Increased Control of Squeeze-Film Performance with Magnetohydrodynamics and Surface Roughness: Theory and Modeling

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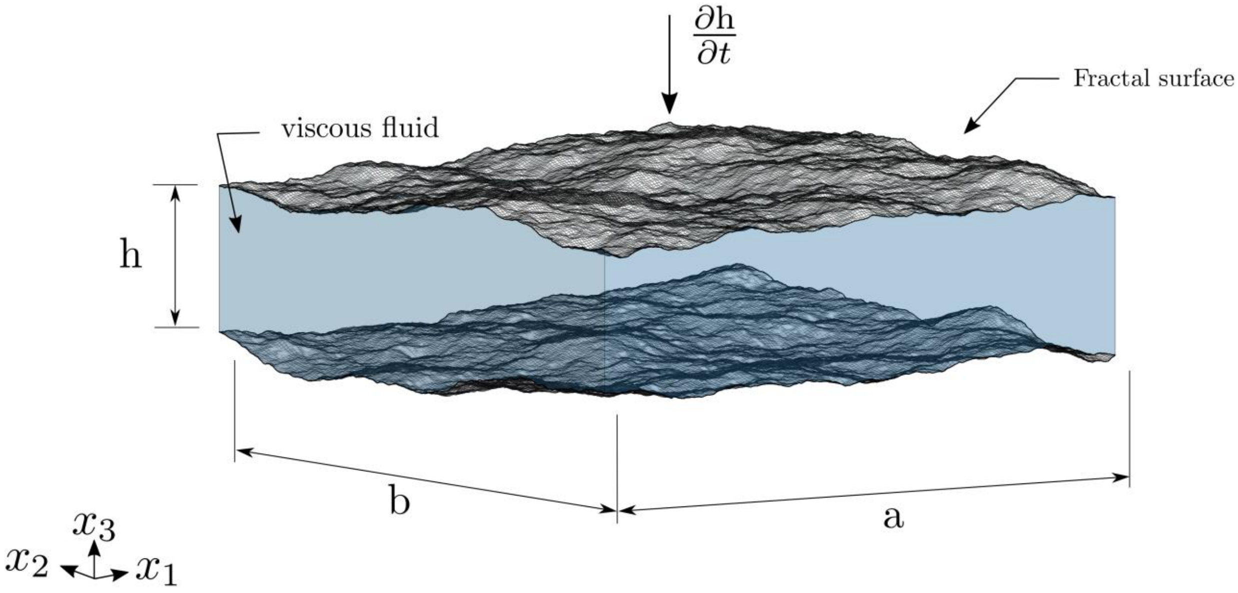
Overview of Squeeze-Films

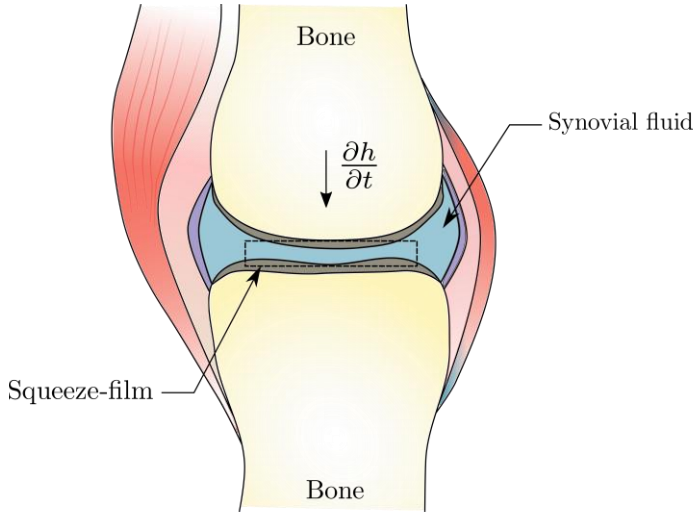
Squeeze-film flows

- Relative normal motion of surfaces separated by a thin film of *viscous* fluid
- Surfaces trying to squeeze fluid out of the interface (and vice versa)
- Induced hydrodynamic pressure tends to oppose motion of surfaces

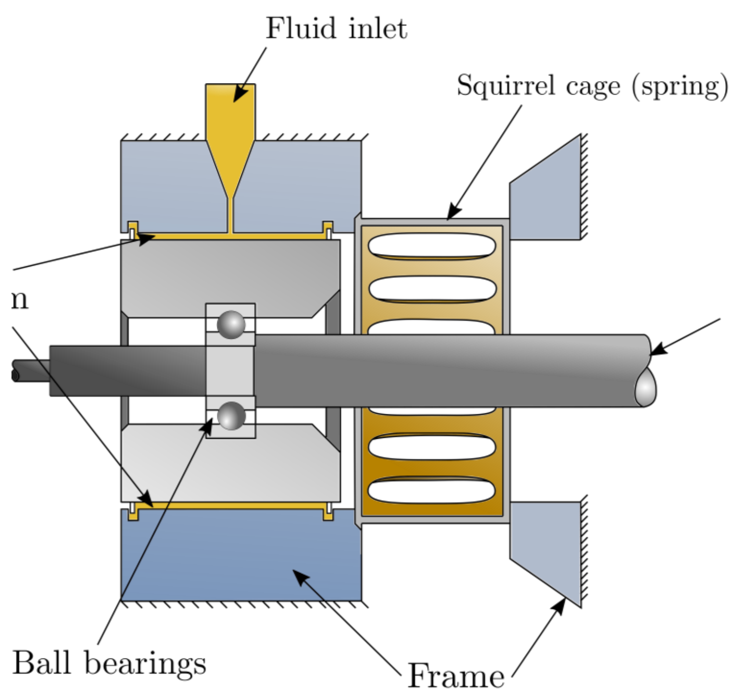


Squeeze-film dampers (SFDs)

- Squeeze effect often used in mechanical vibration dampers
- Common applications:
  - High-performance turbojet and turboshaft engines
  - Microelectromechanical systems (MEMS)
  - Nature (e.g. synovial joints)



Schematic of a squeeze-film in the knee



Schematic of SFD in turbojet aircraft engine

Performance and Modelling Challenges

Damping via viscous dissipation

- Viscosity of the fluid is crucial for effectiveness
- **Problem:** viscosity typically diminishes with increased temperature
- **Potential solution**: leverage magnetohydrodynamic forces

Small length scales

- Classical lubrication theory assumes negligible inertia
- High-frequency motion/decreased viscosity brings this assumption into question
- Small length scale of flow amplifies effect of surface roughness
- How does roughness structure of the surfaces affect the flow?

