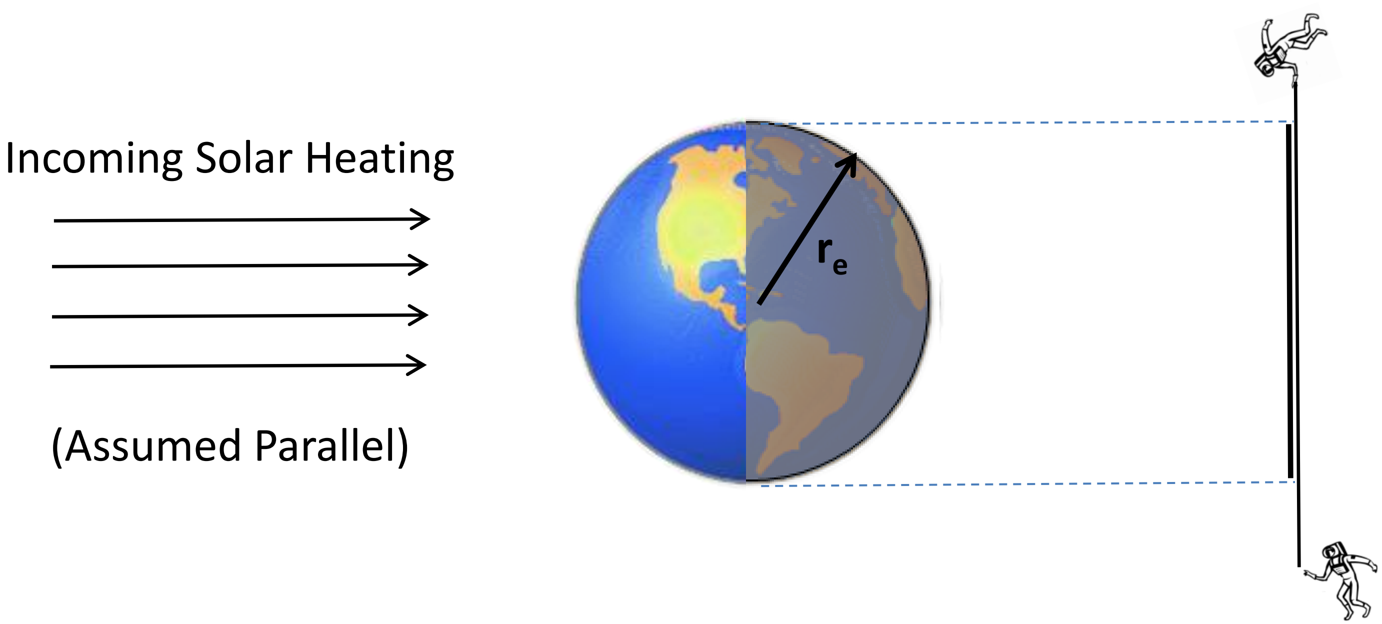
But it's easy to visualize how much sunlight is intercepted by the planet, even with this non-uniform illumination.

Earth as Seen from Apollo 8 (NASA Photo)

Aside: Projected Area

Imagine that you could lower a white screen behind an illuminated planet and observe from afar;

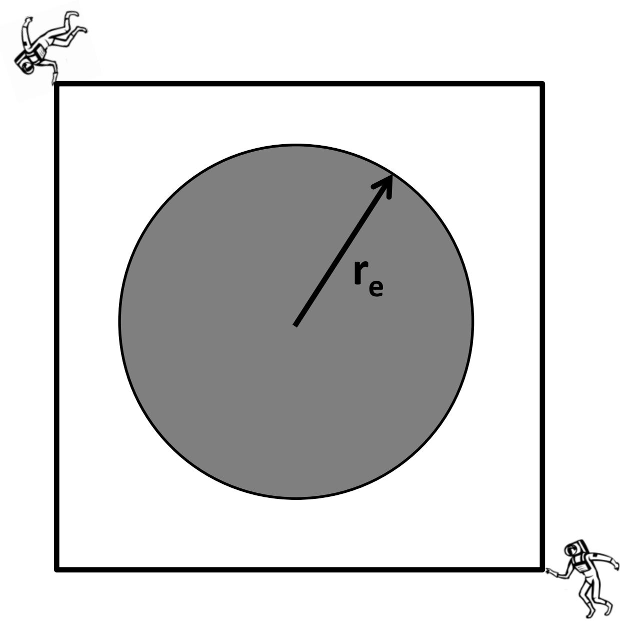
How much sunlight would be missing?



Aside: Projected Area

Looking at the screen head-on, we see that a circle of sunlight is missing;

The projected area of a sphere is simply that of a circle.



$$A_{proj} = \pi r_e^2$$

The Overall Planetary Heat Balance

The heat emitted is assumed uniform over the entire planet;

We can express the heat rejected using the familiar Stefan-Boltzmann law:

$$\dot{Q}_{out} = 4\pi r_e^2 \varepsilon \sigma T^4$$

where $4\pi r_e^2$ is recognized as the surface area of the planetary sphere.

The Overall Planetary Heat Balance

Next, we equate the outgoing and incoming energy, and substituting for the projected area:

$$\dot{q}_{sol} A_{proj} (1-\rho) = 4\pi r_e^2 \varepsilon \sigma T^4$$

Solving for the planetary temperature, *T* yields:

$$T = \sqrt[4]{\frac{\dot{q}_{sol}(1-\rho)}{4\varepsilon\sigma}}$$

The Overall Planetary Heat Balance

We can also find the flux emitted by the planet at temperature, *T*:

$$\dot{q}_{pla} = \frac{\dot{q}_{sol}(1-\rho)}{4}$$

For a mean solar flux of 1367 W/m^2, albedo of 0.3 and an assumed Earth emittance of 1.0:

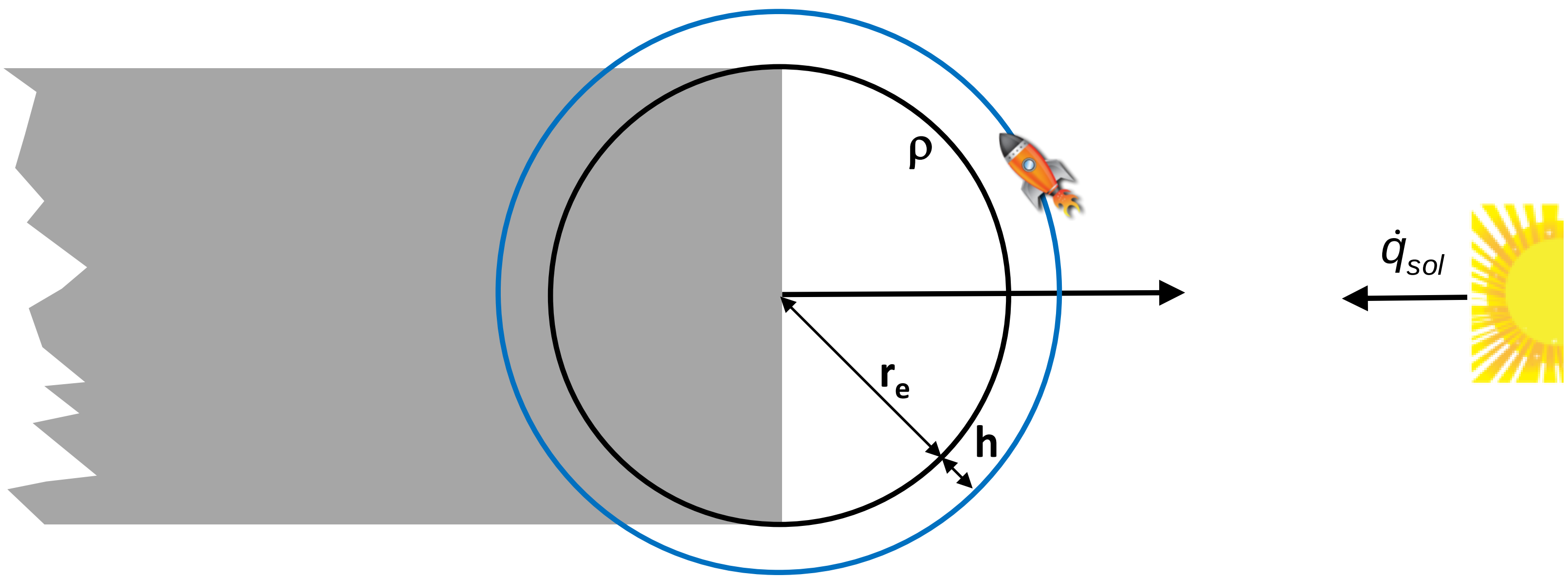
$T =$ 255 K

$\dot{q}_{pla} =$ 239 W/m^2

The Overall Planetary Heat Balance

The flux incident on the spacecraft is scaled by the local form factor to the planet for a nadir-facing plate:

$$(\dot{q}_{pla})_{nadir} = \frac{\dot{q}_{sol}(1-\rho)}{4}\left(\frac{r_e}{r_e+h}\right)^2$$

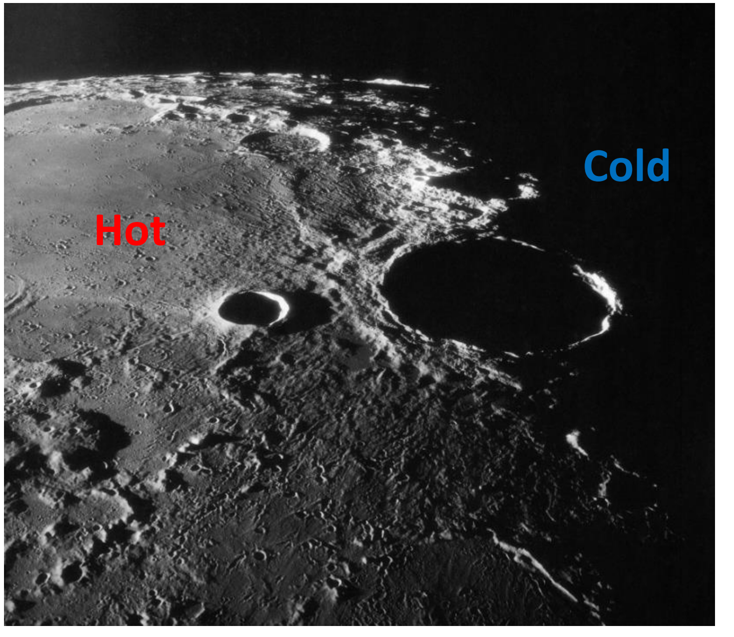


Why Won't This Calculation Work for the Moon?

Recall our simplifying assumptions:

Earth has an atmosphere to transport heat over the globe -- the Moon does not;

Earth rotates rapidly with respect to the sun when compared to the Moon.



Moon as Seen from Apollo 11 (NASA Photo)

Why Won't This Calculation Work for the Moon?

Because of this, assuming the Moon is isothermal is not a good assumption.

Moon as Seen from Apollo 11 as it was Homeward Bound (NASA Photo)

Albedo and Planetary Flux Combinations

Combination of Natural Environmental Parameters

The previous developments were used to show how reasonable estimates of natural environmental parameters could be obtained using some simplifying assumptions;

Now, it's time to explore "reality".

Local Variation vs. the Planet-Wide Heat Balance

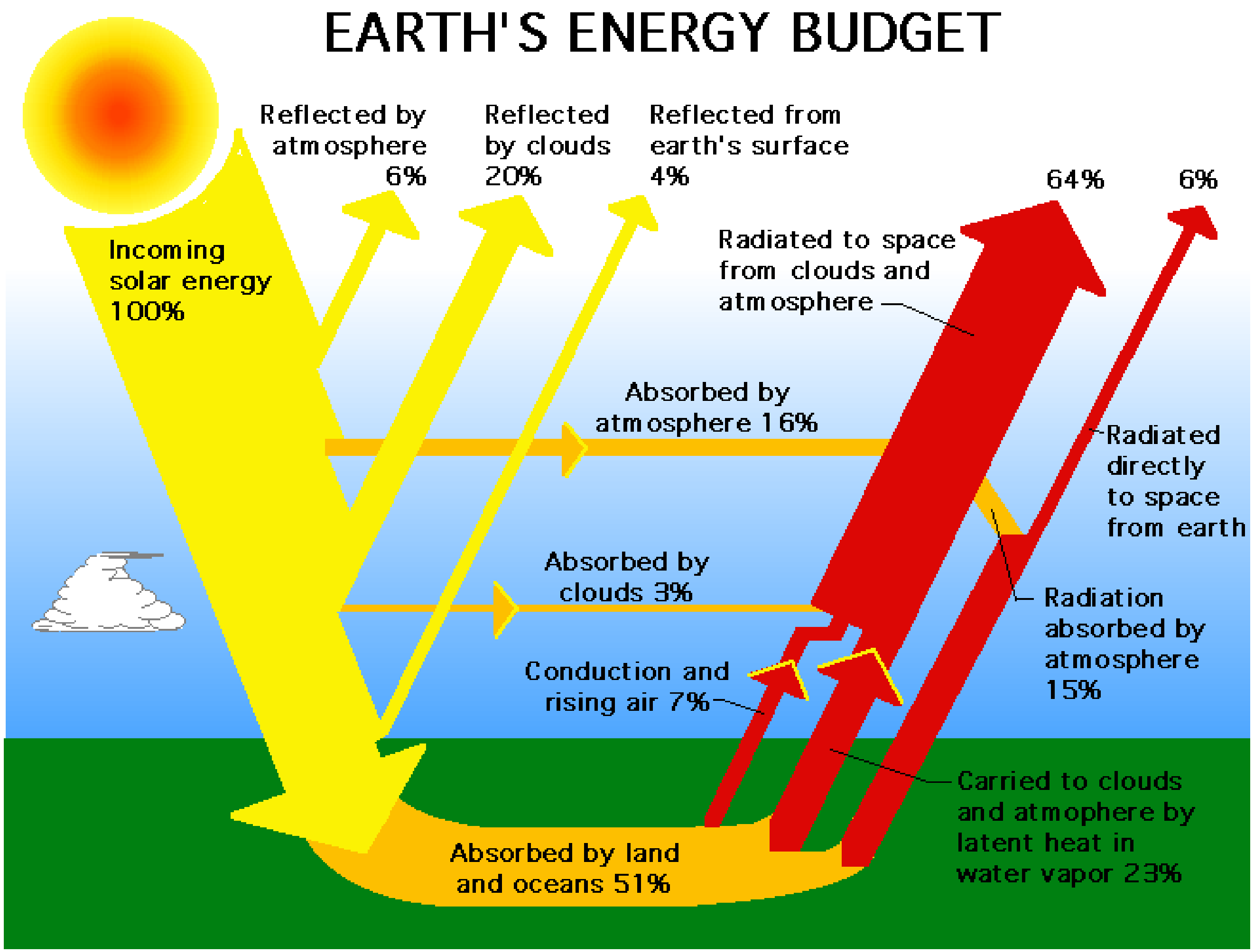
Our earlier derivation for planetary OLR assumed a planet-wide heat balance;

This isn't necessarily true for local conditions;

Seasonal variation in vegetation, snow cover, cloud cover, etc., can all affect local albedo and OLR components.

So how do engineers account for this?

Local Variation vs. the Planet-Wide Heat Balance (Ref. 7)

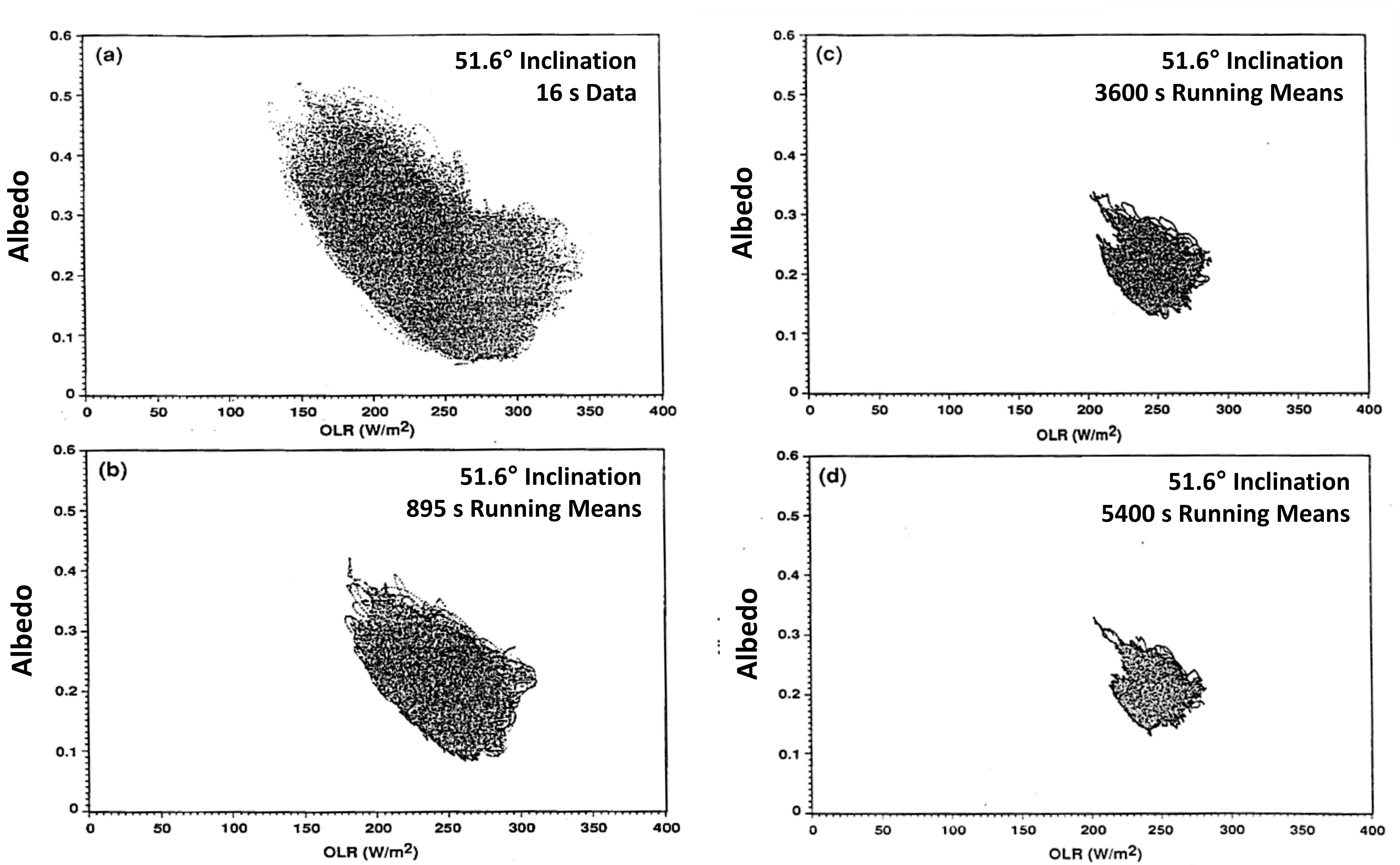


Earth Radiation Budget Experiment (ERBE)

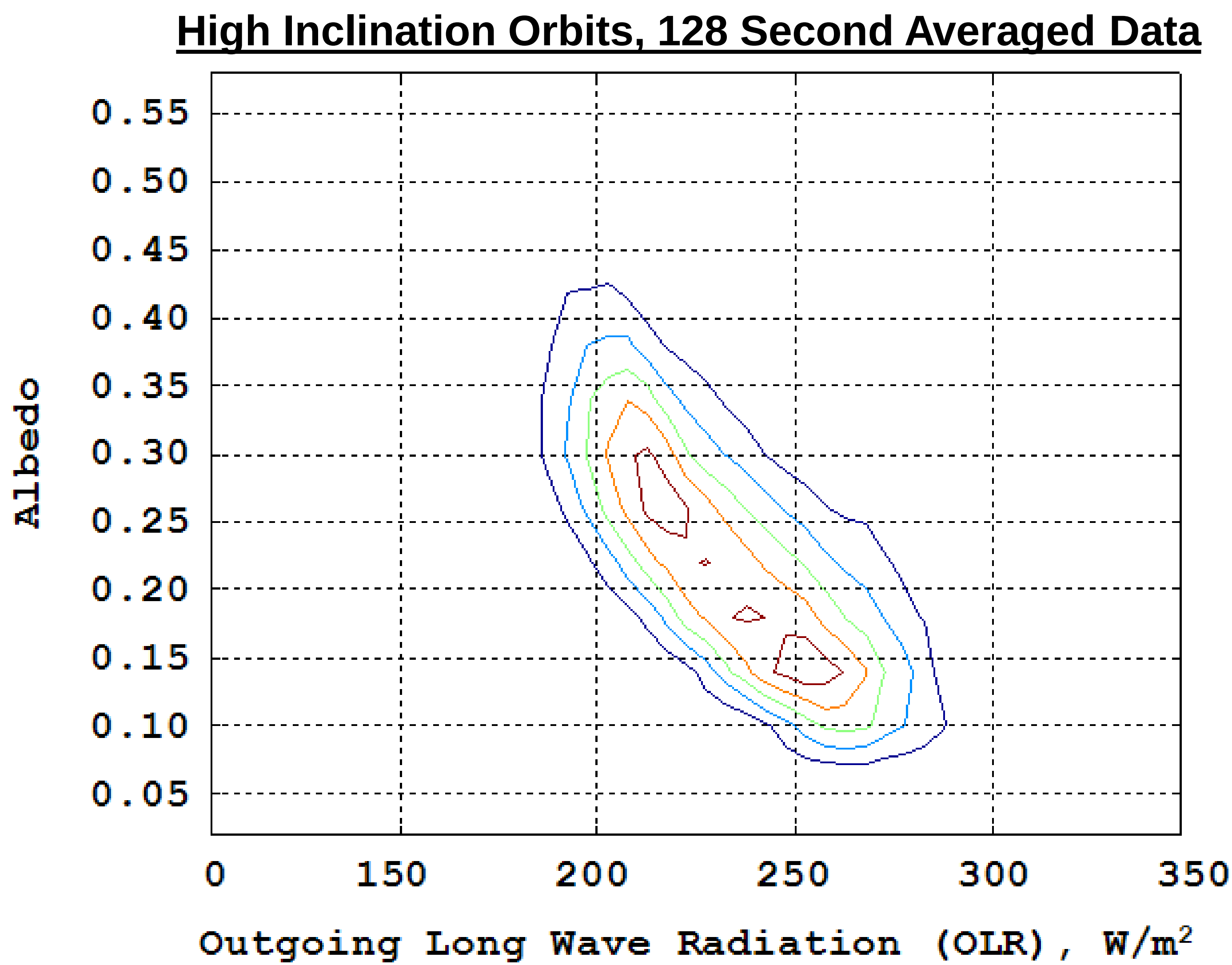
The Earth Radiation Budget Experiment (ERBE) used a new generation of instrumentation to make accurate regional and global measurements of the components of the radiation budget. (Ref. 8)

Earth Radiation Budget Satellite (ERBS)

Albedo and Planetary OLR Data (Ref. 9)



Albedo vs.Planetary OLR (Ref. 9)



Aside: Time Constant

As the thermal environment changes, so does an object's temperature;

Objects with low thermal inertia react faster than objects with a higher thermal inertia;

Objects with high heat transfer to or from the environment will react more rapidly than those with lower heat transfer.