Outline

Part I: quasi-steady analysis

1. Develop a general governing equation for MHD squeeze-films
2. Introduce fractals for modeling real surface topography
3. Apply the FEM to solve the flow problem
4. Conduct quasi-steady numerical studies

Part II: transient analysis

1. Incorporate MHD squeeze-film model into a nonlinear mass-spring-damper model
2. Apply implicit time-integration to solve nonlinear equation of motion
3. Conduct time-domain numerical studies to evaluate MHD damper performance

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Governing Flow Equations

Classical Reynolds equation

- Famously derived by O. Reynolds in 1886
- Reduction of the Navier-Stokes equations based on arguments of scale
- Assumptions:
  - $\mathrm{Re} \ll 1$
  - Newtonian fluid and incompressible flow
  - Gravity negligible
  - Pressure invariant over depth (i.e. the thin-film assumption)
- Poisson-type PDE for pressure:

$$\nabla \cdot \left(\frac{h^3}{12\eta} \nabla p \right) = \frac{\partial h}{\partial t}$$

$$\begin{aligned} p &\equiv \text{hydrodynamic pressure} \\ h &\equiv \text{film thickness} \\ \frac{\partial h}{\partial t} &\equiv \text{squeeze velocity} \\ \eta &\equiv \text{viscosity} \end{aligned}$$

Note: This is a ‘squeeze’ variant of Reynolds’ original derivation

Governing Flow Equations

Magnetohydrodynamics (MHD)

- Interaction between conducting fluids and magnetic fields
- Based on the Lorentz force

$$\boldsymbol{f}_L = \boldsymbol{J} \times \boldsymbol{B} \qquad \text{where:} \quad \begin{aligned} \boldsymbol{B} &\equiv \text{magnetic field} \\ \boldsymbol{J} &\equiv \text{current density} \end{aligned}$$

- Appears as a body force in the Navier-Stokes equations
- Couples the fluid dynamics and electrodynamics

pre-Maxwell equations

$$\begin{aligned}
&\nabla \cdot \boldsymbol{E} = 0 && \text{(Gauss' electricity law)} \\
&\nabla \cdot \boldsymbol{B} = 0 && \text{(Gauss' magnetism law)} \\
&\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B} && \text{(Faraday's law)} \\
&\nabla \times \boldsymbol{B} = \mu \boldsymbol{J} && \text{(Ampere's law)}
\end{aligned}$$

Couple

$$\boldsymbol{f}_L = \boldsymbol{J} \times \boldsymbol{B}$$

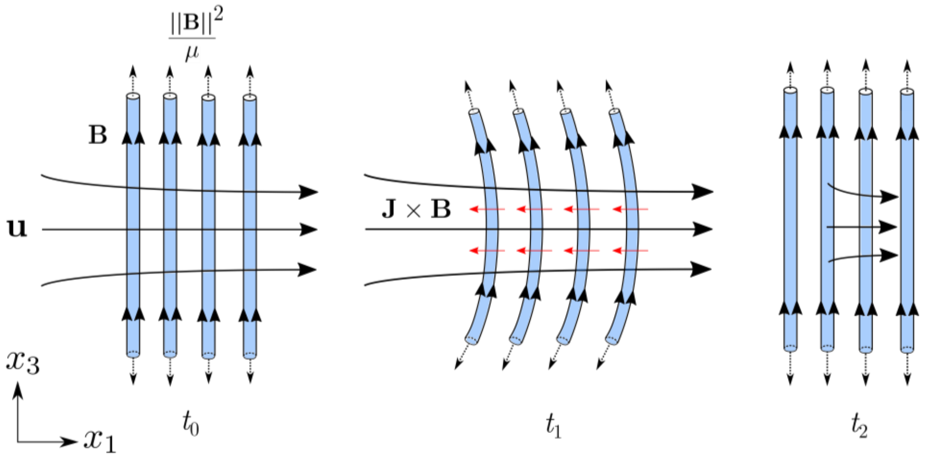
Navier-Stokes equations

$$\begin{aligned}
\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \frac{1}{\rho}\nabla p - \nu \nabla^2 \boldsymbol{u} - \boldsymbol{f} = \boldsymbol{0} \quad & \text{(momentum)} \\
\nabla \cdot \mathbf{u} = 0 \quad & \text{(mass)}
\end{aligned}$$

Governing Flow Equations

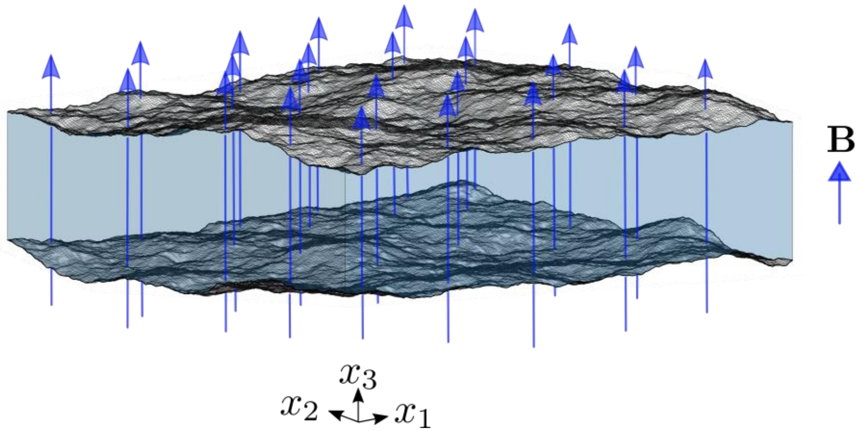
Magnetic Damping

- Lorentz force a result of magnetic stress
- Magnetic field lines deformed by flow
- "Tension" in the field line acts opposite of the flow



Augmenting role of viscosity in SFDs

- Assume the fluid is an electrical conductor
- Apply vertical magnetic field across film



Governing Flow Equations

Derivation of the MHD Reynolds equation with temporal inertia

1. Point-of-departure

MHD Equations

$$
\begin{aligned}
\rho\big(\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\big) + \nabla p - \eta\nabla^2\boldsymbol{u} - \boldsymbol{J}\times\boldsymbol{B} &= \boldsymbol{0} && \text{(momentum)}\\
\partial_t \boldsymbol{B} - \nabla\times(\boldsymbol{u}\times\boldsymbol{B}) - \alpha\nabla^2\boldsymbol{B} &= \boldsymbol{0} && \text{(induction)}\\
\nabla\cdot\boldsymbol{u} &= 0 && \text{(mass)}\\
\nabla\cdot\boldsymbol{J} &= 0 && \text{(charge)}\\
\boldsymbol{J} - \varsigma(\boldsymbol{u}\times\boldsymbol{B}) &= \boldsymbol{0} && \text{(closure: Ohm's law)}
\end{aligned}
$$

2. Perform dimensional analysis

Parameter	Definition	Description
Aspect ratio	$\varepsilon = \frac{h_0}{L}$	Ratio of film thickness to lateral dimension
Reynolds number	$\mathrm{Re} = \frac{\rho h_0 U}{\eta}$	Ratio of inertial to viscous forces
Squeeze Reynolds number	$\mathrm{Re}_s = \frac{\rho h_0^2 \omega}{\eta}$	Frequency-based Reynolds number
Magnetic Reynolds number	$\mathrm{Re}_m = \lVert\boldsymbol{u}\rVert \mu \varsigma h_0$	Ratio of advection to magnetic diffusion
Hartmann number	$\mathrm{Ha} = \lVert\boldsymbol{B}\rVert h_0 \sqrt{\frac{\varsigma}{\eta}}$	Ratio of Lorentz to viscous forces

Derivation of the MHD Reynolds equation with temporal inertia

3. Impose assumptions
   i. Newtonian fluid and incompressible flow
   ii. Flow domain is a thin film (i.e. $\varepsilon \ll 1$)
   iii. Magnetic field is quasi-steady (i.e. $\mathrm{Re}_m \ll 1$)
   iv. Temporal inertia dominates convective inertia (i.e. $\frac{\mathrm{Re}}{\mathrm{Re}_s} \to 0, \quad \mathrm{Re}_s > 1$)

Thin-film MHD Equations

$$\mathrm{Re}_s \frac{\partial \tilde{u}_1}{\partial \tilde{t}} + \frac{\partial \tilde{p}}{\partial \tilde{x}_1} - \frac{\partial^2 \tilde{u}_1}{\partial \tilde{x}_3^2} + \mathrm{Ha}^2 \tilde{u}_1 = 0$$

$$\mathrm{Re}_s \frac{\partial \tilde{u}_2}{\partial \tilde{t}} + \frac{\partial \tilde{p}}{\partial \tilde{x}_2} - \frac{\partial^2 \tilde{u}_2}{\partial \tilde{x}_3^2} + \mathrm{Ha}^2 \tilde{u}_2 = 0$$

$$\frac{\partial \tilde{p}}{\partial \tilde{x}_3} = 0$$

$$\frac{\partial \tilde{u}_1}{\partial \tilde{x}_1} + \frac{\partial \tilde{u}_2}{\partial \tilde{x}_2} + \frac{\partial \tilde{u}_3}{\partial \tilde{x}_3} = 0$$

Governing Flow Equations

Derivation of the MHD Reynolds equation with temporal inertia

4. Integrate continuity equation over film thickness

$$\tilde{\nabla} \cdot \int_0^{\tilde{h}(\tilde{\boldsymbol{x}},\tilde{t})} \tilde{\boldsymbol{u}} \, d\tilde{x}_3 = -\frac{\partial \tilde{h}}{\partial \tilde{t}}$$

where:

$$\tilde{\boldsymbol{x}} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix}, \quad \tilde{\nabla} = \begin{pmatrix} \frac{\partial}{\partial \tilde{x}_1} \\ \frac{\partial}{\partial \tilde{x}_2} \end{pmatrix}, \quad \tilde{\boldsymbol{u}} = \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{pmatrix}$$

5. Use the momentum equations to evaluate above integral

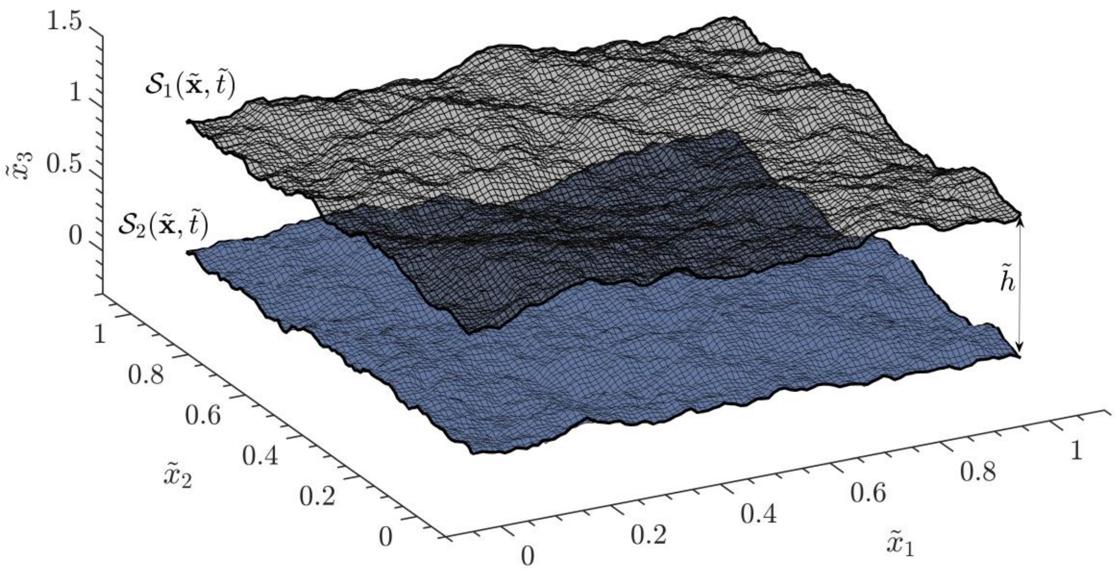
Generalized MHD Reynolds Equation

$$\tilde{\nabla} \cdot \left(\kappa \tilde{\nabla} \tilde{p}\right) = \mathrm{Ha}^3 \frac{\partial \tilde{h}}{\partial \tilde{t}} + \mathrm{Re}_s \kappa \frac{\partial^2 \tilde{h}}{\partial \tilde{t}^2}$$

$$(\kappa \circ \tilde{h})(\tilde{\boldsymbol{x}}, \tilde{t}) = \mathrm{Ha}\, \tilde{h}(\tilde{\boldsymbol{x}}, \tilde{t}) - 2 \tanh\left(\mathrm{Ha} \frac{\tilde{h}(\tilde{\boldsymbol{x}},\tilde{t})}{2}\right) \qquad (\text{“flow conductivity”})$$

Surface Characterization

Surface roughness in the Reynolds equation



$$\tilde{h}(\tilde{\boldsymbol{x}}, t) = \mathcal{S}_1(\tilde{\boldsymbol{x}}, \tilde{t}) - \mathcal{S}_2(\tilde{\boldsymbol{x}}, \tilde{t}))$$

$\mathcal{S}_1 \equiv$ top surface

$\mathcal{S}_2 \equiv$ bottom surface

- Film thickness depends on the topographies of the bounding surfaces
- Digital representation of real surfaces is not trivial
- Properties change with resolution of measuring device
- Scale-independent characterization parameters are desired