Aside: Time Constant

The effect of thermal inertia and heat transfer into and out of an object may be understood through investigation of the time constant, τ.

Mathematically, for a mass, m, the time constant, τ is given by:

$$\tau = \frac{mC_p}{\sum G}$$

where C_p is the specific heat and G is conductance or a *linearized* conductance.

Aside: Time Constant

Real-life problems require more than a single set of environmental parameters:

Objects with short time constants react to short-term changes in the thermal environment -- examples: components such as radiators and insulation surfaces;

Objects with longer time constants react more slowly to changes -- examples: massive components with small area and insulated structure.

Albedo vs. OLR (Ref. 9)

Engineering Extreme Cases for High Inclination Orbits

	COLD CASES		
Averaging Time	**Minimum Albedo** Alb ⇔ OLR (W/m^2)	**Combined Minimum** Alb – OLR (W/m^2)	**Minimum OLR** Alb – OLR (W/m^2)
16 second	0.06 ⇔ 273	0.16 ⇔ 212	0.40 ⇔ 97
128 second	0.06 ⇔ 273	0.16 ⇔ 212	0.38 ⇔ 102
896 second	0.09 ⇔ 264	0.17 ⇔ 218	0.33 ⇔ 141
30 minute	0.13 ⇔ 246	0.18 ⇔ 218	0.31 ⇔ 171
90 minute	0.16 ⇔ 231	0.19 ⇔ 218	0.26 ⇔ 193
6 hour	0.18 ⇔ 231	0.20 ⇔ 224	0.27 ⇔ 202
24 hour	0.18 ⇔ 231	0.20 ⇔ 224	0.24 ⇔ 205
	HOT CASES		
Averaging Time	**Maximum Albedo** Alb – OLR (W/m^2)	**Combined Maximum** Alb – OLR (W/m^2)	**Maximum OLR** Alb – OLR (W/m^2)
16 second	0.50 ⇔ 180	0.32 ⇔ 263	0.22 ⇔ 350
128 second	0.49 ⇔ 184	0.31 ⇔ 262	0.22 ⇔ 347
896 second	0.35 ⇔ 202	0.28 ⇔ 259	0.20 ⇔ 304
30 minute	0.33 ⇔ 204	0.27 ⇔ 260	0.20 ⇔ 280
90 minute	0.28 ⇔ 214	0.26 ⇔ 244	0.22 ⇔ 231
6 hour	0.27 ⇔ 218	0.24 ⇔ 233	0.22 ⇔ 221
24 hour	0.24 ⇔ 224	0.23 ⇔ 232	0.20 ⇔ 217
Mean Albedo: 0.21		**Mean OLR:** 211	

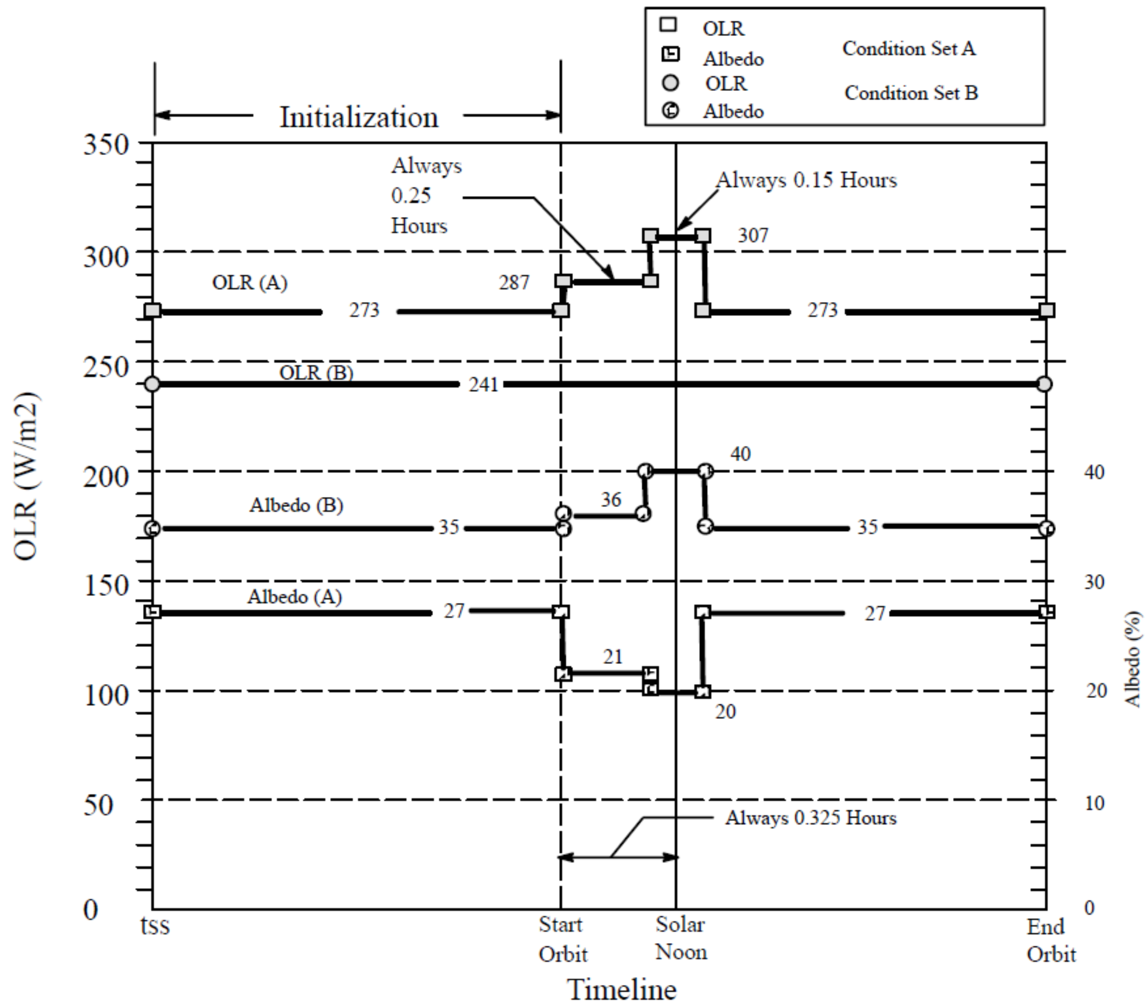
Albedo and OLR Combinations (Ref. 10)

Condition[2]	Orbit Time[4]					
	0 to 0.25 hr		0.25 to 0.4 hr		After 0.4 hr	
	Albedo	OLR (W/m^2)	Albedo	OLR (W/m^2)	Albedo	OLR (W/m^2)
Cold A	(3)	207	(3)	177	0.27	217
B	(3)	207	(3)	177	0.22	241
Mean					0.27	241
Hot A (5)	0.21	287	0.20	307	0.27	273
B	0.36	241	0.40	241	0.35	241
Solar Constants (W/m^2)						
Cold 1321						
Mean 1371						
Hot 1423						

Notes:

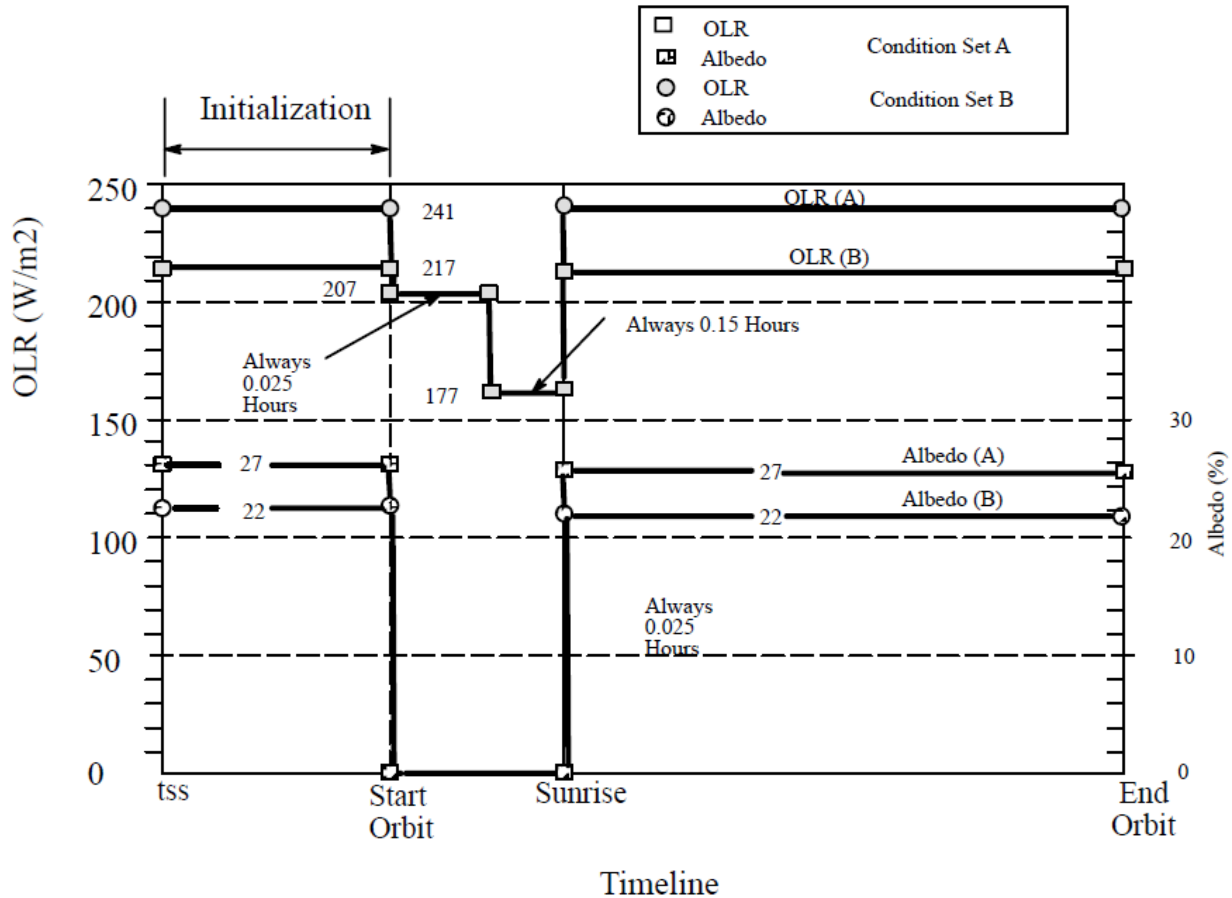
1. Values in this table are expected to be exceeded no more than 0.5% of the time. Albedo and OLR are adjusted to the top of the atmosphere (30 km altitude).
2. Both Set A and Set B are design requirements.
3. No Albedo value, extreme cold case occurs in eclipse.
4. Referenced to orbit location per Figures 7–A and 7–B.
5. Line A as noted on this Table was developed from the Probability Table as specified in SSP 50094, section 13.

Albedo and OLR Combinations (Ref. 10)



Design Hot Case Thermal Environment Profile

Albedo and OLR Combinations (Ref. 10)

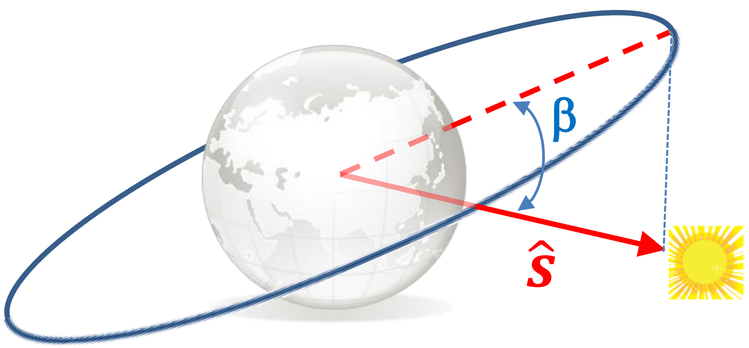


Design Cold Case Thermal Environment Profile

Beta Angle

The Beta Angle

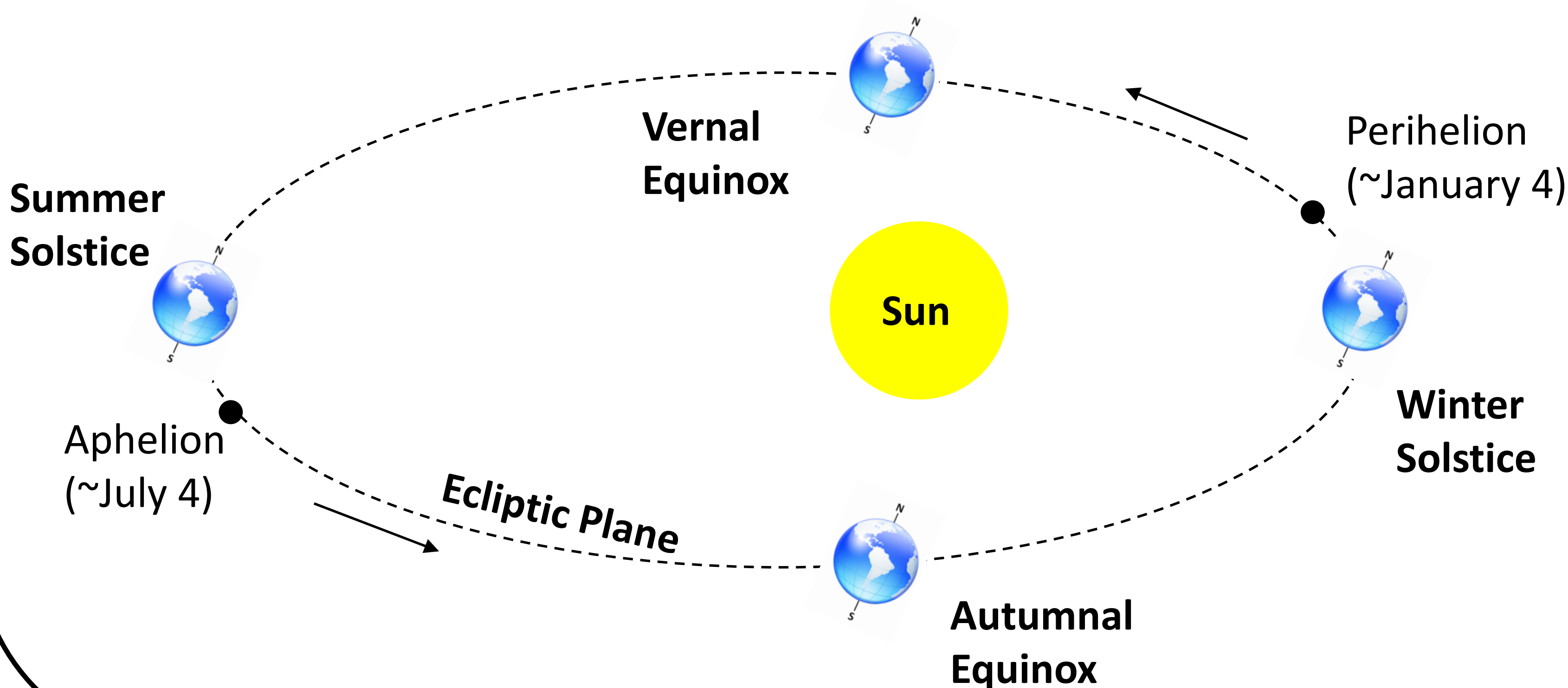
The beta angle, β is the angle between the solar vector, **s**, and its projection onto the orbit plane;



We're going to calculate the beta angle but before we do, we need to explain some concepts.

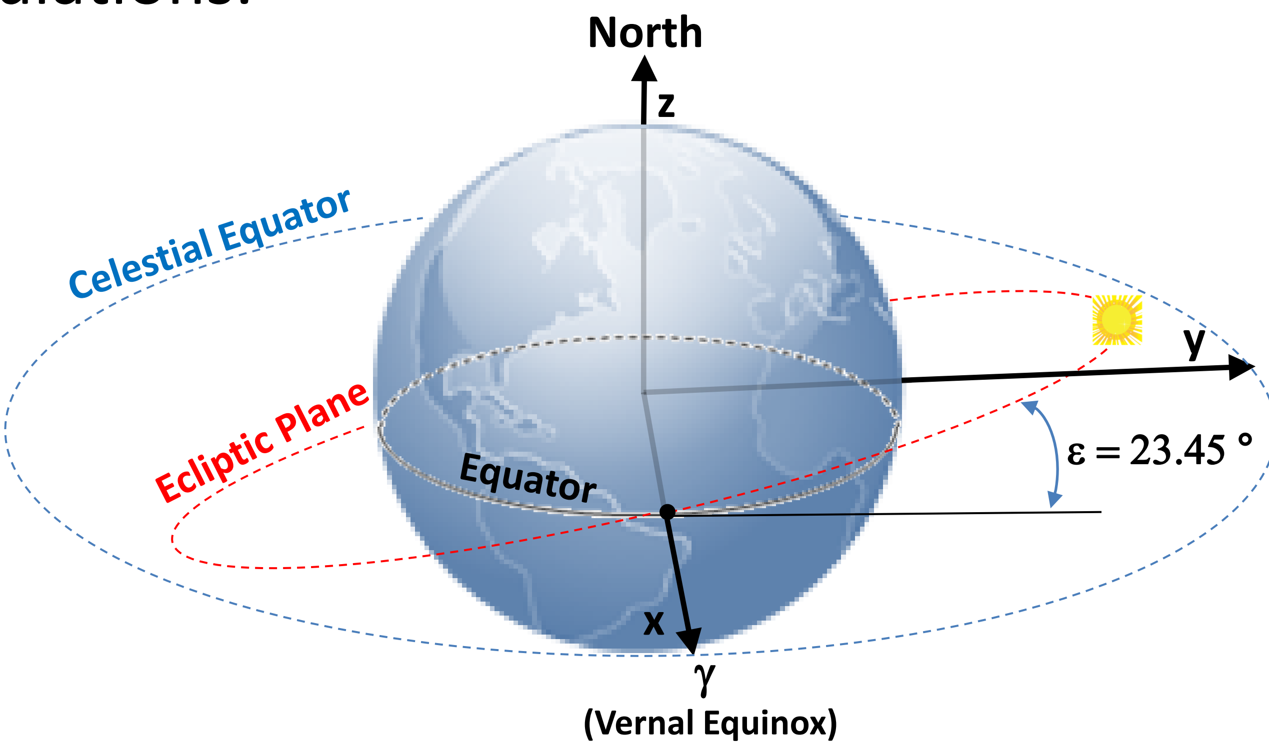
Aside: The Celestial Inertial Coordinate System

A simplified representation of Earth's orbit about the sun is shown below:



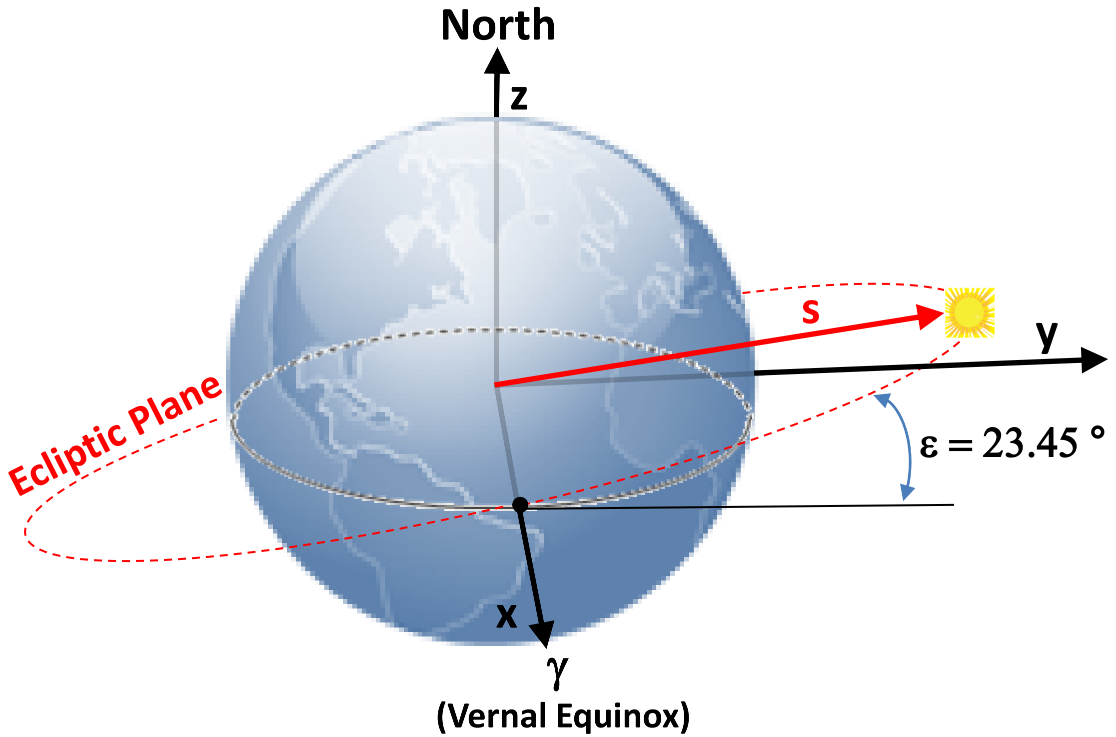
Aside: The Celestial Inertial Coordinate System

The celestial inertial coordinate system is convenient for performing on-orbit thermal environment calculations.



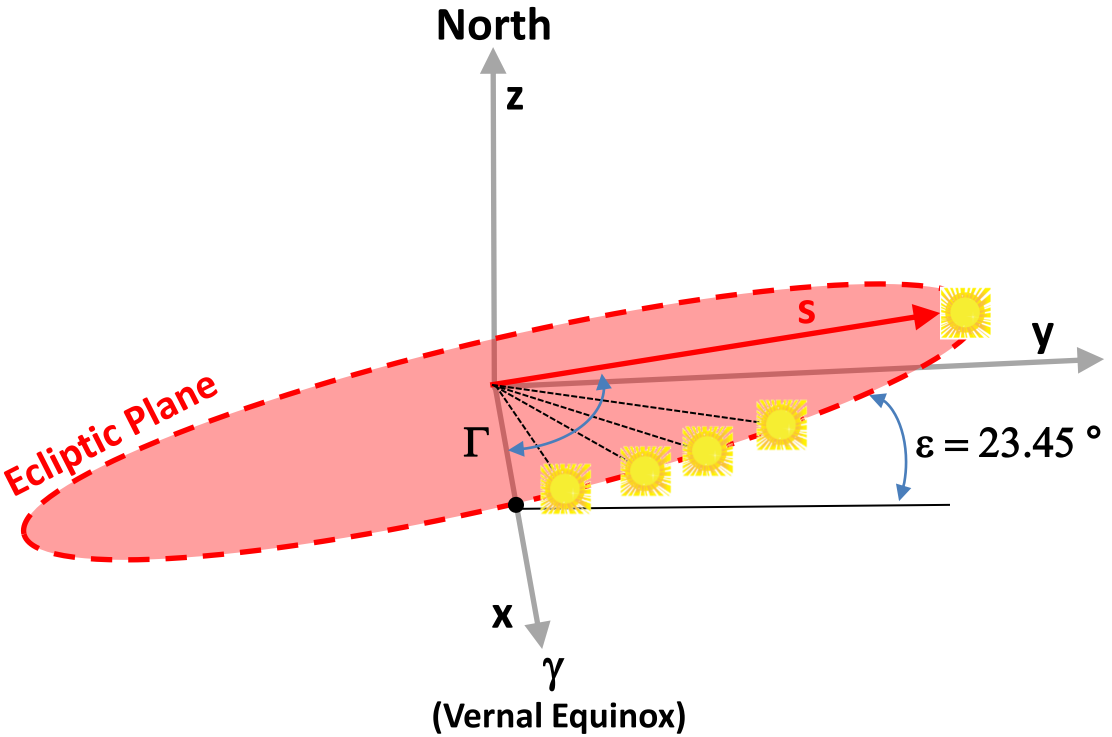
The Solar Vector

We define the solar vector, **s**, as a unit vector in the celestial inertial coordinate system that points toward the sun.



The Solar Vector

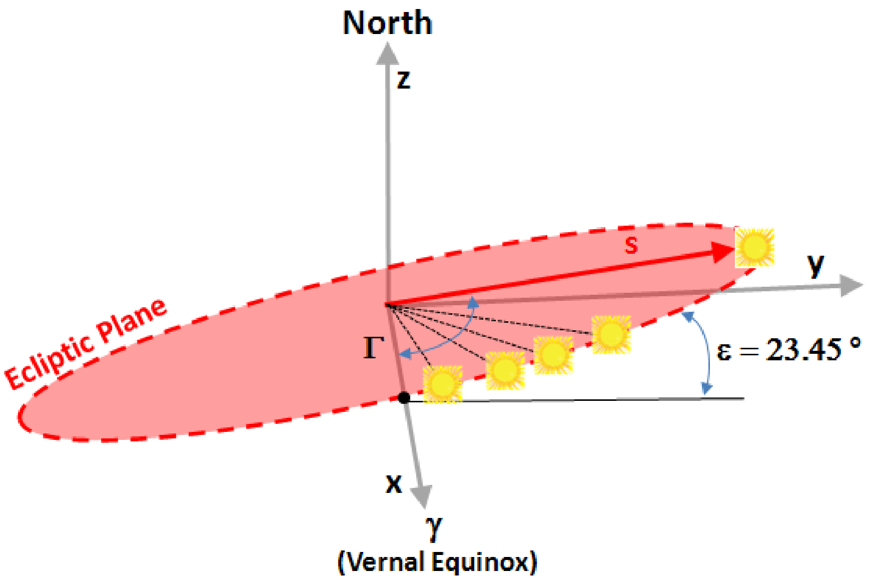
The apparent motion of the sun is constrained to the Ecliptic Plane and is governed by two parameters: Γ and ε.



The Solar Vector

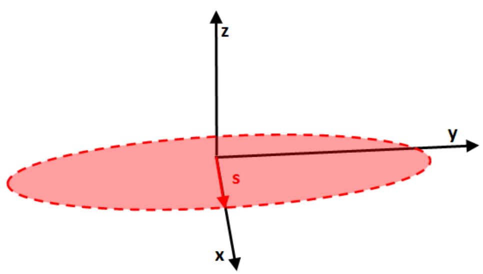
ε is the **Obliquity of the Ecliptic** and, for Earth, is presently 23.45°;

Γ is the **Ecliptic True Solar Longitude** and changes with date -- Γ is 0° when the sun it at the Vernal Equinox.

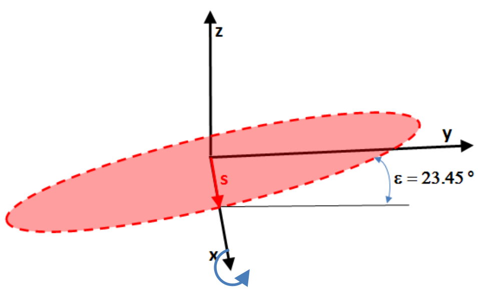


The Solar Vector

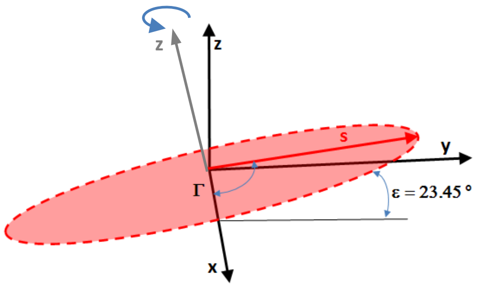
We can form the solar vector via two Euler angle transformations: first a rotation of the unit vector of ε about the x-axis and then a rotation of Γ about the *new* z-axis.



Unit Vector,
No Rotation



First Rotation,
ε about x-axis

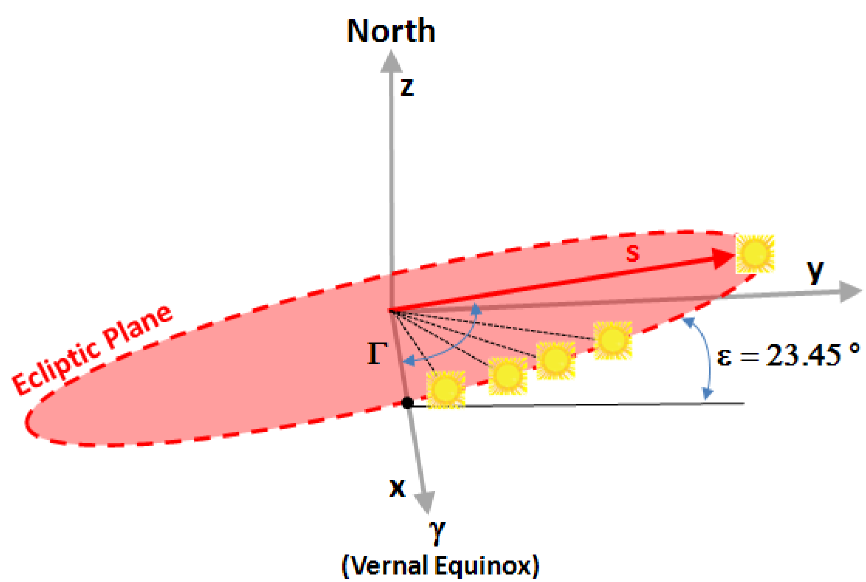


Second Rotation,
Γ about new z-axis

The Solar Vector

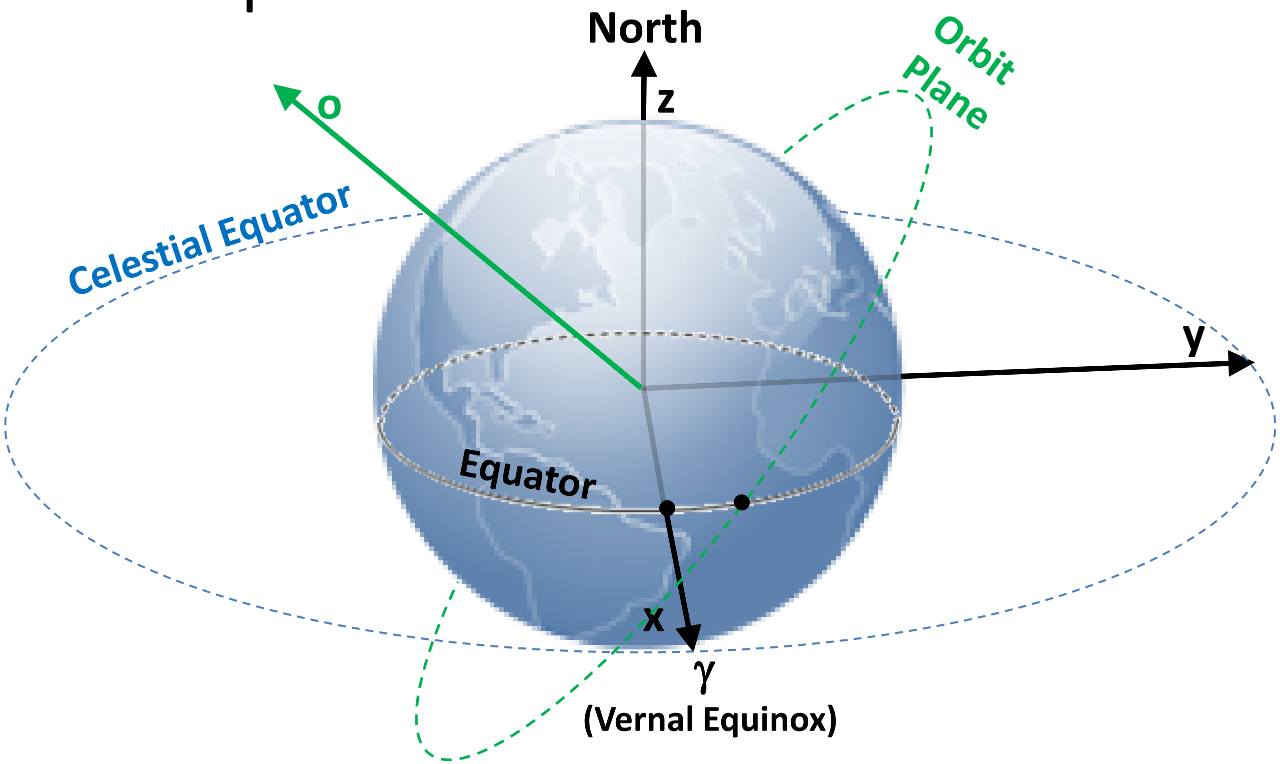
Mathematically, the transformation is expressed as:

$$\hat{s} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & -\sin\varepsilon \\ 0 & \sin\varepsilon & \cos\varepsilon \end{bmatrix} \begin{bmatrix} \cos\Gamma & -\sin\Gamma & 0 \\ \sin\Gamma & \cos\Gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \cos\Gamma \\ \sin\Gamma\cos\varepsilon \\ \sin\Gamma\sin\varepsilon \end{Bmatrix}$$



The Orbit Normal Vector

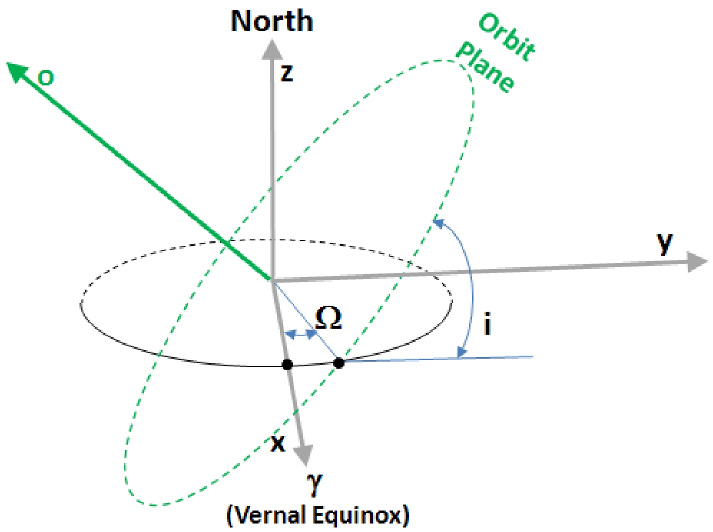
In the same celestial inertial coordinate system, we define the vector, **o**, as a unit vector pointing normal to the orbit plane.



The Orbit Normal Vector

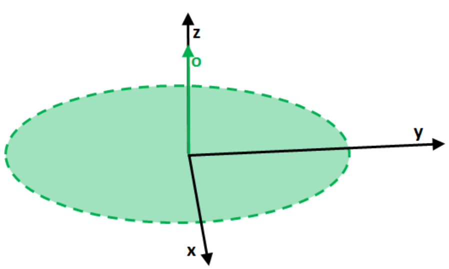
i is the **Orbit Inclination** -- a measure of angular tilt from the equatorial plane;

Ω is the **Right Ascension of the Ascending Node** -- a measure of angle between the x-axis at the point where the orbit cross the equatorial plane going from south to north.

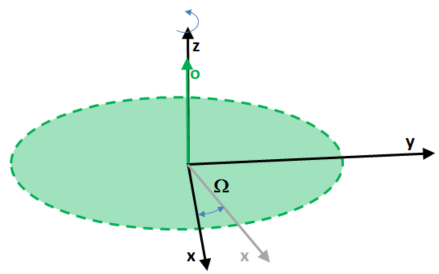


The Orbit Normal Vector

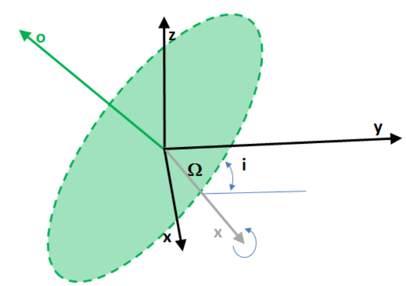
We can form the orbit normal vector via two Euler angle transformations: first a rotation of the unit vector of Ω about the z-axis and then a rotation of i about the *new* x-axis.



Unit Vector, No Rotation



First Rotation, Ω about z-axis

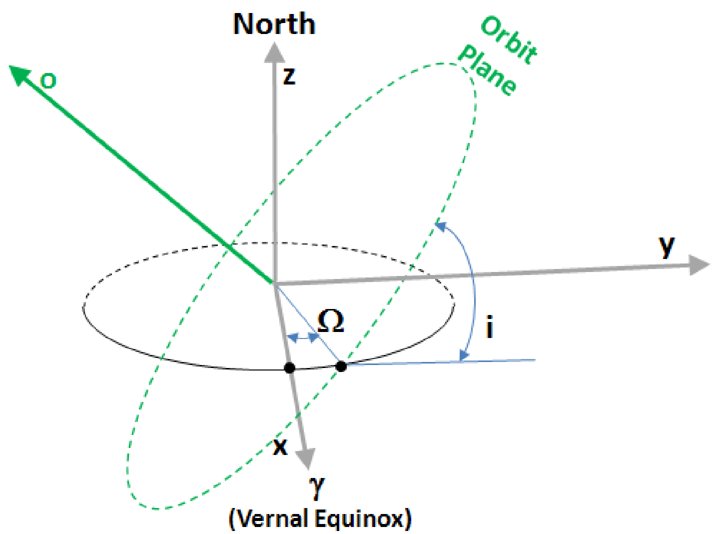


Second Rotation, i about new x-axis

The Orbit Normal Vector

Mathematically, the transformation is expressed as:

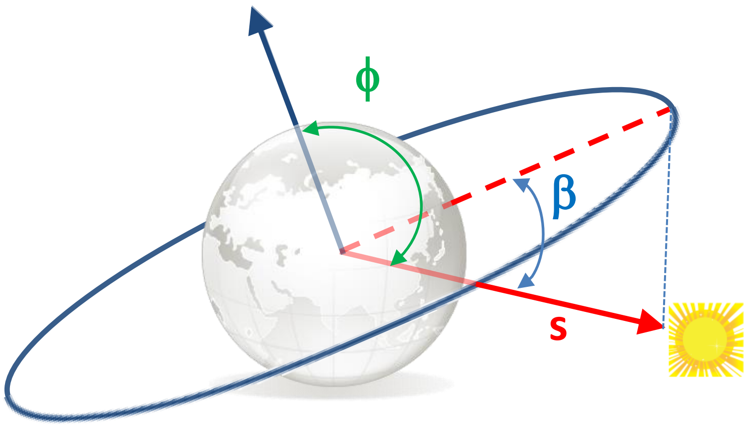
$$\hat{o} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} \sin\Omega \sin i \\ -\cos\Omega \sin i \\ \cos i \end{Bmatrix}$$



Calculating the Beta Angle

To most easily calculate the angle between a vector and a plane, it is necessary to determine the angle between the vector and a vector normal to the plane, denoted here by ϕ;

We note that $\beta = \phi - (\pi/2)$ radians.



Calculating the Beta Angle

The beta angle, then, is given by:

$$\cos\phi = \hat{o}\cdot\hat{s} = \begin{Bmatrix} \sin\Omega\sin i \\ -\cos\Omega\sin i \\ \cos i \end{Bmatrix}^T \begin{Bmatrix} \cos\Gamma \\ \sin\Gamma\cos\varepsilon \\ \sin\Gamma\sin\varepsilon \end{Bmatrix}$$

$$\cos\phi = \cos\Gamma\sin\Omega\sin i - \sin\Gamma\cos\varepsilon\cos\Omega\sin i + \sin\Gamma\sin\varepsilon\cos i$$

But, since β = ϕ - (π/2) radians:

$$\beta = \sin^{-1}(\cos\Gamma\sin\Omega\sin i - \sin\Gamma\cos\varepsilon\cos\Omega\sin i + \sin\Gamma\sin\varepsilon\cos i)$$

Calculating the Beta Angle

We see that β is limited by:

$$\beta = \pm(\varepsilon + |i|)$$

Beta angles where the sun is north of the orbit plane are considered positive -- beta angles where the sun is south of the orbit are considered negative.

Consequences of Beta Angle Variation

Variation of the Beta Angle

Our expression for β, repeated here for convenience is:

$$\beta = \sin^{-1}(\cos\Gamma \sin\Omega \sin i - \sin\Gamma \cos\varepsilon \cos\Omega \sin i + \sin\Gamma \sin\varepsilon \cos i)$$

The beta angle is not static and varies constantly;

Two factors that affect β variation the most:

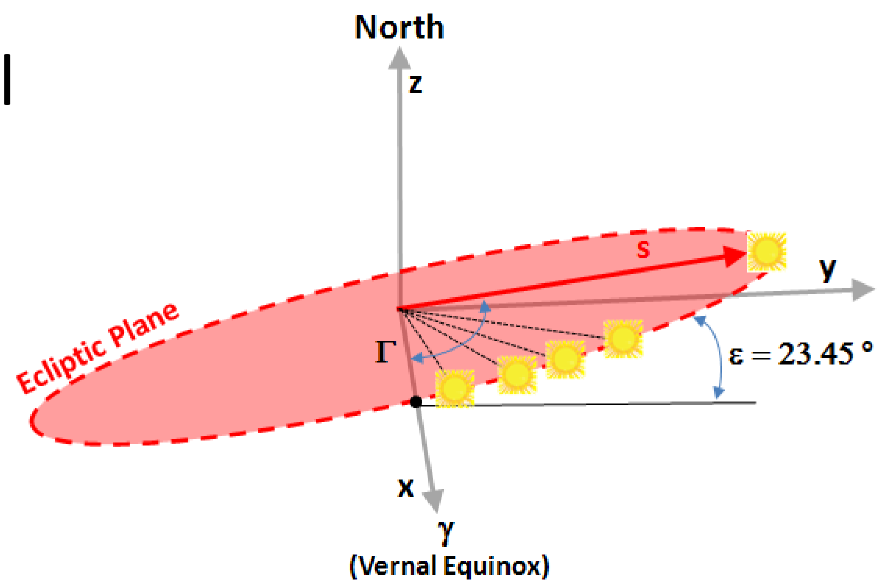
1. The change of seasons (variation in Γ);
2. Perturbation of the orbit due to the oblateness of the planet (variation in Ω).