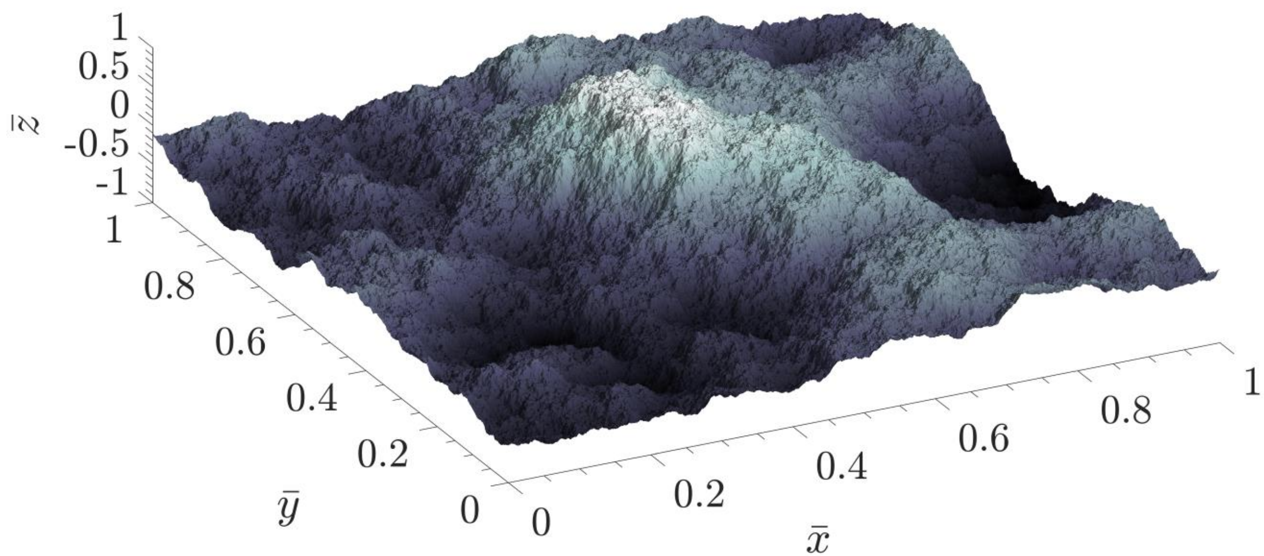
Surface Characterization

Weierstrass-Mandelbrot fractal

- Fractals can be used to generate scale-invariant topographies
- Possess self-similar structure (asperities upon asperities)
- Construction similar to a Fourier series:

$$\mathcal{S}(x) = \Re\left[\sum_{n=-\infty}^{\infty} \gamma^{(D-2)n}(1 - \mathrm{e}^{i\gamma^n x})\mathrm{e}^{i\phi_n}\right]$$

$D \equiv$ fractal dimension
$\gamma \equiv$ frequency density



Weak form

- FEM based on the weak formulation
- No longer have to differentiate κ
- Dirichlet boundary conditions built into the solution space

Solution and Test Spaces

$$\mathcal{U} = \left\{ \tilde{p}(\tilde{\mathbf{x}}) \in \mathcal{H}^1(\Omega) : \tilde{p}(\tilde{\mathbf{x}}) = g_D \quad \forall \tilde{\mathbf{x}} \in \Gamma_D \right\},$$

$$\mathcal{V} = \left\{ v(\tilde{\mathbf{x}}) \in \mathcal{H}^1(\Omega) : v(\tilde{\mathbf{x}}) = 0 \quad \forall \tilde{\mathbf{x}} \in \Gamma_D \right\},$$

$$\Omega \subset \mathbb{R}^2 \quad \text{(Domain)}$$
$$\Gamma_D = \partial\Omega \quad \text{(Boundary)}$$
$$\mathcal{H}^1(\Omega) = \{ u : \nabla u \in L^2(\Omega) \}$$
$$L^2(\Omega) = \left\{ u : \int_\Omega u^2 \, d\Omega < \infty \right\}$$

Weak form of the MHD Reynolds equation

Find $\tilde{p} \in \mathcal{U}$ such that $\forall v \in \mathcal{V}$:

$$\int_\Omega \kappa \tilde{\nabla} \tilde{p} \cdot \tilde{\nabla} v \, d\Omega - \int_\omega v \mathrm{Ha}^3 \frac{\partial \tilde{h}}{\partial \tilde{t}} \, d\Omega - \int_\Omega v \mathrm{Re}_s \kappa \frac{\partial^2 \tilde{h}}{\partial \tilde{t}^2} \, d\Omega = 0$$

Galerkin FEM

- Solution and test functions projected onto finite element space with linear basis functions
- Numerical integration via Gauss quadratures
- Results in the linear system:

$$\mathbf{A}\mathbf{p} = \mathbf{L}$$

where:

$$A_{ij} = \int_{\Omega} \kappa \tilde{\nabla} N_j \cdot \tilde{\nabla} N_i \; d\Omega$$

$$L_i = \int_{\omega} N_i \mathrm{Ha}^3 \frac{\partial \tilde{h}}{\partial \tilde{t}} \; d\Omega + \int_{\Omega} N_i \mathrm{Re}_s \kappa \frac{\partial^2 \tilde{h}}{\partial \tilde{t}^2} \; d\Omega$$