Variation Due to the Change of Seasons

It takes the sun just over one year to make one circuit around the celestial sphere;

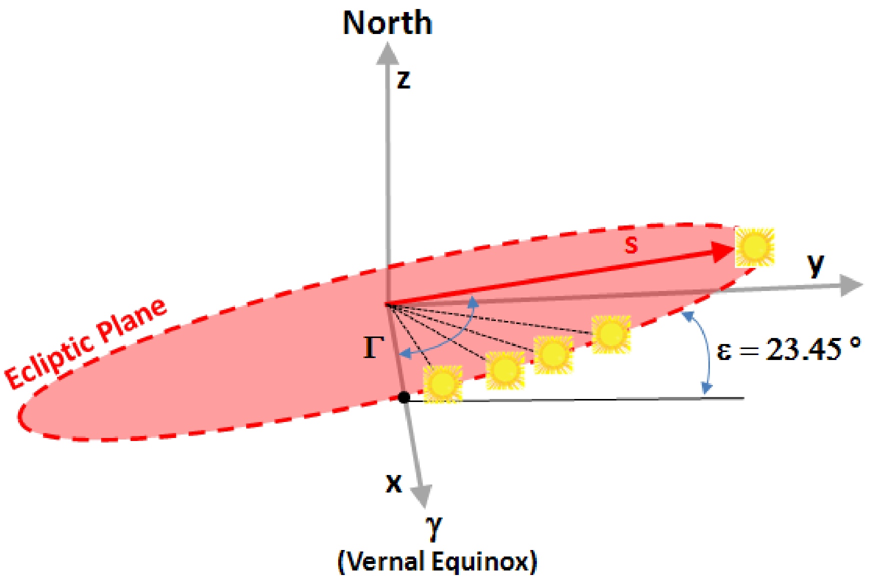
We measure the sun's progress by Γ;

Strictly speaking, the rate that the sun makes this circuit is *not constant*.



Variation Due to the Change of Seasons

Since Earth's orbit about the sun is nearly circular, an approximation of the variation of Γ with time is sufficient.



$$\dot{\Gamma} \approx \frac{360°}{365.25 \text{ days}} = \frac{0.986°}{\text{day}}$$

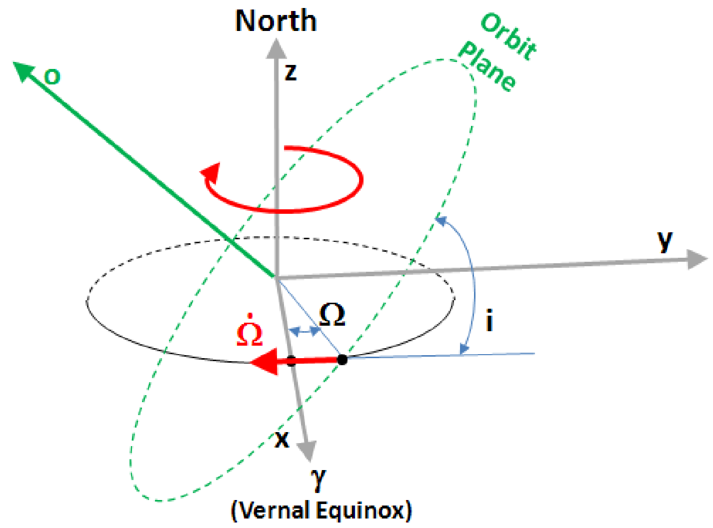
Variation Due to Precession of the Orbit

Earth is not a perfect sphere;

Equatorial bulge produces a torque on the orbit;

Effect is a precession of the orbit ascending node;

Precession is a function of orbit altitude and inclination.

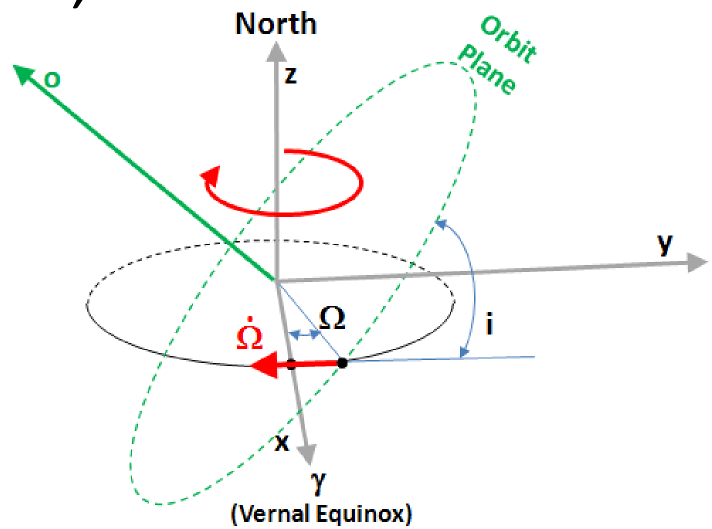


Variation Due to Precession of the Orbit

This variation is called the Ascending Node Angular Rate, $\dot{\Omega}$, and is given by:

$$\dot{\Omega} = -\frac{3}{2} J_2 \left(\frac{r_{eq}}{p}\right)^2 \sqrt{\frac{\mu}{r^3}} \cos i$$

where J_2 is the oblateness perturbation, r_{eq} is the planet equatorial radius, p is the "parameter", μ is the planet mass $\times$ G, r is the orbit radius and i is inclination.

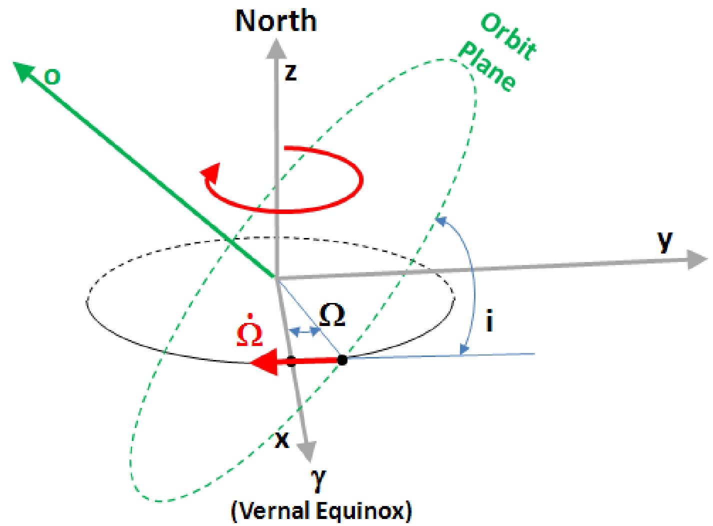


Variation Due to Precession of the Orbit

For Earth, typical values for these parameters are (Ref. 4):

$J_2 = 1082.62 \times 10^{-6}$
$r_{eq} = 6378.1$ km
$\mu = 0.3986 \times 10^6$ km^3/s^2

For a typical ISS orbit...
r = 6378.1 km + 408 km
Circular (e = 0, a = r)
i = 51.6°



Variation Due to Precession of the Orbit

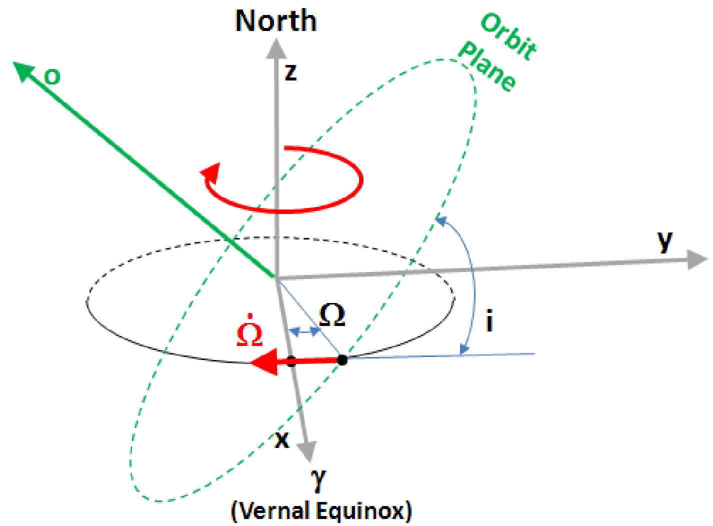
The "parameter" is calculated as:

$$p = a(1 - e^2) = r$$

The resulting precession is:

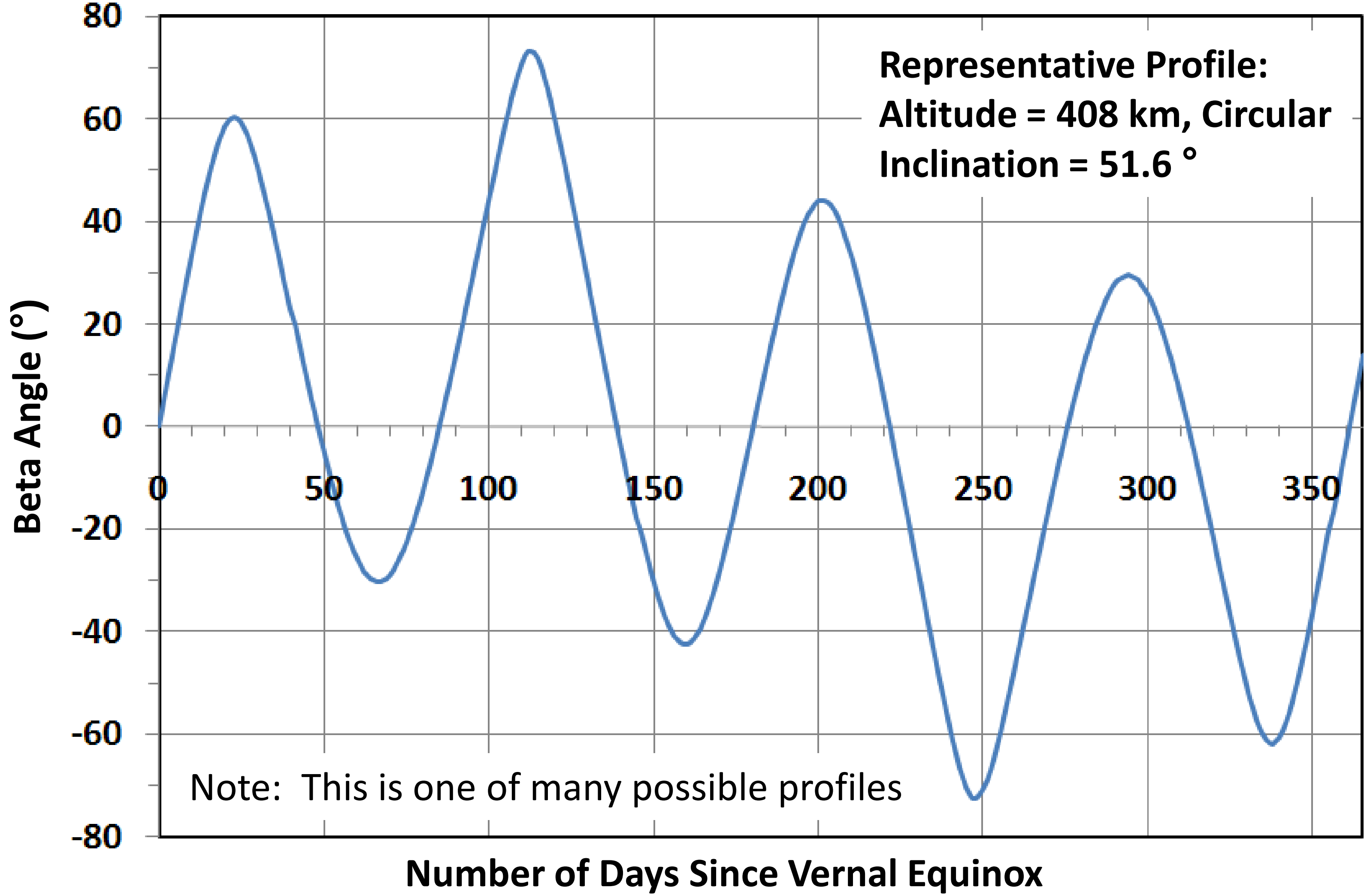
$\dot{\Omega}$ = -1.00634×10^{-6} rad/s

$\dot{\Omega}$ = -4.98 °/day



$$\dot{\Omega} = -\frac{3}{2} J_2 \left(\frac{r_{eq}}{p}\right)^2 \sqrt{\frac{\mu}{r^3}} \cos i$$

Variation of the Beta Angle Due to Seasonal Variation and Orbit Precession



Consequences of Beta Angle Variation

As β changes, there are two consequences of interest to thermal engineers:

1) The time spent in eclipse (i.e., planet shadow) varies;

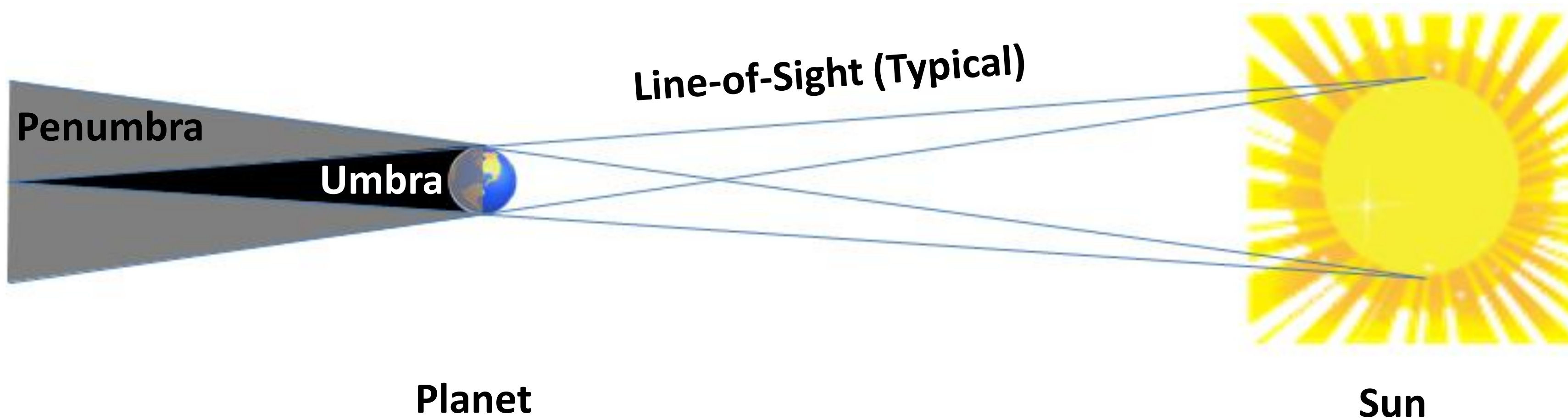
2) The intensity and direction of heating incident on spacecraft surfaces changes;

Let's explore each of these effects.

Eclipse: Umbra and Penumbra

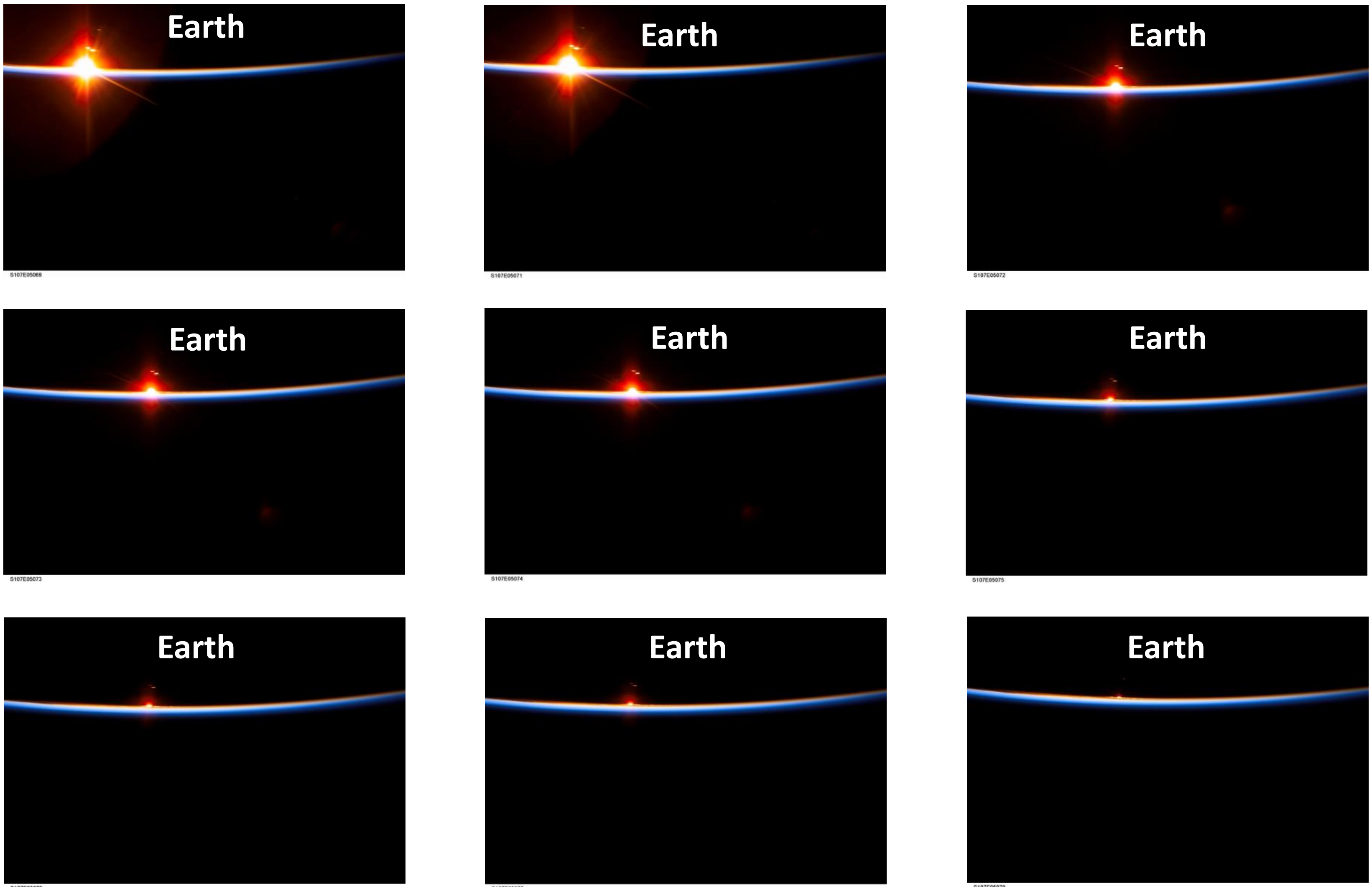
Umbral region - sunlight is completely obscured;

Penumbral region - sunlight is partially obscured.