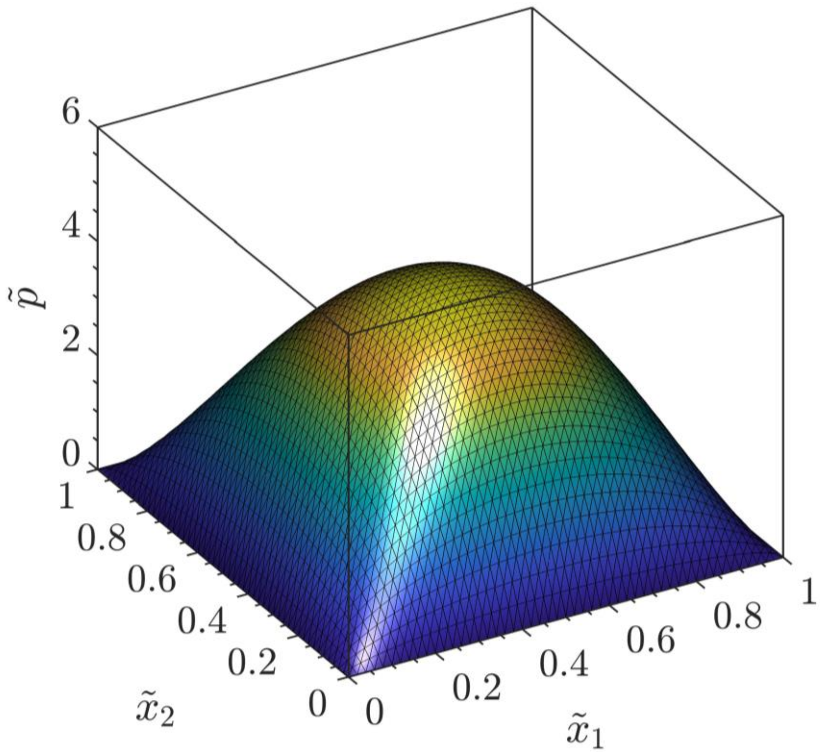
$$p_i = p(\mathbf{x}_i)$$

$$N_i \equiv \text{Nodal basis}$$

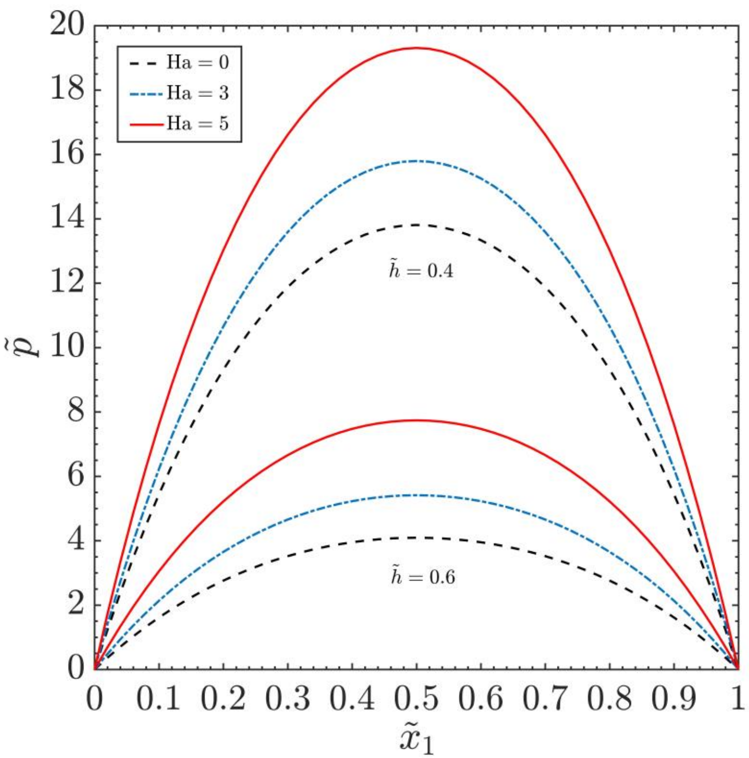
Quasi-steady Results

Problem 1: Smooth surfaces, varying magnetic field strength

Parameters:
$\Omega = \mathbb{R}(0,1)^2$ (unit square), $\quad \frac{\partial \tilde{h}}{\partial \tilde{t}} = 1, \quad \mathrm{Re} = 0$
Boundary conditons: $\tilde{p}(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \Gamma_D$



Pressure field solution



Pressure solution along a slice

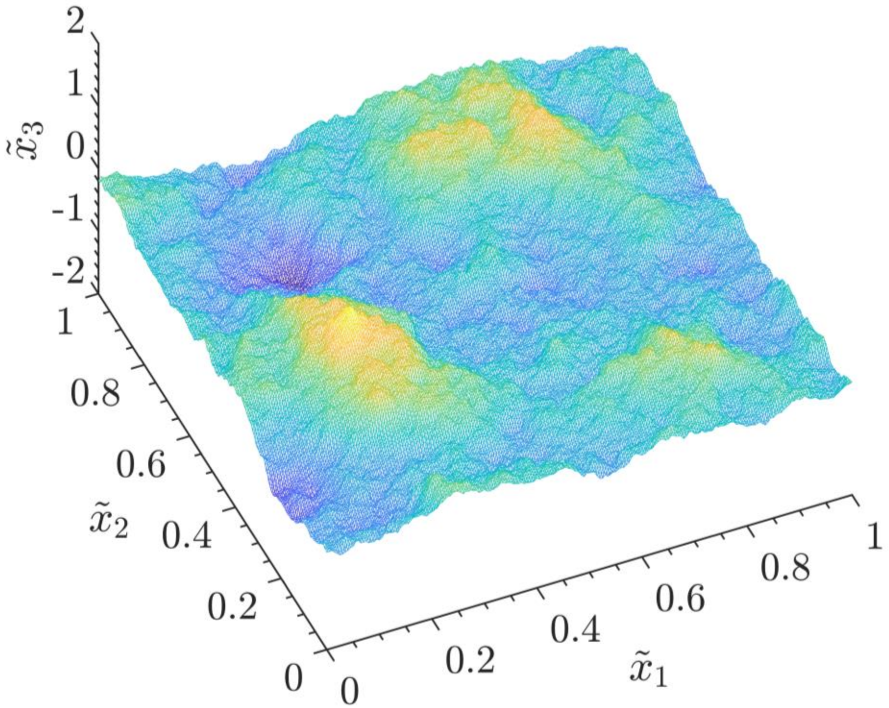
Quasi-steady Results

Problem 2: Rough surfaces, varying fractal dimension

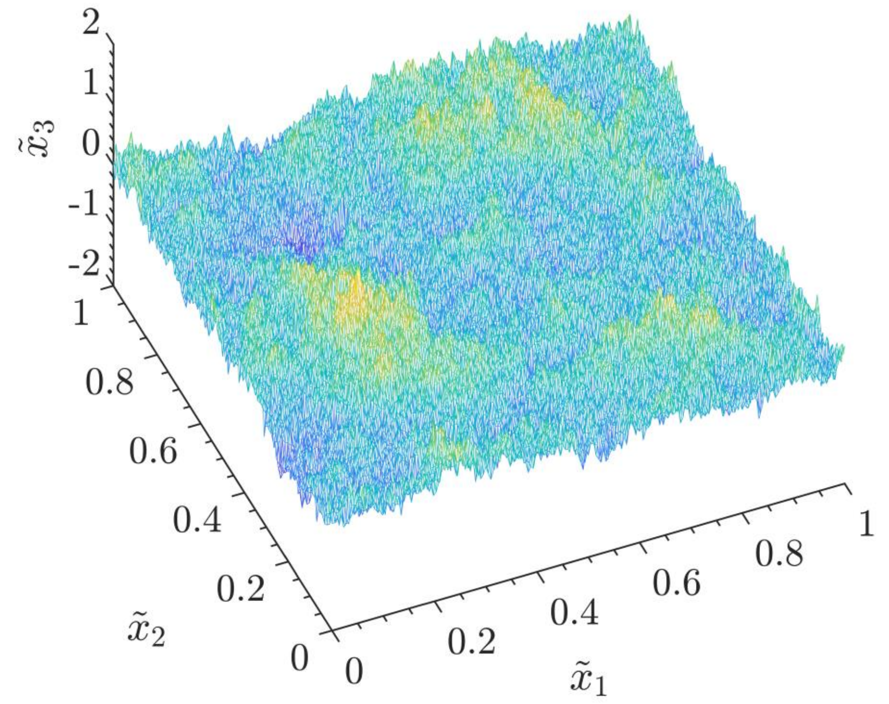
Parameters:

$\Omega = \mathbb{R}(0,1)^2$ (unit square), $\quad \frac{\partial \tilde{h}}{\partial \tilde{t}} = 1, \quad \mathrm{Re} = 0, \quad \mathrm{Ha} = 0, \quad \tilde{h} = 1$

Boundary conditons: $\tilde{p}(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \Gamma_D$