Not to Scale

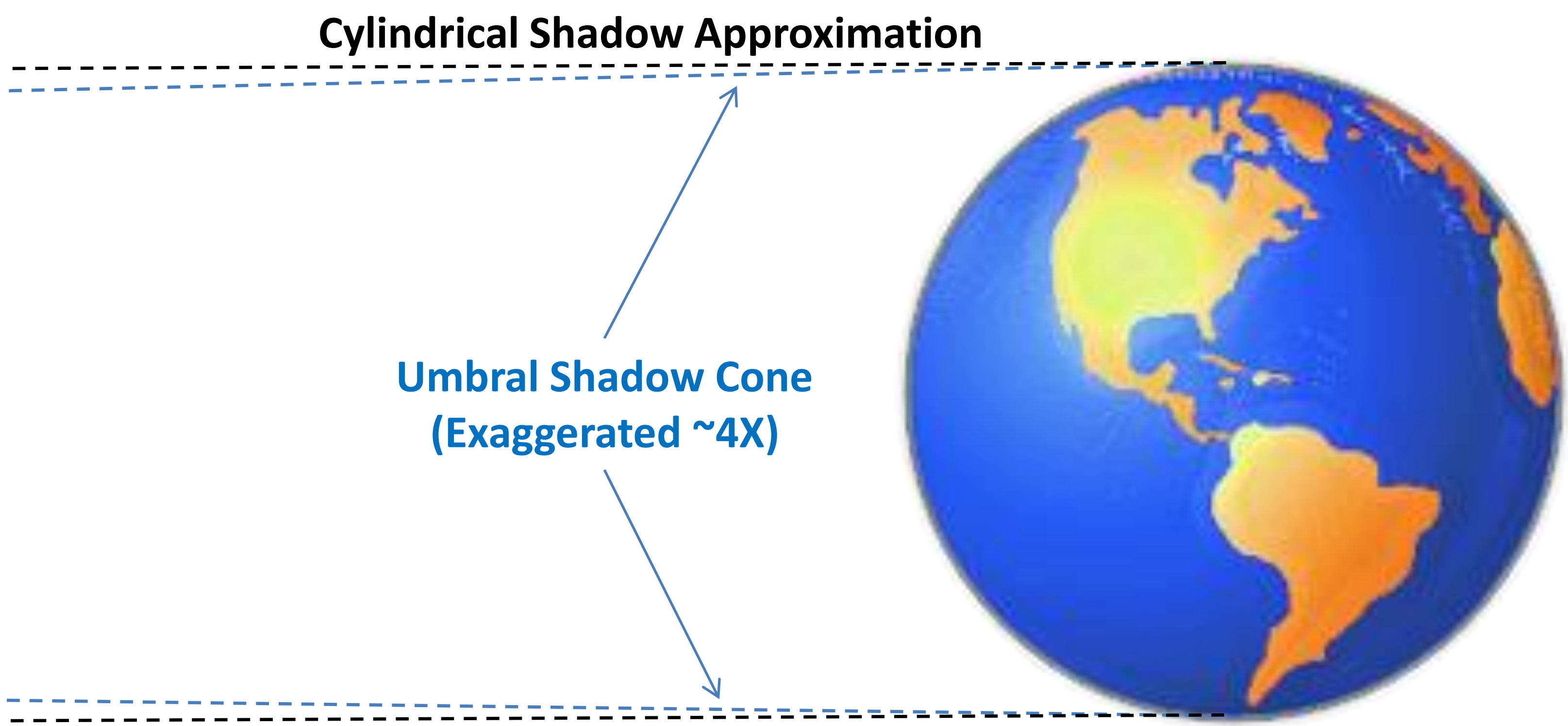
Orbital Sunset: From Penumbra to Umbra



NASA Photos

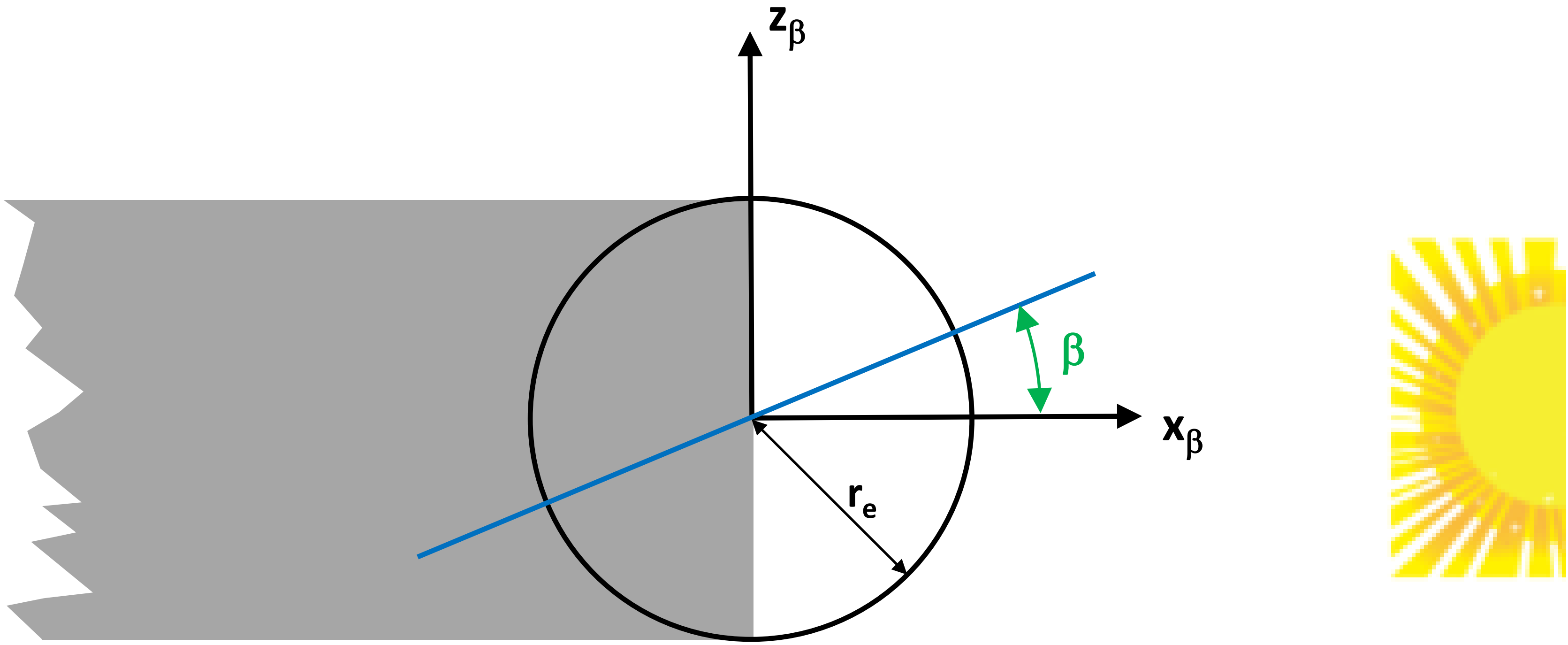
Eclipse: Umbra and Penumbra

If time in penumbra is minimal, analysis may be simplified using a cylindrical shadow assumption.



Geometry for Eclipse Calculation (Low, Circular Orbit Only)

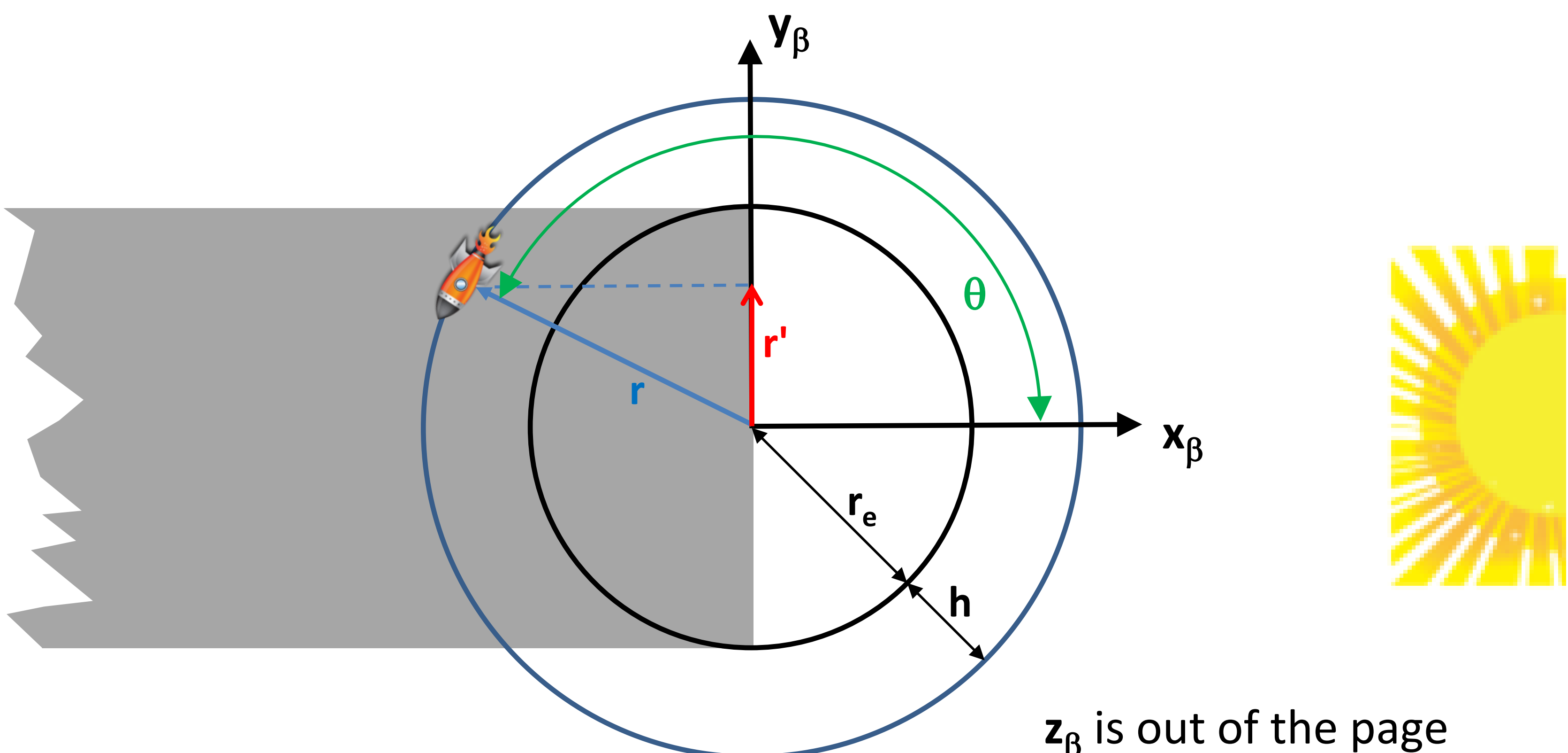
We create a new coordinate system where the sun is always in the xy-plane and the orbit is inclined β;



y_β is in to the page

Geometry for Eclipse Calculation (Low, Circular Orbit Only)

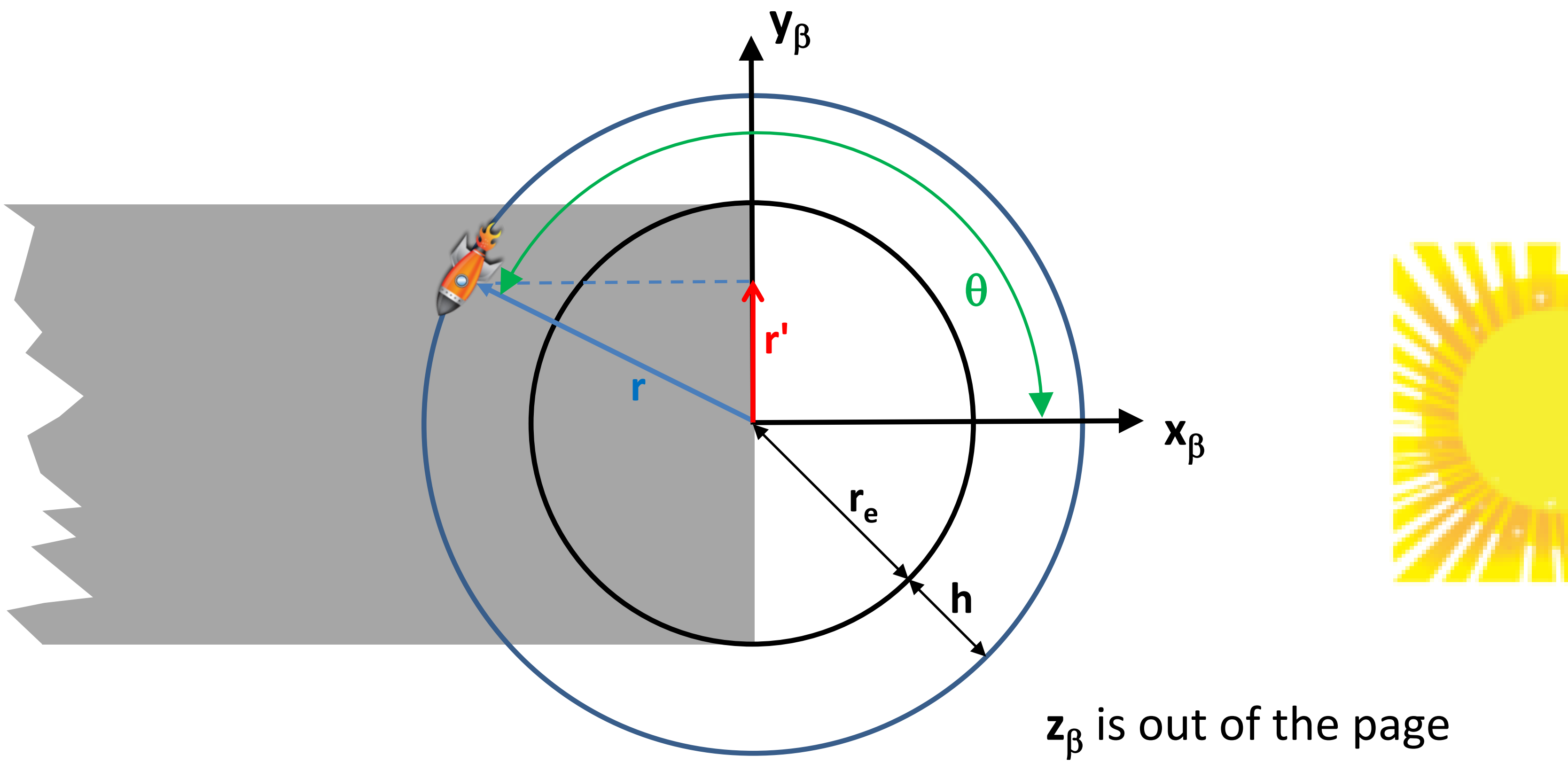
Looking down onto the orbit plane gives us this geometry (when $\beta = 0°$).



z_β is out of the page

Geometry for Eclipse Calculation (Low, Circular Orbit Only)

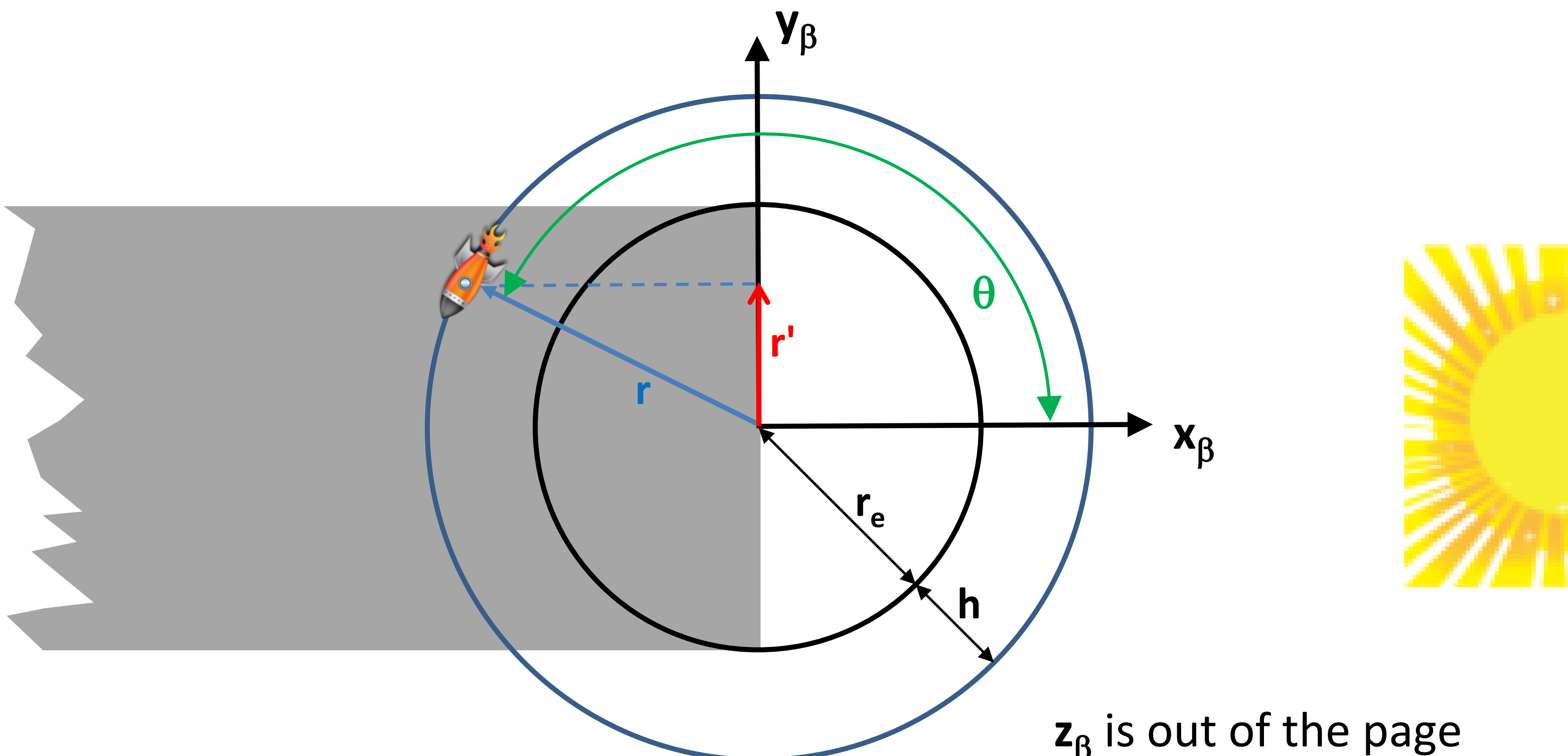
We seek an expression for **r'** which is a projection of **r** onto the $\mathbf{y}_\beta \mathbf{z}_\beta$-plane.



$\mathbf{z}_\beta$ is out of the page

Geometry for Eclipse Calculation (Low, Circular Orbit Only)

When |**r'**| < r_e, the spacecraft is in the umbral shadow.



z_β is out of the page

Calculating Umbral Eclipse Entry (Low, Circular Orbit Only)

Spacecraft position vector, **r**, can be expressed as a function of altitude above planet, h, planet radius, r_e, angle from orbit noon, θ, and beta angle, β:

$$\vec{r} = (r_e + h)\left[\cos\theta\cos\beta\,\hat{\imath} + \sin\theta\,\hat{\jmath} + \cos\theta\sin\beta\,\hat{k}\right]$$

The magnitude of this vector is:

$$|\vec{r}| = (r_e + h)\sqrt{\cos^2\theta\cos^2\beta + \sin^2\theta + \cos^2\theta\sin^2\beta}$$

This reduces to:

$$|\vec{r}| = r_e + h$$

Calculating Umbral Eclipse Entry (Low, Circular Orbit Only)

The projection of this vector onto the $y_\beta z_\beta$-plane is given by:

$$\vec{r}' = (r_e + h)\left[\sin\theta\,\hat{j} + \cos\theta\sin\beta\,\hat{k}\right]$$

And the magnitude is given by:

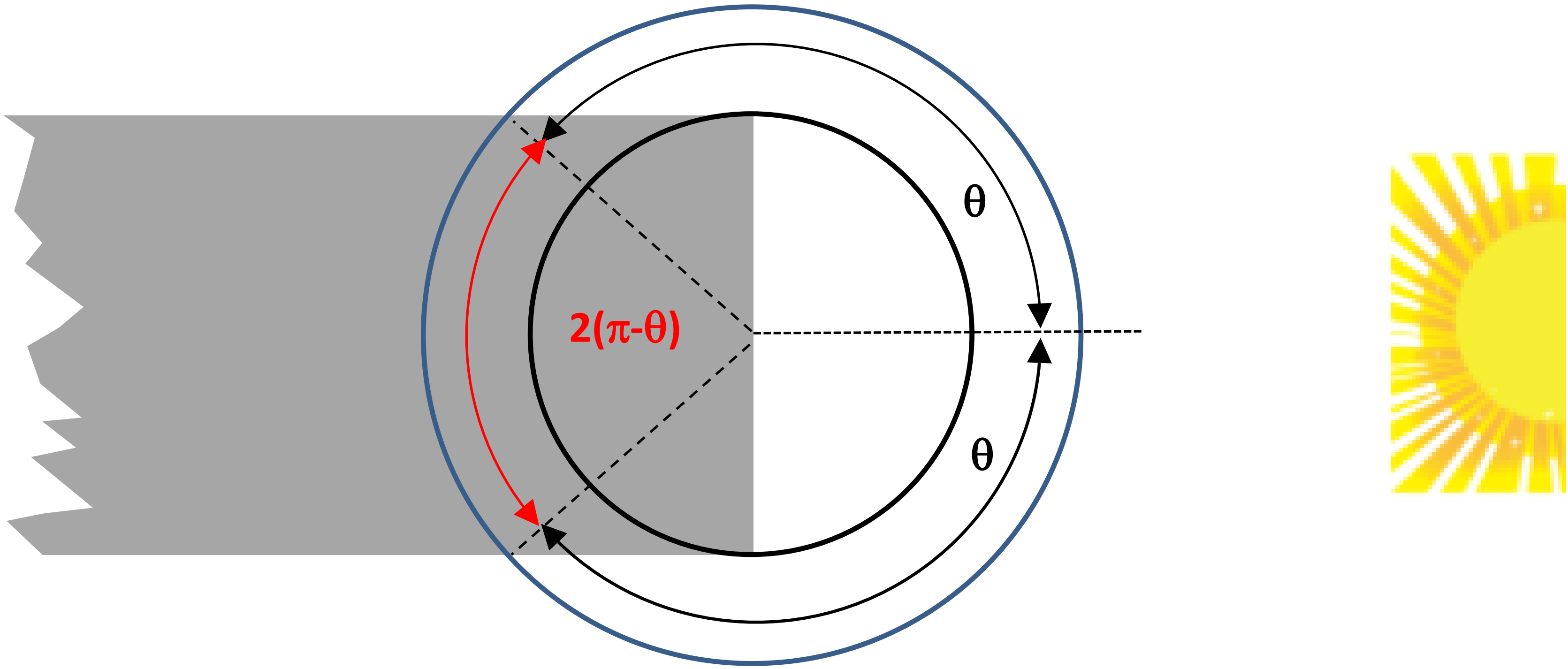
$$|\vec{r}'| = (r_e + h)\sqrt{\sin^2\theta + \cos^2\theta\sin^2\beta}$$

The onset of shadowing occurs when $|\mathbf{r}'| < r_e$:

$$\sin\theta = \sqrt{\frac{1}{\cos^2\beta}\left[\left(\frac{r_e}{r_e + h}\right)^2 - \sin^2\beta\right]}$$