Fractal top surface mesh, 80,000 finite elements, D = 2.3



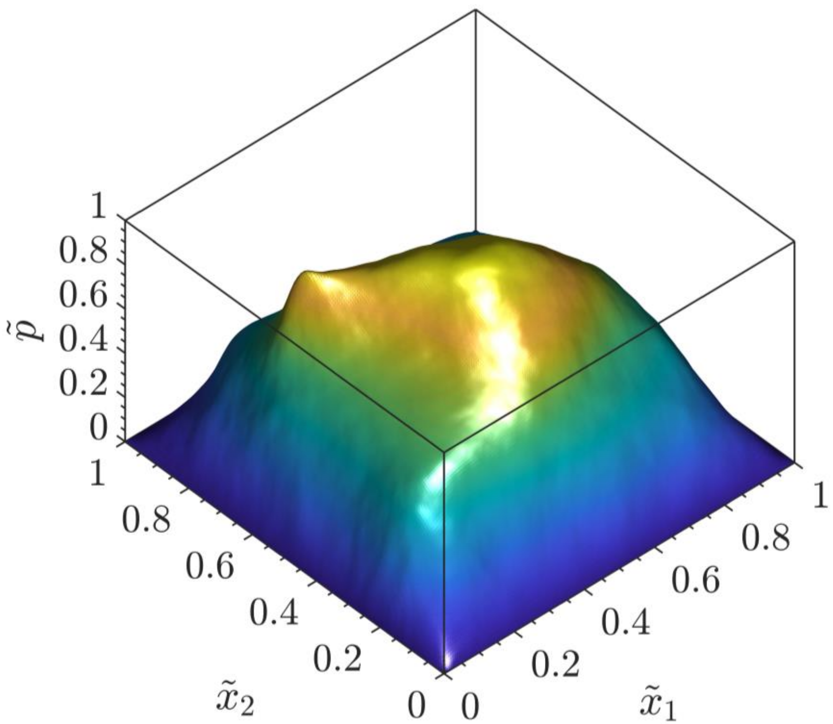
Fractal top surface mesh, 80,000 finite elements, D = 2.8

Problem 2: Rough surfaces, varying fractal dimension

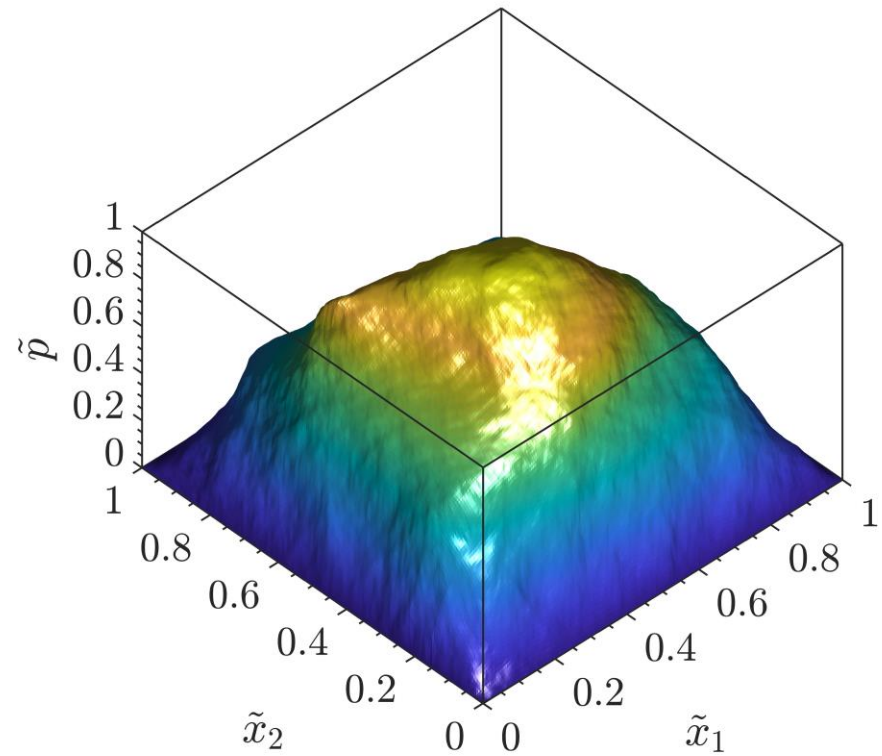
Parameters:

$$\Omega = \mathbb{R}(0,1)^2 \text{ (unit square)}, \quad \frac{\partial \tilde{h}}{\partial \tilde{t}} = 1, \quad \mathrm{Re} = 0, \quad \mathrm{Ha} = 0, \quad \tilde{h} = 1$$

Boundary conditons: $\tilde{p}(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \Gamma_D$



Pressure field solution, D = 2.3



Pressure field solution, D = 2.8

Quasi-steady Results

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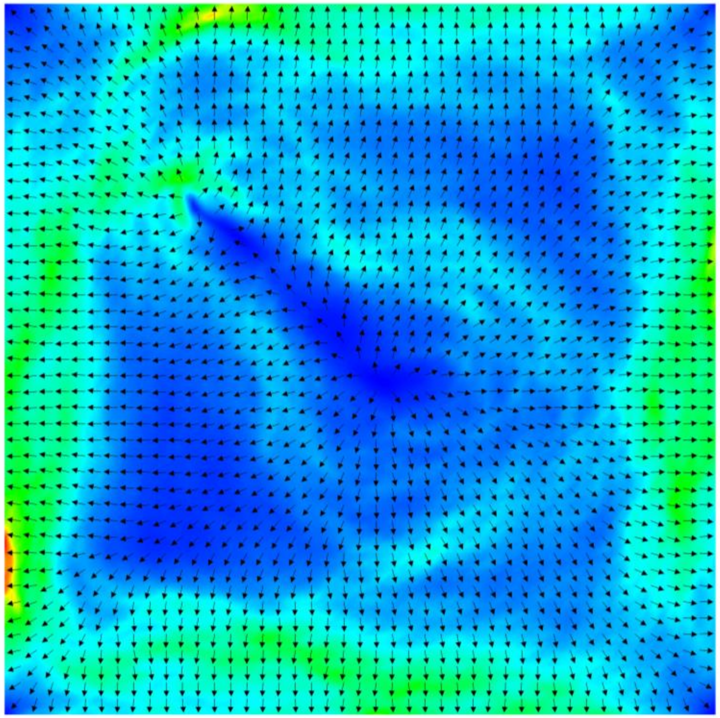
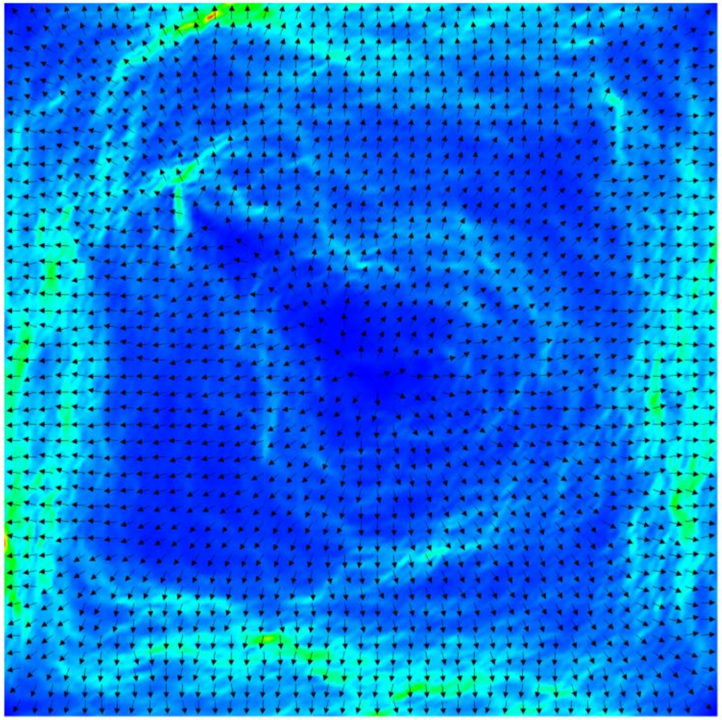
Boundary conditons: $\tilde{p}(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \Gamma_D$



Velocity field, D = 2.3



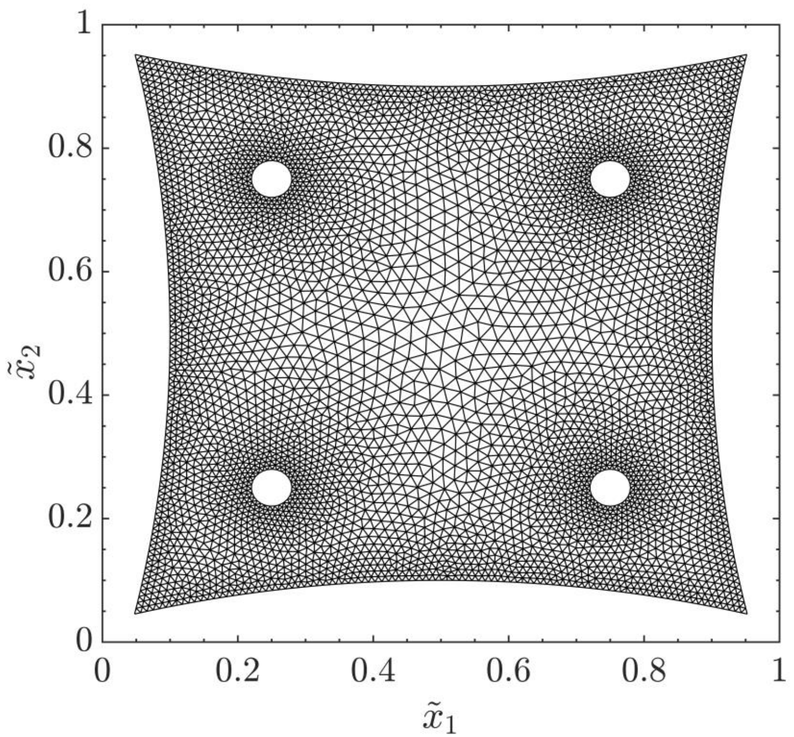
Velocity field, D = 2.8

Problem 3: Arbitrary surface geometry/topology

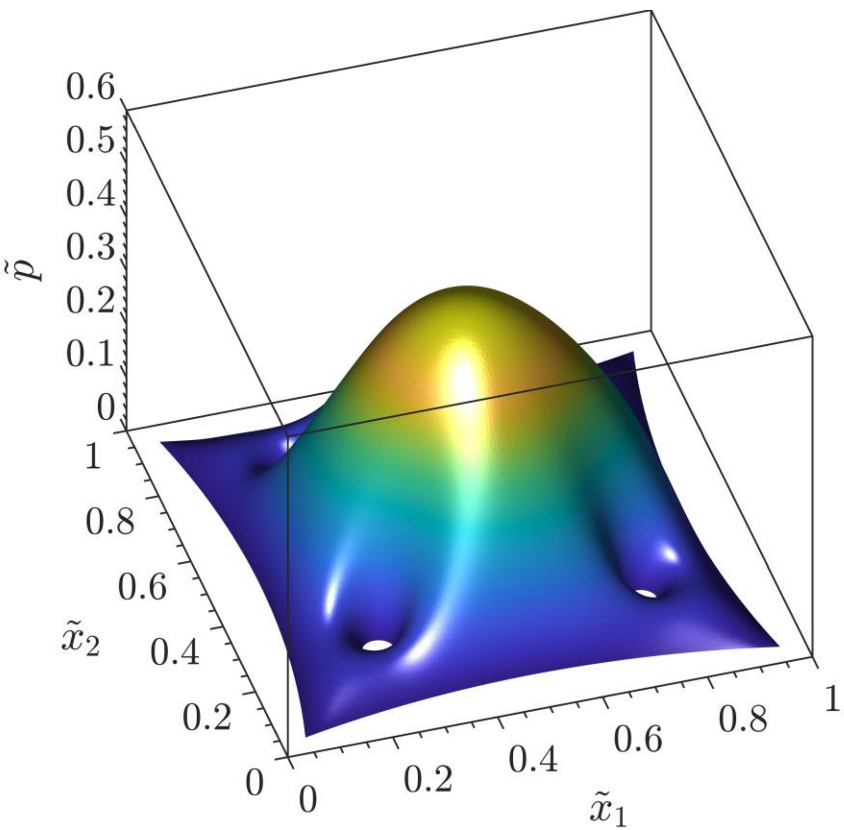
Parameters:

$$\frac{\partial \tilde{h}}{\partial \tilde{t}} = 1, \quad \mathrm{Re} = 0, \quad \mathrm{Ha} = 0, \quad \tilde{h} = 1$$

Boundary conditons: $\tilde{p}(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \Gamma_D$