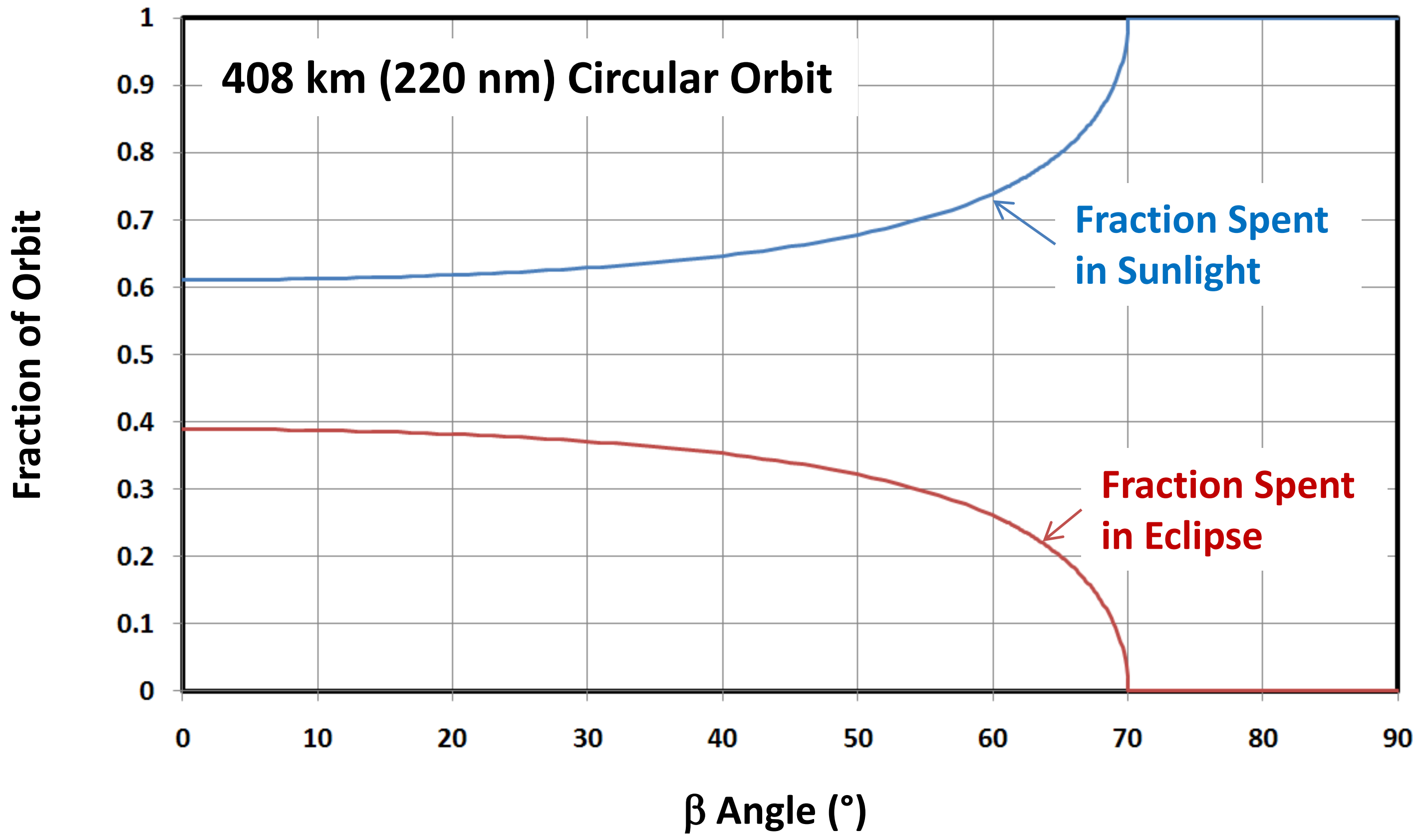
Calculating Umbral Eclipse Entry/Exit (Low, Circular Orbit Only)

Now that the θ of eclipse onset is known, it is a simple matter to determine the entire eclipse period by noting that the total angle shadowed is $2(\pi-\theta)$:

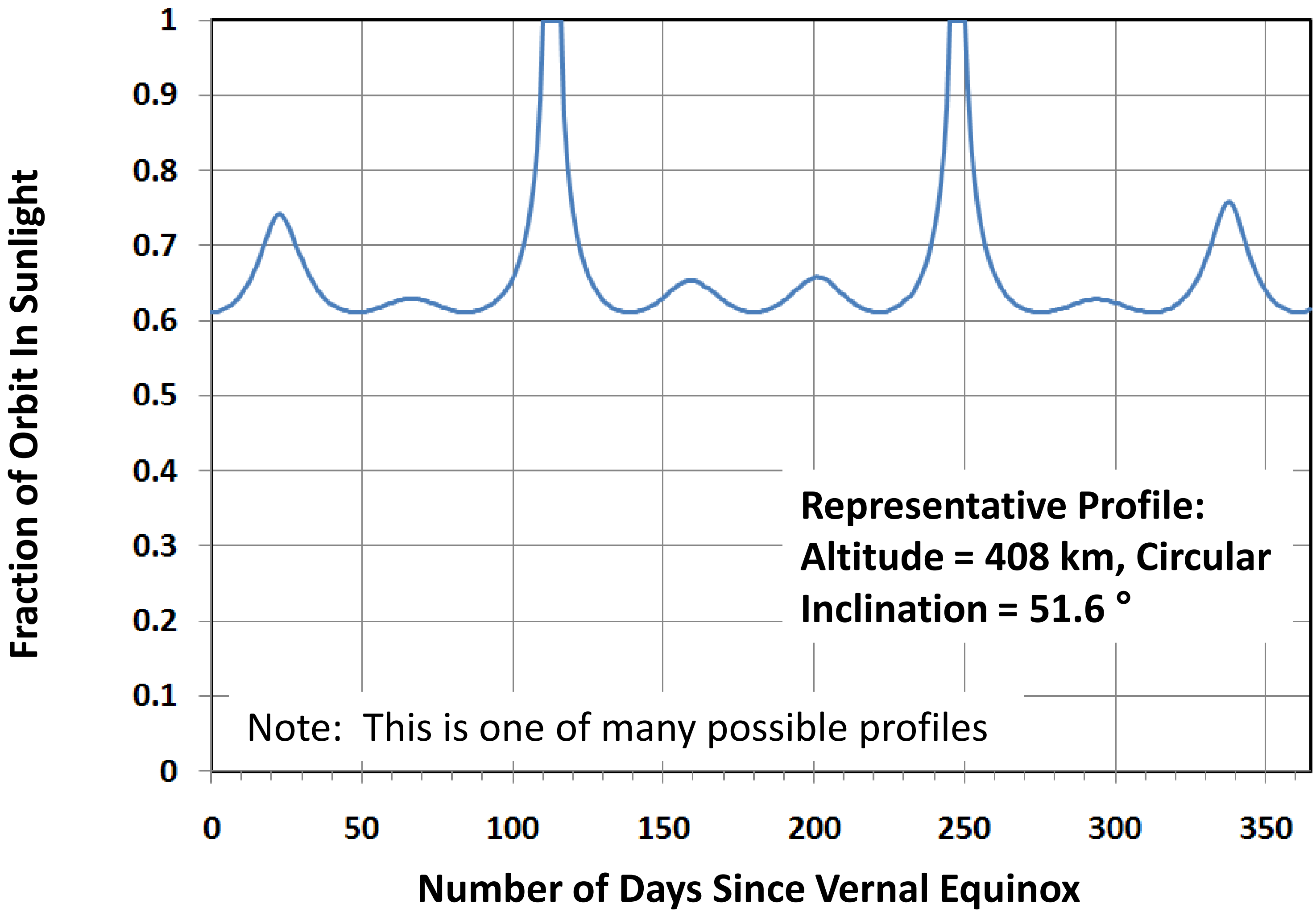


Fraction of Orbit Spent in Sunlight/Eclipse

The fraction of orbit spent in sunlight and eclipse for a circular orbit is clearly related to β:

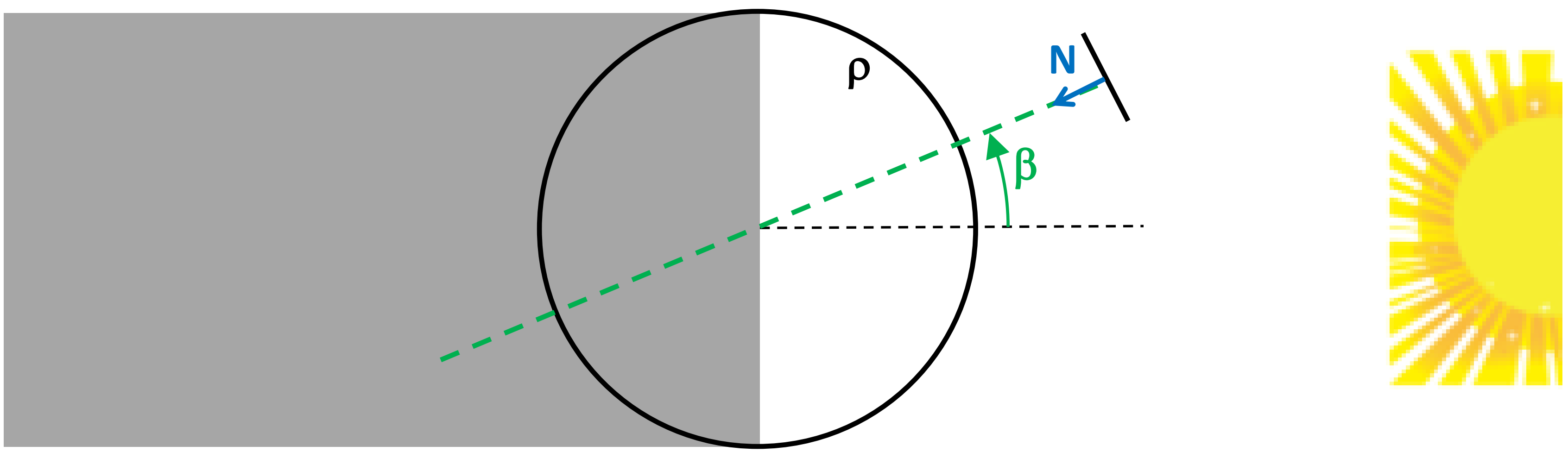


Variation of the Orbit Eclipse Period as a Function of Beta Angle



Variation of the Heating to Surfaces as Function of Beta Angle

To study this, we will consider an orbiting plate and look at the effect of β angle on heating to a planet-facing (**N** = nadir) surface.

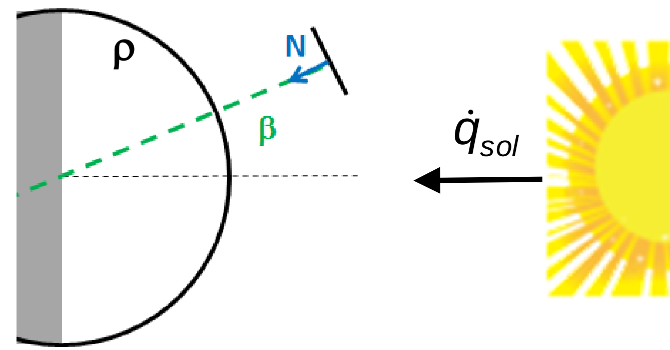


Variation of the Solar Heating to a Nadir-Facing Surface as a Function of Beta Angle

At orbit noon (θ = 0°), the nadir-facing surface has no view to the incoming solar flux;

As θ varies past 90°, the heating scales with -cos θ.

As β increases, the projected area scales with cos β;



$$(\dot{q}_{sol})_n = -\dot{q}_{sol} \cos\theta \cos\beta$$

for:

$$+90° \leq \theta \leq \theta_{term\ entry}$$

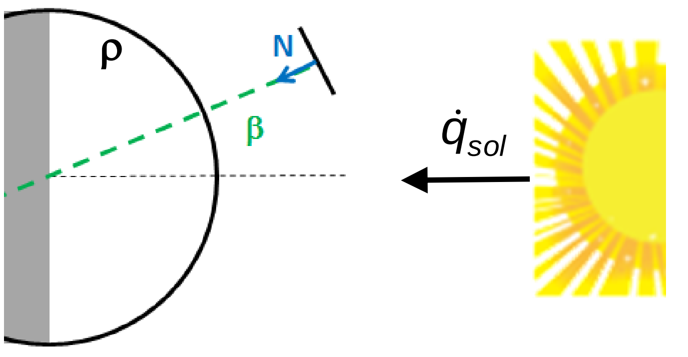
$$\theta_{term\ exit} \leq \theta \leq +270°$$

Variation of the Albedo Heating to a Nadir-Facing Surface as a Function of Beta Angle

For a given β, at orbit noon (θ = 0°), the nadir-facing surface has a maximum view to the sun lit planet;

As β increases and θ increases from orbit noon, the solar zenith angle, ξ, increases.

Overall, the heating scales with cos ξ.



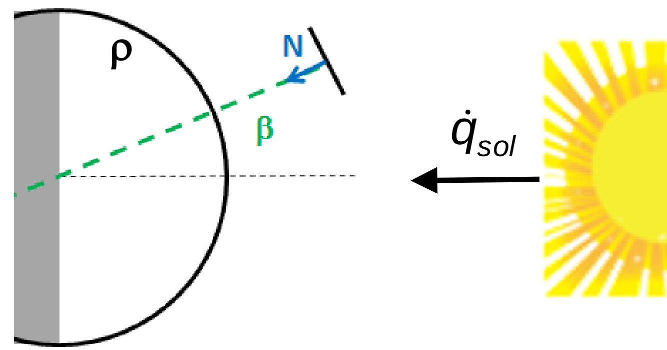
$$(\dot{q}_{alb})_n = \dot{q}_{sol}\rho\left(\frac{r_e}{r_e + h}\right)^2 \cos\xi$$

for:

$$-90° \leq \xi \leq +90°$$

Variation of the Planetary Heating to a Nadir-Facing Surface as a Function of Beta Angle

From our assumption of a constant planetary flux, the planetary infrared heating to the plate is easily calculated as the product of the planetary flux, scaled by the form factor to the planet.

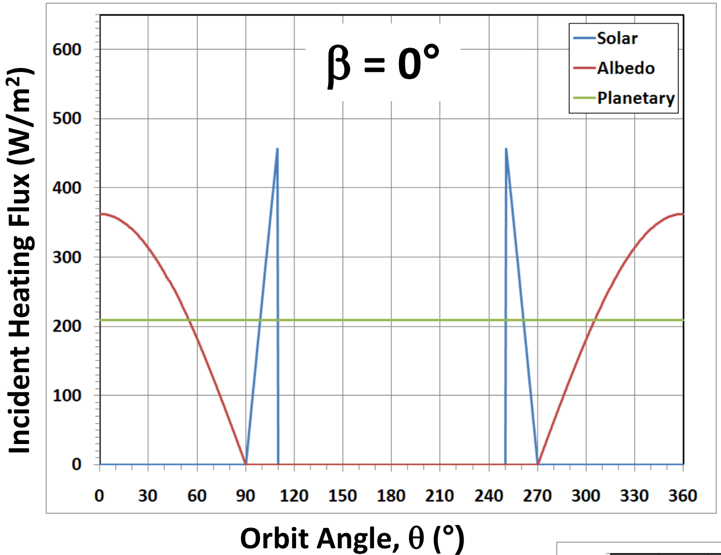


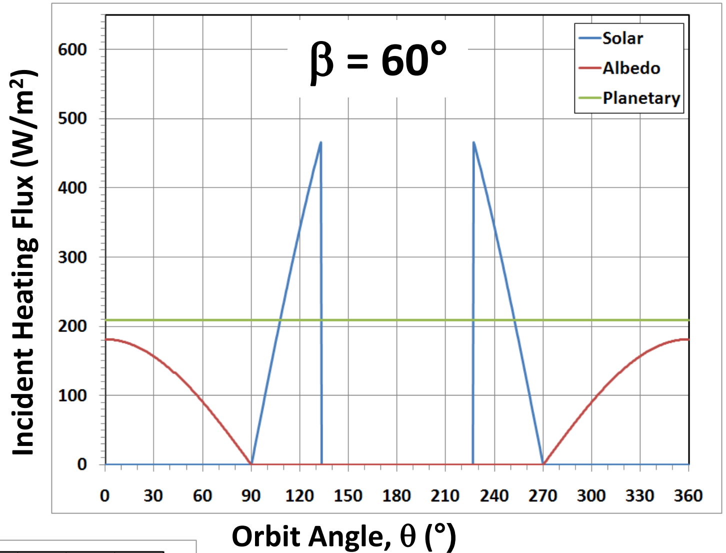
$$\left(\dot{q}_{pla}\right)_n = \dot{q}_{pla}\left(\frac{r_e}{r_e + h}\right)^2$$

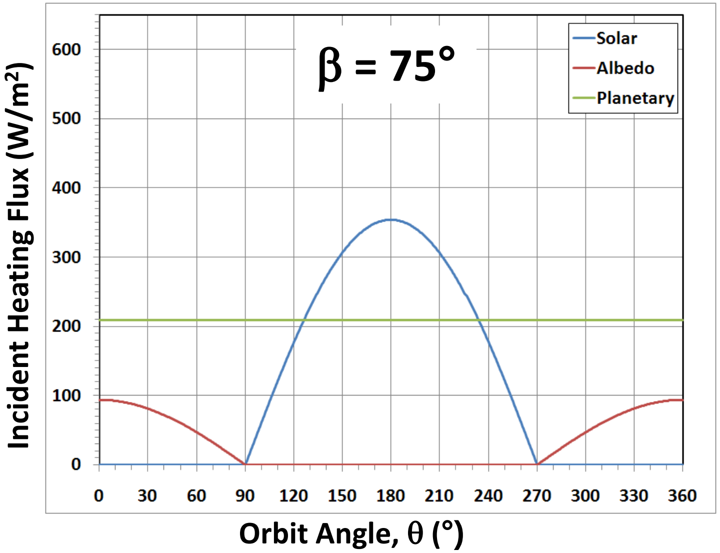
for:

$$0° \leq \theta \leq +360°$$

Variation of the Heating to a Nadir-Facing Surface as a Function of Beta Angle







h = 408 km (Circular)
$\dot{q}_{sol}$ = 1367 W/m^2
ρ = 0.3
$\dot{q}_{pla}$ = 236 W/m^2
r_e = 6378.14 km

Putting It All Together

Putting It All Together

We've looked separately at:

The solar, albedo and planetary infrared heating components;

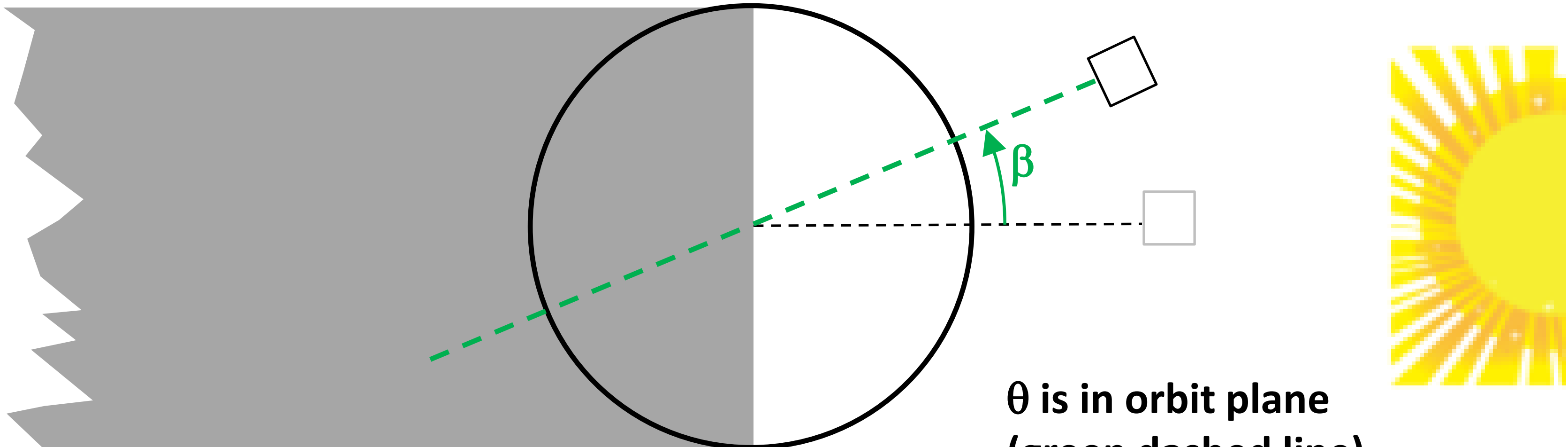
Beta angle and its effect on heating flux and on-orbit eclipse.

Let's pull it all together with an example problem.

Example: The Orbiting Box

Consider a box-like spacecraft orbiting Earth:

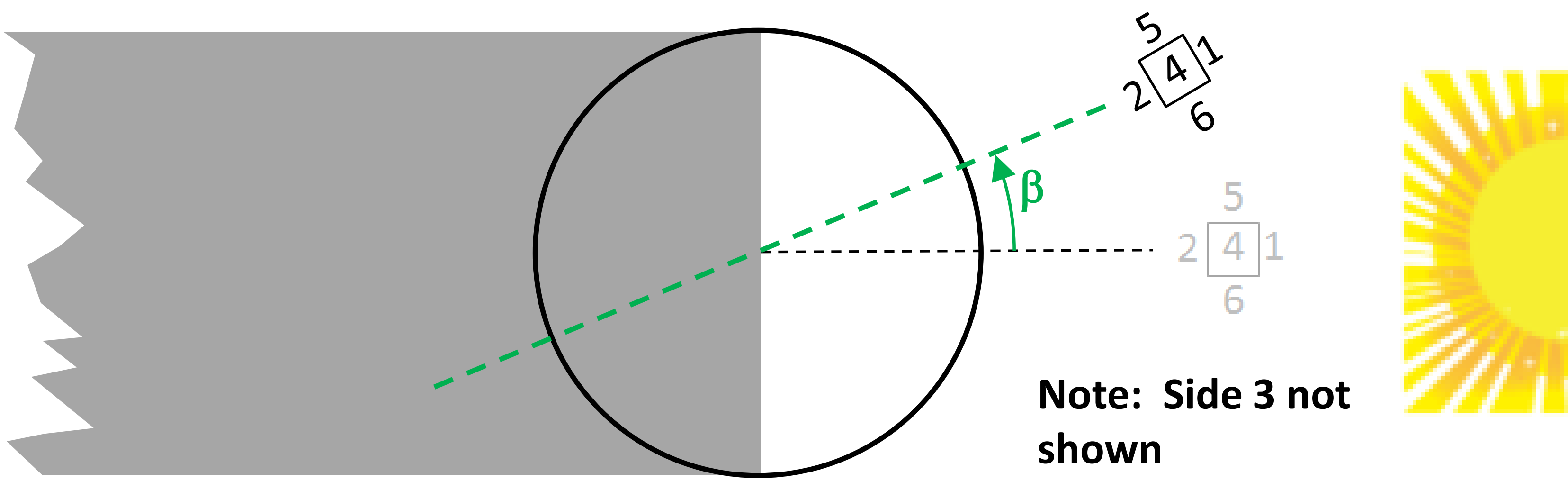
h = 408 km (Circular)
$\dot{q}_{sol} = 1367\ W/m^2$
$\rho = 0.3$
$\dot{q}_{pla} = 236\ W/m^2$
$r_e = 6378.14$ km



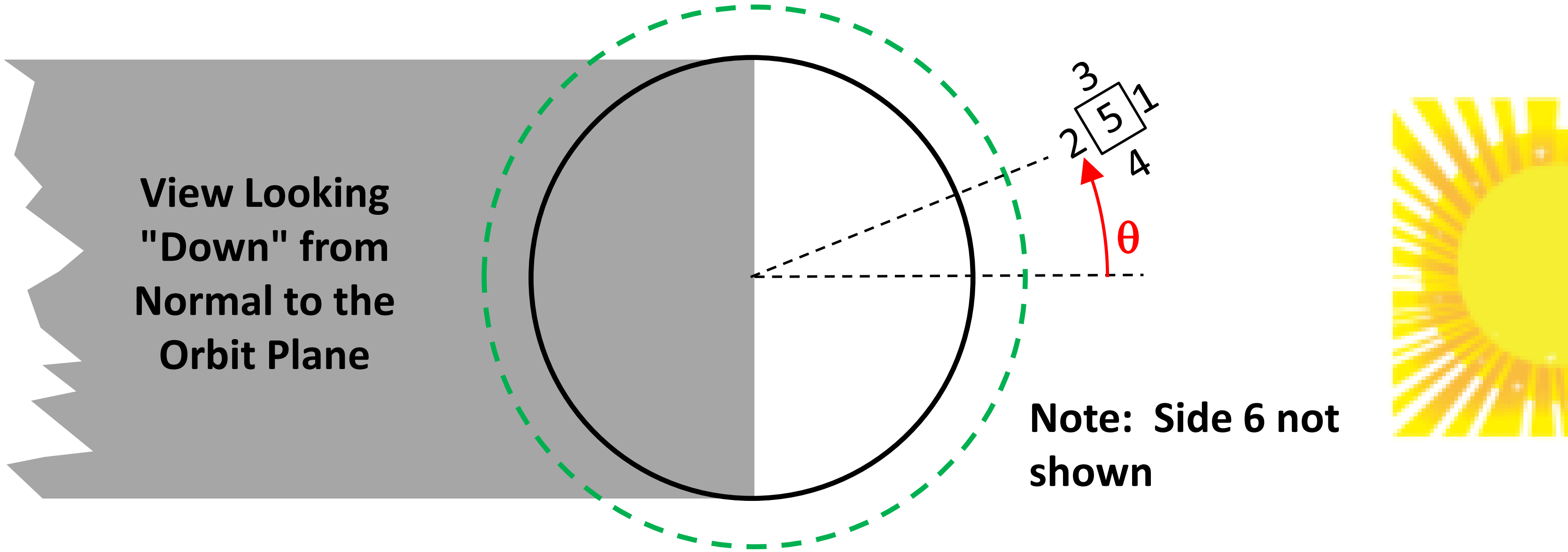
θ is in orbit plane (green dashed line)

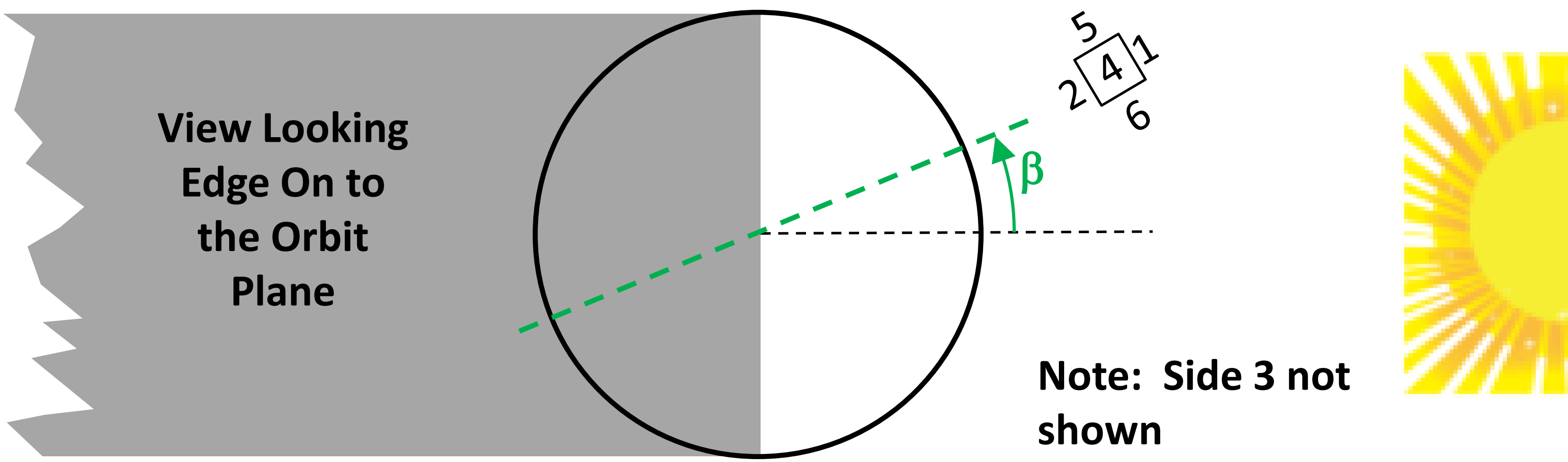
Example: The Orbiting Box

Side 1 - faces away from the planet (zenith-facing)
Side 2 - faces the planet (nadir-facing)
Side 3 - faces forward (velocity vector-facing)
Side 4 - faces aft (anti-velocity vector-facing)
Side 5 - faces north
Side 6 - faces south



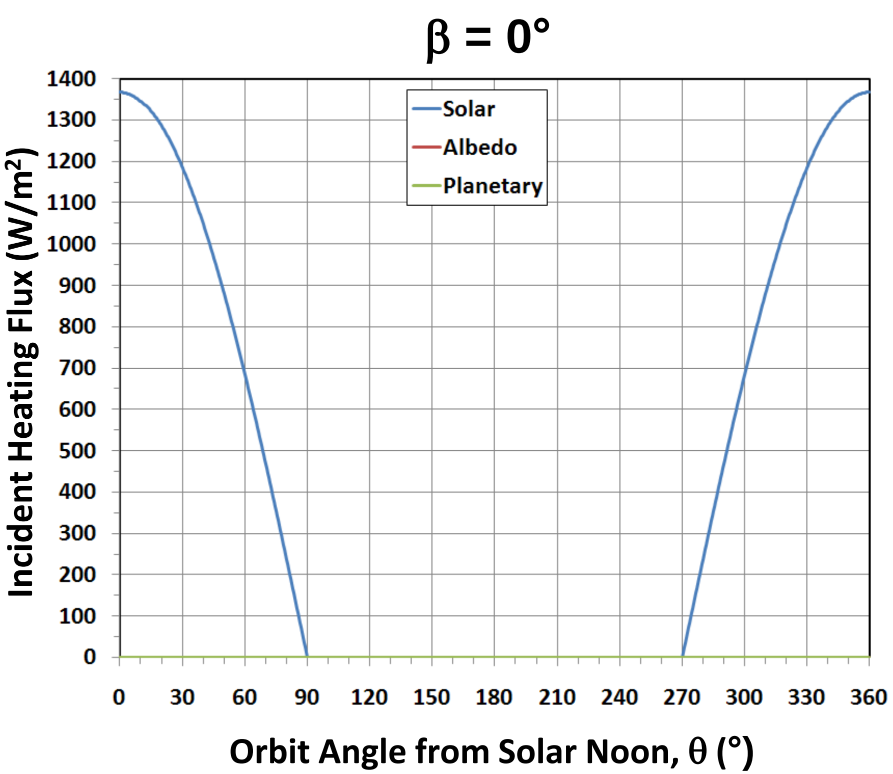
Example: The Orbiting Box

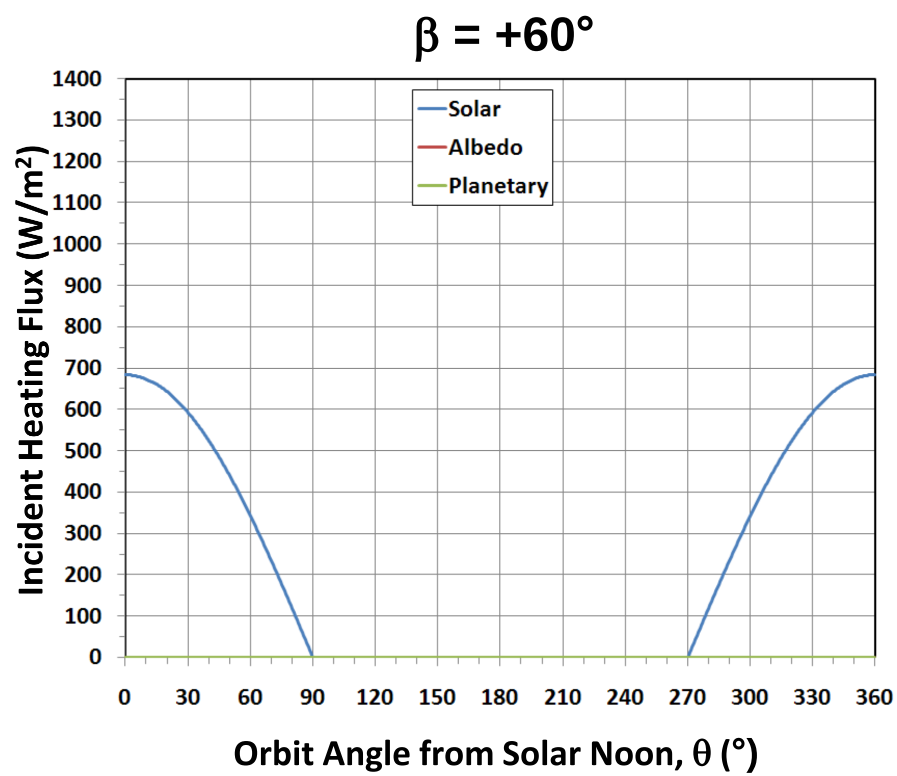




Example: Side 1 - Zenith-Facing Surface

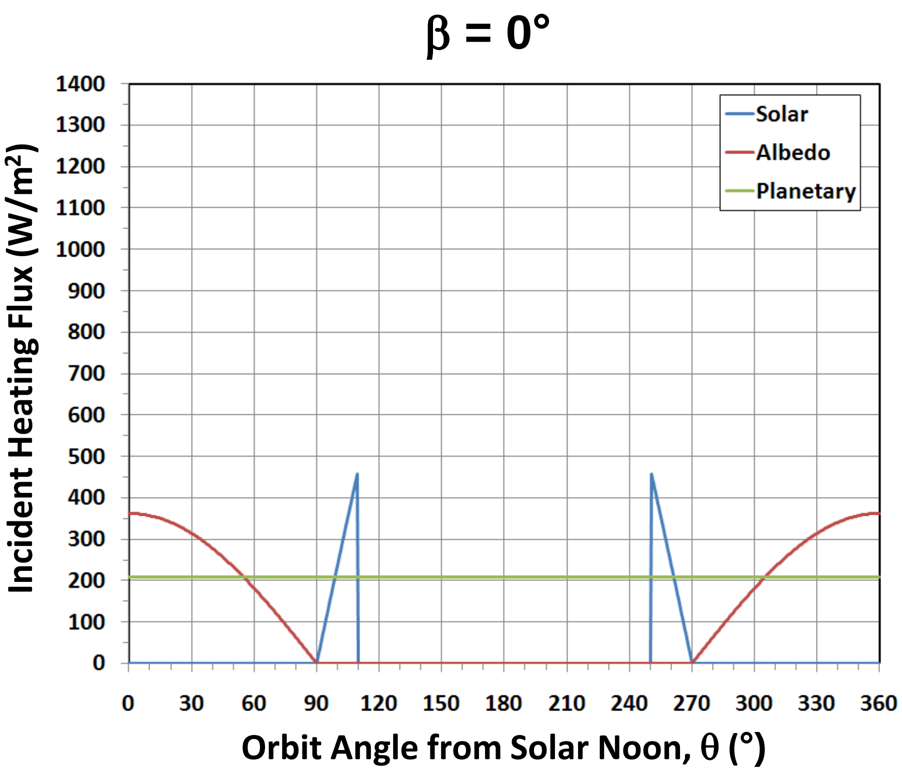
The zenith-facing plate "sees" only sunlight.

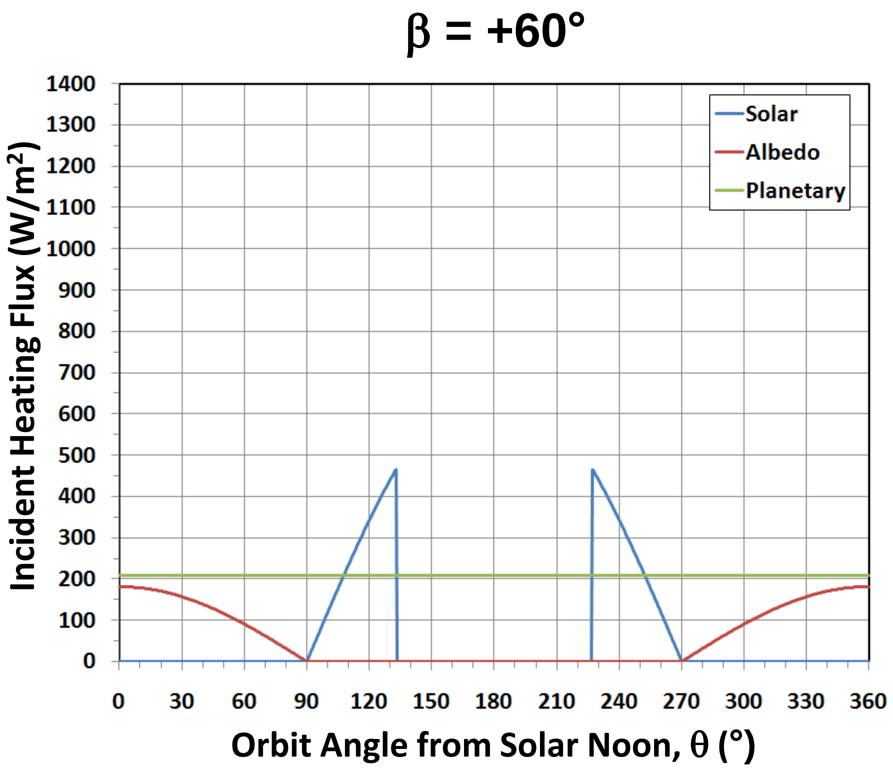




Example: Side 2 - Nadir-Facing Surface

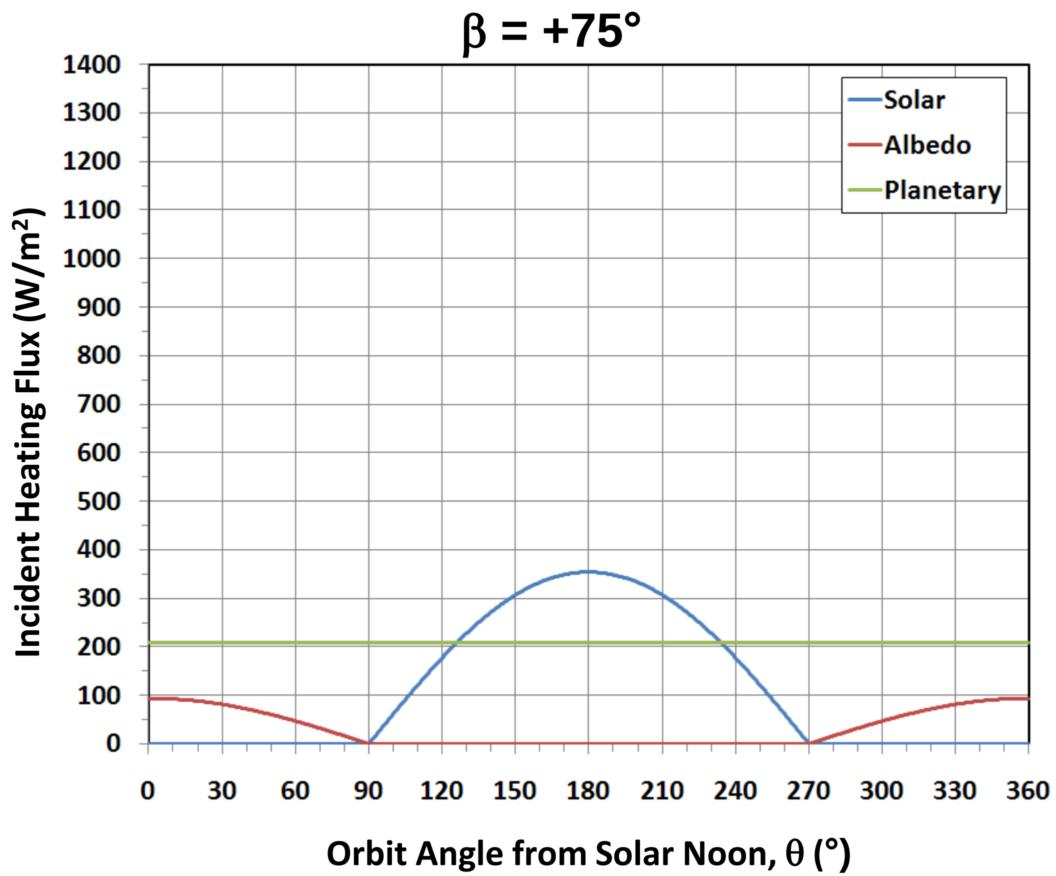
The nadir-facing plate experiences solar, albedo and planetary heating.





Example: Side 2 - Nadir-Facing Surface

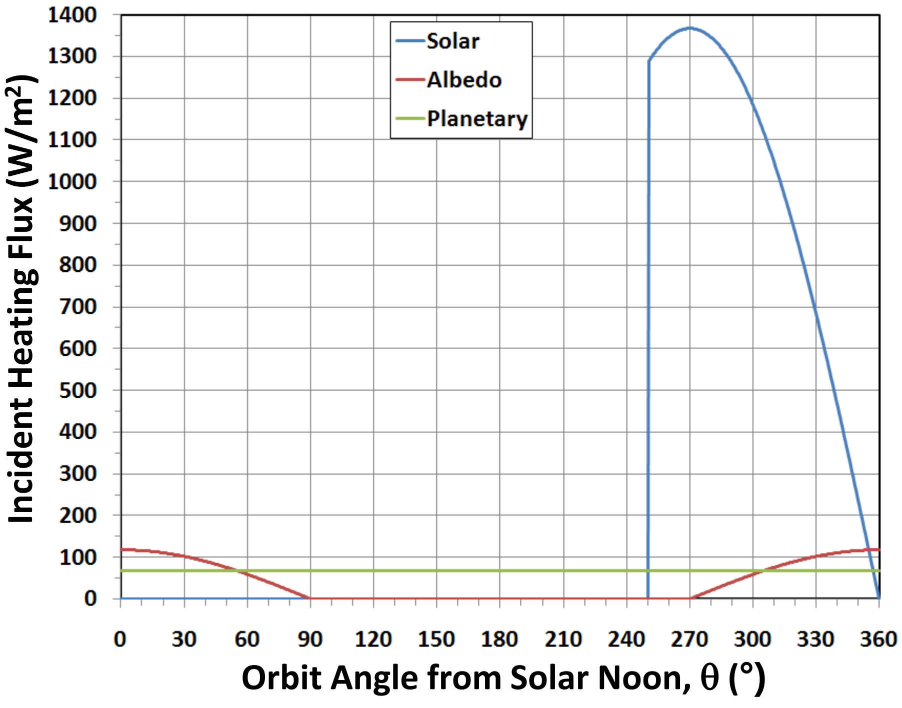
For this orbit, above |β| = ~71°, no eclipse is experienced.



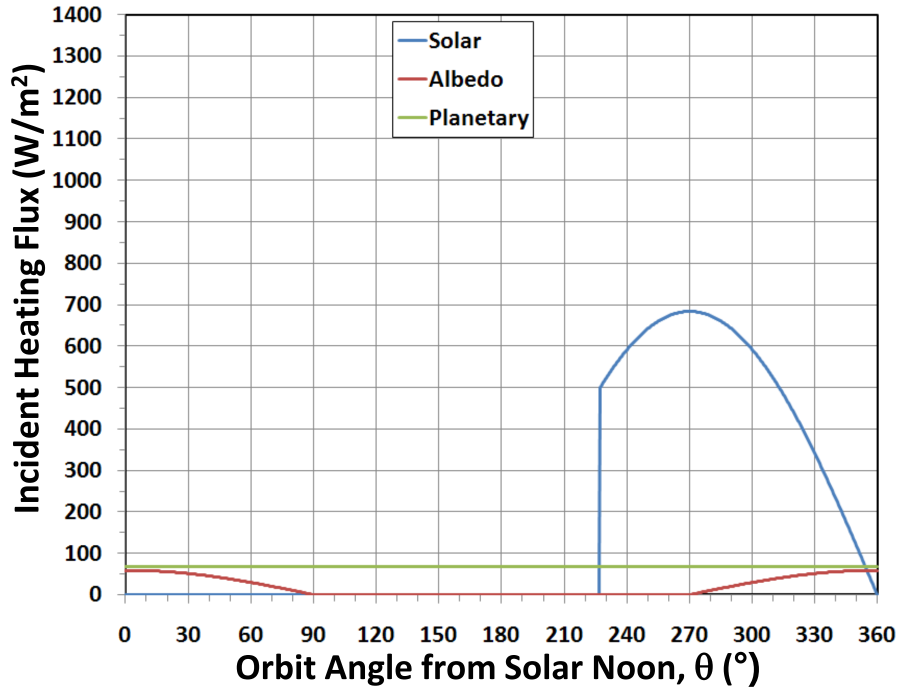
Example: Side 3 - Forward-Facing Surface

Solar flux phase is shifted due to orientation of surface;

$\beta = 0°$



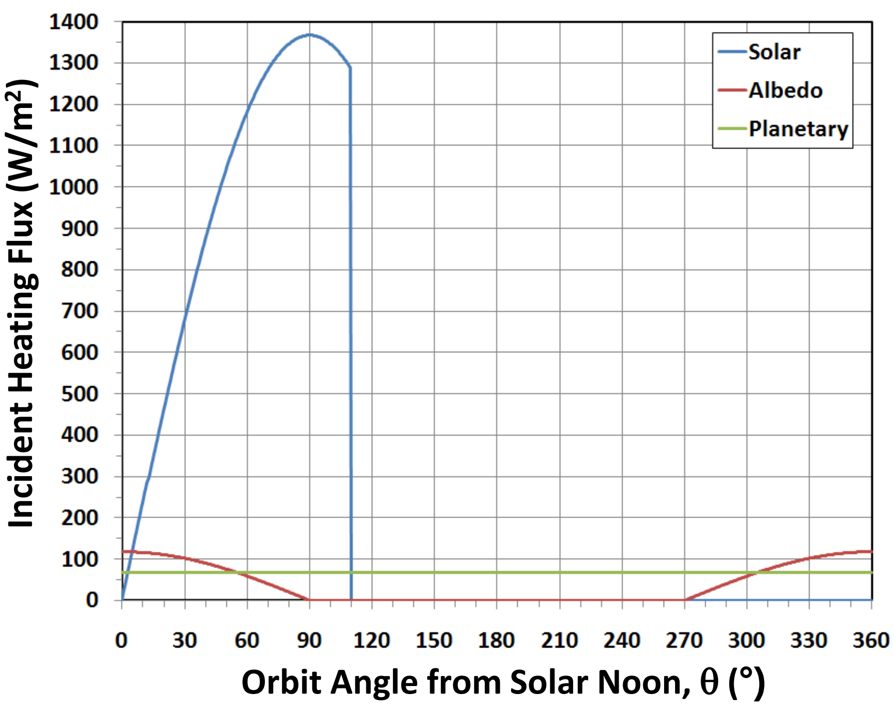
$\beta = +60°$



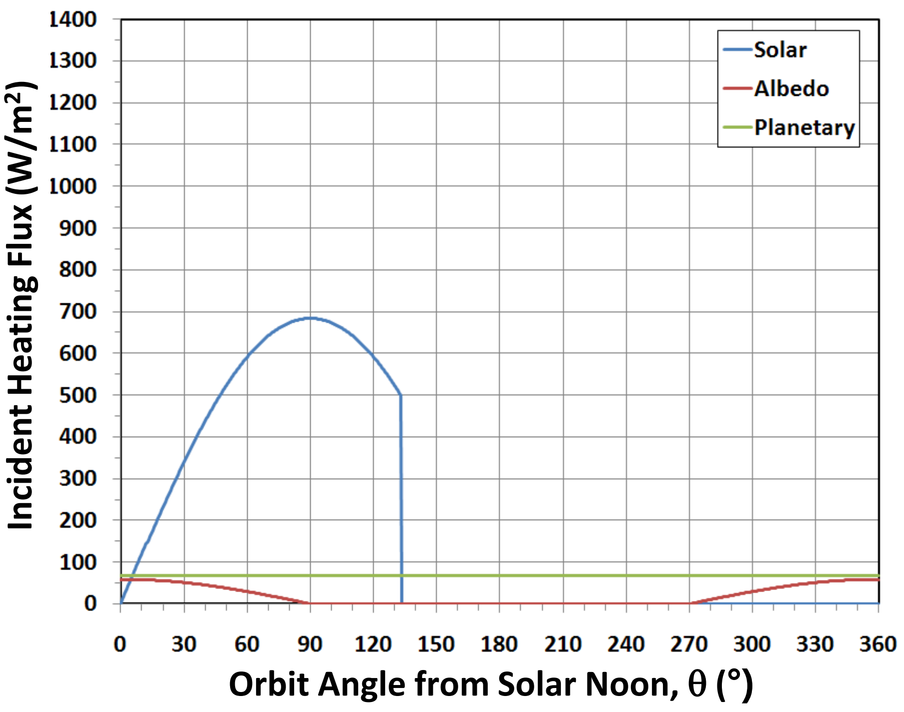
Example: Side 4 - Aft-Facing Surface

Solar flux phase is shifted again due to orientation of surface -- mirror image of Forward-Facing surface;

β = 0°

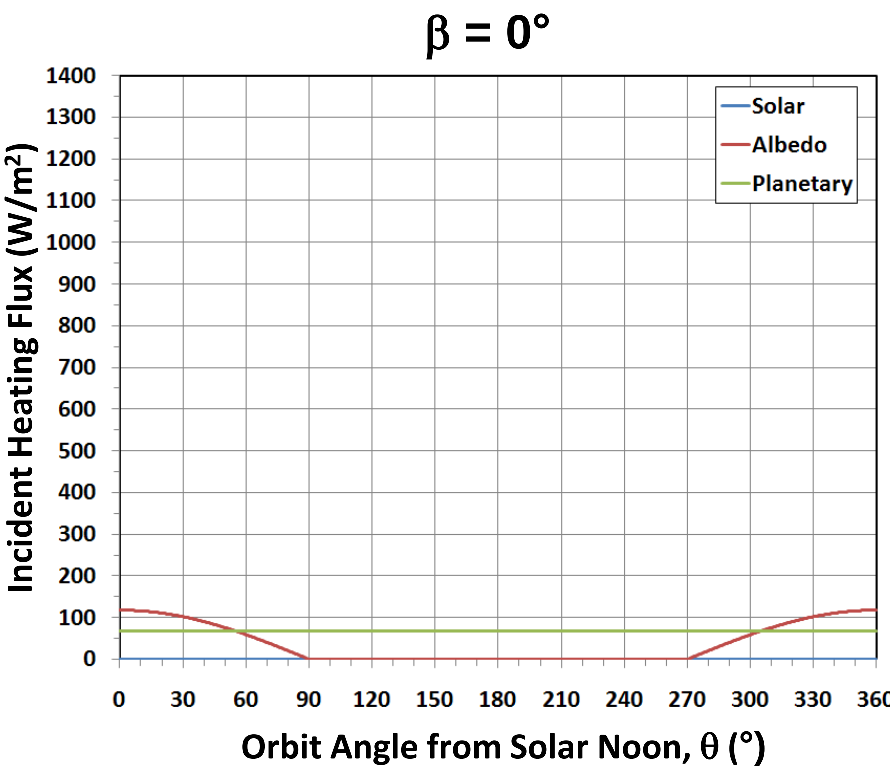


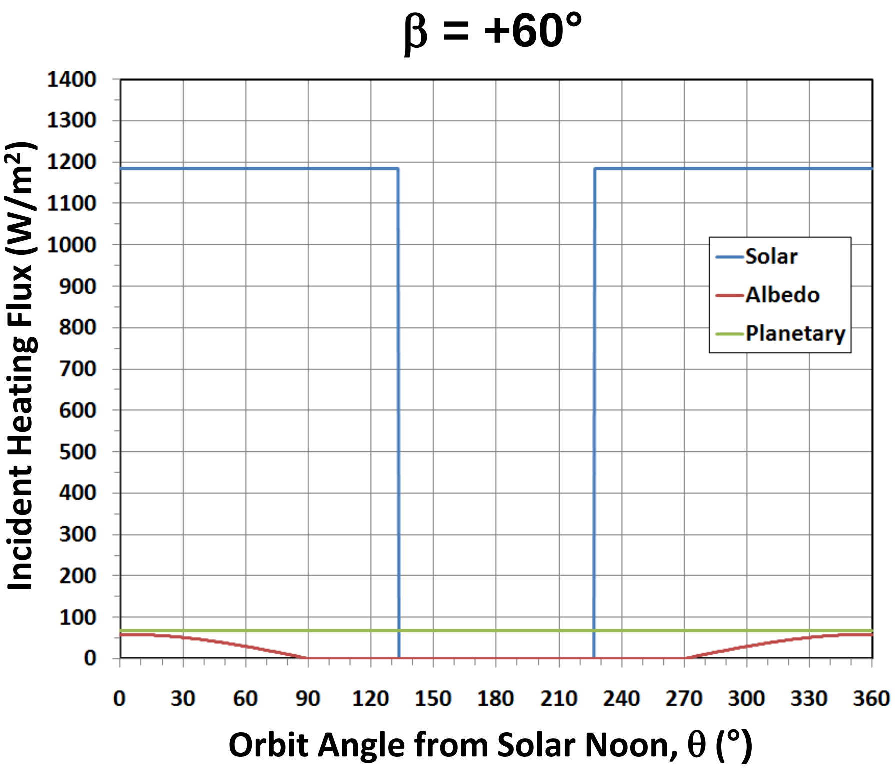
β = +60°



Example: Side 5 - North-Facing Surface

At β = 0°, surface is edge on to sun and solar flux is zero -- for *negative* β, solar flux is zero;

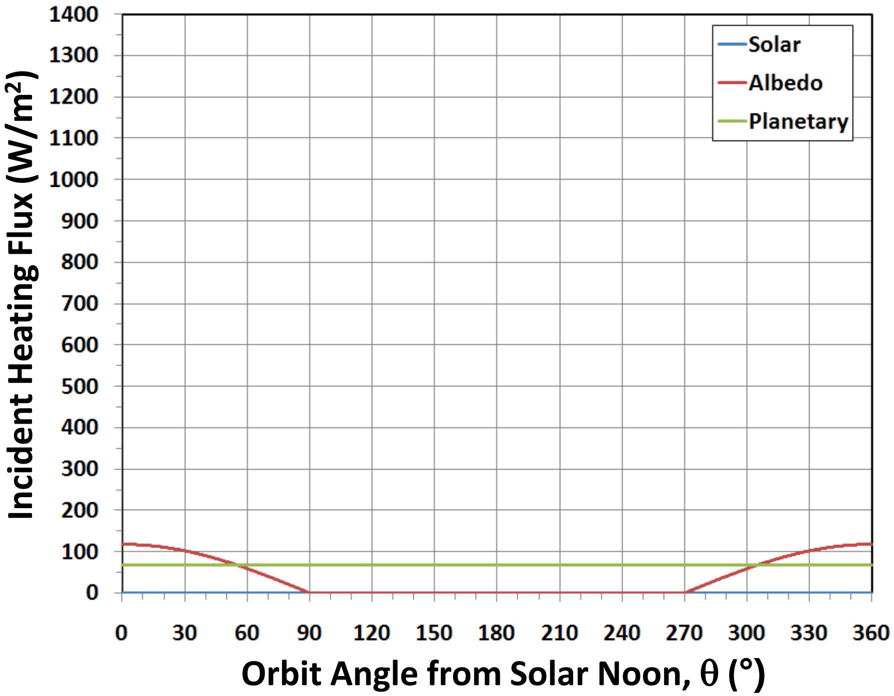




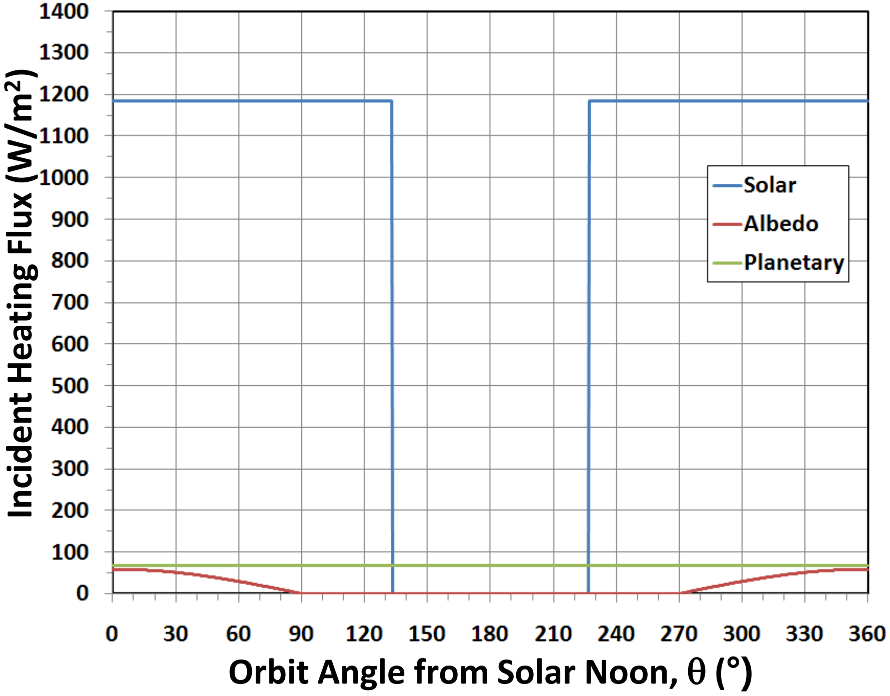
Example: Side 6 - South-Facing Surface

At $\beta = 0°$, the surface is edge on to sun and solar flux is zero -- for *positive* β, solar flux is zero;

$\beta = 0°$



$\beta = -60°$



Other Heating Environments

Free Molecular Heating (Refs. 11 and 12)

We define the Stanton number, St to be:

$$St = \frac{\dot{q}_w}{\rho_e u_e (h_{aw} - h_w)}$$

where $\dot{q}_w$ is the convective heat flux, ρ_e is the flow density at the edge of the boundary layer, u_e is the flow velocity at the edge of the boundary layer, h_{aw} is the enthalpy at the wall assuming an adiabatic wall temperature and h_w is the enthalpy at the wall assuming the actual wall temperature.

Free Molecular Heating (Ref. 11)

St can be thought of is the efficiency of heat transfer from the flow field to a surface;

The numerator is the actual heating applied while the denominator is the total available convective energy;

From this definition, we see that:

$$St \leq 1$$

Free Molecular Heating (Ref. 11)

We can rearrange our equation for St to solve for the surface convective heating flux:

$$\dot{q}_w = St \rho_e u_e (h_{aw} - h_w)$$

We assume that:

$$h \approx C_p T$$

This is true for a calorically perfect gas (i.e., C_p and C_v are functions of T only) and may be true for a thermally perfect gas (C_p and C_v are constant) assuming C_p is a function of T only.

Free Molecular Heating (Ref. 11)

From this, we can say that:

$$h_w \approx C_p T_w$$

For a high energy flow field, it is reasonable to assume that:

$$T_w \ll T_{aw}$$

and this implies:

$$h_w \ll h_{aw}$$

Free Molecular Heating (Ref. 11)

With this information, we can further simplify our expression for the convective heating flux at the wall:

$$\dot{q}_w = St\rho_e u_e h_{aw}$$

But, this equation can be simplified even further because if viscous effects are ignored:

$$\rho_e \approx \rho_\infty$$
$$u_e \approx u_\infty$$

Free Molecular Heating (Ref. 11)

Our expression becomes:

$$\dot{q}_w = St \rho_\infty u_\infty h_{aw}$$

Also,

$$T_{aw} \approx T_0 \rightarrow h_{aw} \approx h_0$$

so...

$$\dot{q}_w = St \rho_\infty u_\infty h_0$$

Free Molecular Heating (Ref. 11)

By definition,

$$h_0 = h_\infty + \frac{1}{2}u_\infty^2$$

But for high velocity flows,

$$h_\infty \ll \frac{1}{2}u_\infty^2$$

because

$$h_\infty \approx C_p T_\infty$$

Free Molecular Heating (Ref. 11)

T_∞ is typically low. The expression becomes:

$$\dot{q}_w = St\rho_\infty u_\infty \frac{1}{2} u_\infty^2$$

For maximum heating, St = 1.0 and the expression becomes:

$$\dot{q}_w = \frac{1}{2}\rho_\infty u_\infty^3$$

which is our desired result.

Free Molecular Heating and Density Dispersions

Ref. 12 describes a methodology to calculate free molecular heating using dispersed density, ρ_{disp}:

$$\dot{q}_w = \frac{1}{2}\rho_{disp}u_\infty^3$$

The dispersed density may be calculated from:

$$\rho_{disp} = K\rho_{76}$$

where K is a density dispersion factor, affected by numerous factors but most notably solar activity, and ρ_{76} is the U.S. Standard 1976 density.

Free Molecular Heating and Density Dispersions

K factors were developed to disperse nominal densities for different altitudes and times in the solar cycle;

The strategy is to calculate a 3-sigma dispersed density. (Ref. 13)

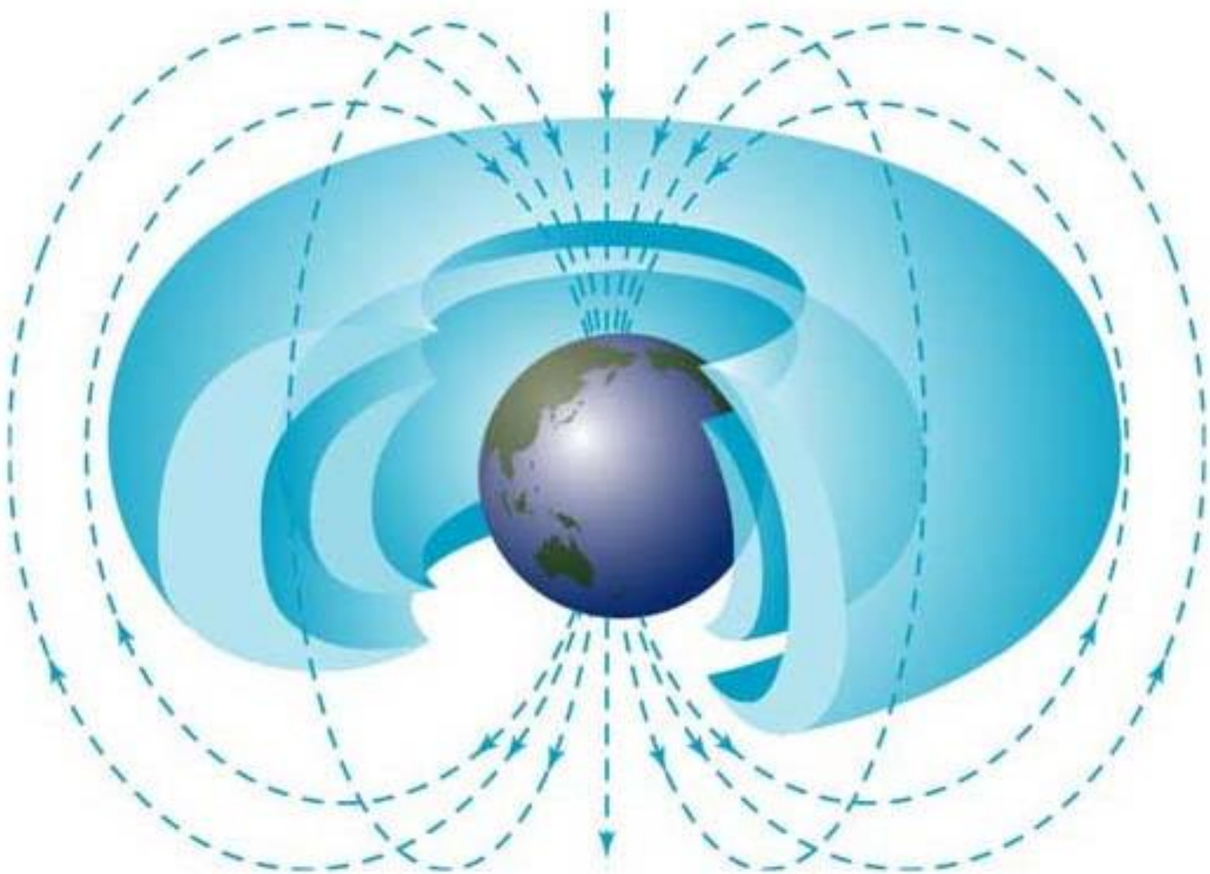
Charged Particle Heating (Ref. 14)

Charged particle heating must be considered in the design of a system operating at cryogenic temperatures;

Focus is to calculate the energy dissipation of charged particles as they pass through an absorbing medium;

The volumetric heating created as a result through interaction with absorbing medium must be factored into the overall heat transfer calculation.

Charged Particle Heating (Ref. 15)



Van Allen Radiation Belts
(NASA Image)

Charged Particle Heating (Ref. 14)

Volumetric heating may be calculated as follows:

$$\dot{Q}_{vol} = \sum_{E=E_{min}}^{E_{max}} \left(\frac{dE}{ds}\right)_E \varphi(t,x)_E$$

Where...

$\left(\frac{dE}{ds}\right)_E$ is the "stopping power"

E is the particle energy

s is the penetration depth

$\varphi(t,x)_E$ is the particle flux.

Concluding Remarks

Overview of radiation heat transfer, the solar and infrared spectra;

Derived expressions for solar, albedo and planetary heating fluxes;

Derived an expression for the beta angle and investigated the effect beta has on orbit environment heating;

Explored free molecular heating and charged particle heating phenomena.

Acknowledgements

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Microsoft® Clip Art was used throughout the presentation.

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