FEM Mesh, 10,000 elements



Velocity field

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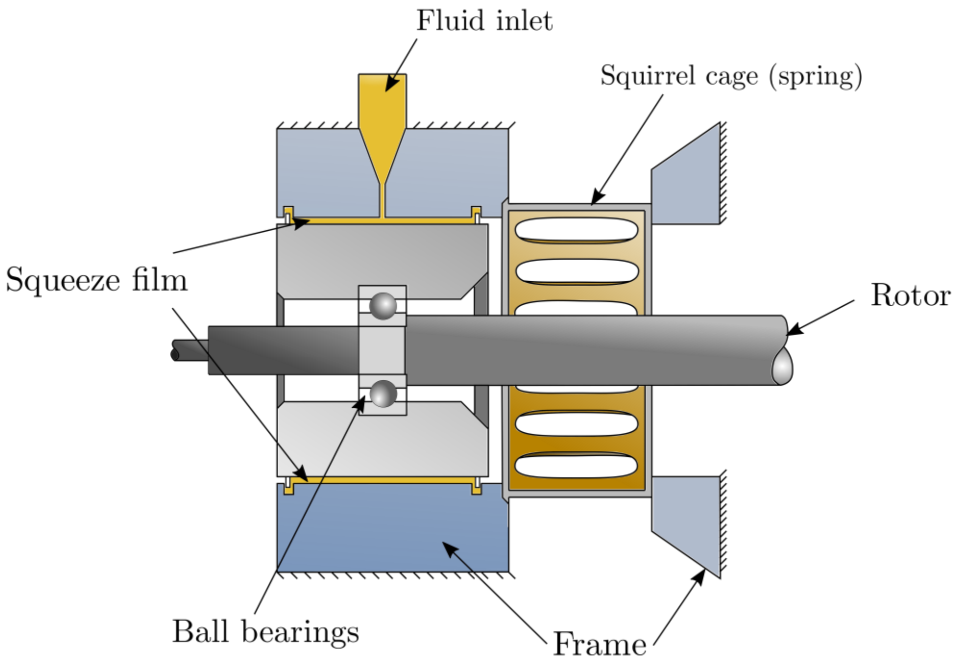
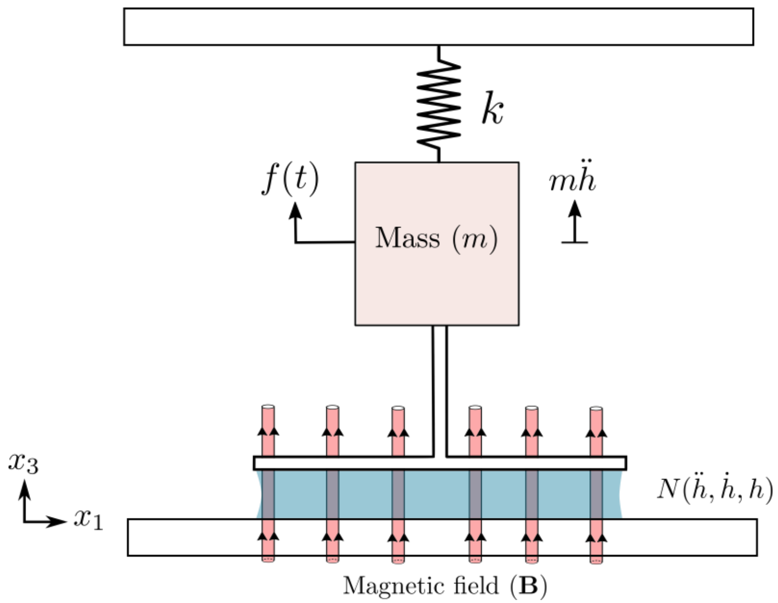
Dynamics with Squeeze-film Dampers

Modeling a SFD on a single DOF oscillator

- Incorporate actual fluid dynamics for the damping in dynamic models
- Modeling the fluid with the MHD Reynolds equation makes the model computationally efficient



SFD in turbo-jet aircraft



Computational model for MHD SFD analysis

Dynamics with Squeeze-film Dampers

Nonlinear mass-spring-damper system

$$m\ddot{h} + N(\ddot{h}, \dot{h}, h) + k(h - h_0) = f(t)$$

- $N(\ddot{h}, \dot{h}, h)$ is the nonlinear damping force from the SFD
- Damping force computed from the Reynolds equation solution

$$N(\ddot{h}, \dot{h}, h) = \int_\Omega p \; d\Omega = \int_\Omega \mathcal{L}^{-1}\left(\mathrm{Ha}^3 \dot{h} + \mathrm{Re}_s \kappa \ddot{h}\right) \; d\Omega$$

where: $\mathcal{L}(\cdot) = \nabla \cdot \big(\kappa \nabla (\cdot)\big)$

- Results in a nonlinear integro-differential equation

$$m\ddot{h} + \int_\Omega \mathcal{L}^{-1}\left(\mathrm{Ha}^3 \dot{h} + \mathrm{Re}_s \kappa \ddot{h}\right) \; d\Omega + k(h - h_0) = f(t)$$

Time integration

Newmark-Beta method with Newton-Raphson iterations

- Choose Newmark parameters corresponding to linear expansion of $\ddot{h}$
- For each time-step n, we solve a nonlinear problem:

Newton-Raphson system in incremental form

$$\mathrm{J}\delta h_{n+1} = \mathrm{R}(\delta\ddot{h}_{n+1}, \delta\dot{h}_{n+1}, \delta h_{n+1})$$

where: $$\mathrm{J} = \left(\frac{6m}{\Delta t^2} + k\right)$$

$$\mathrm{R}(\delta\ddot{h}_{n+1}, \delta\dot{h}_{n+1}, \delta h_{n+1}) = \delta f(t_{n+1}) - \delta N(\ddot{h}_{n+1}, \dot{h}_{n+1}, \dot{h}_{n+1}) + 3m\ddot{h}_n + \frac{6m}{\Delta t}\dot{h}_n$$

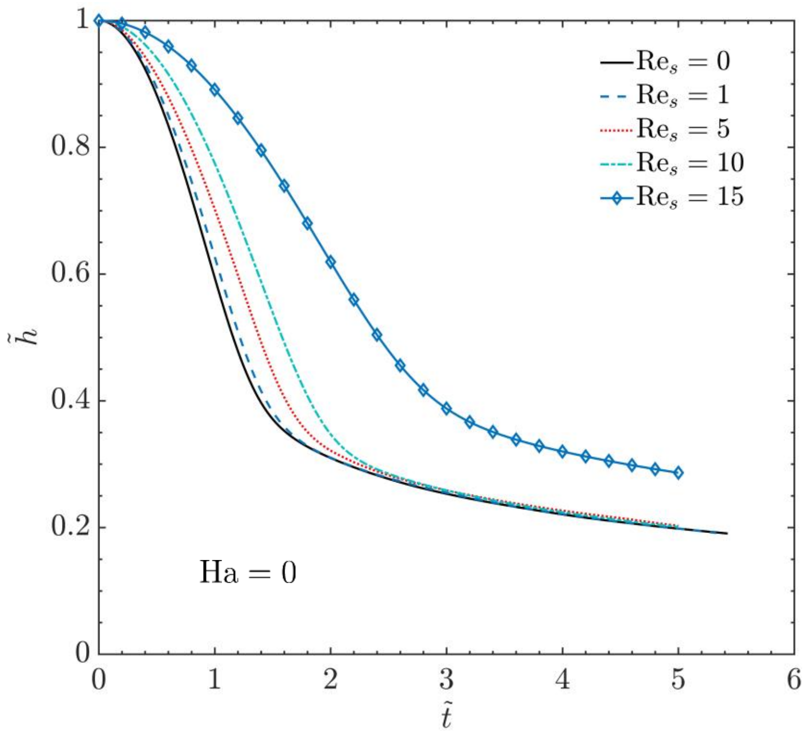
- Once converged, update the solution for next time step

$$h_{n+1} = h_n + \delta h_{n+1}$$
$$\dot{h}_{n+1} = -2\dot{h}_n - \frac{\Delta t}{2}\ddot{h}_n + \frac{3}{\Delta t}\delta h_{n+1}$$
$$\ddot{h}_{n+1} = -m^{-1}\left[k(h_{n+1} - h_0) + n(\ddot{h}_{n+1}, \dot{h}_{n+1}, \dot{h}_{n+1}) - f(t)\right]$$

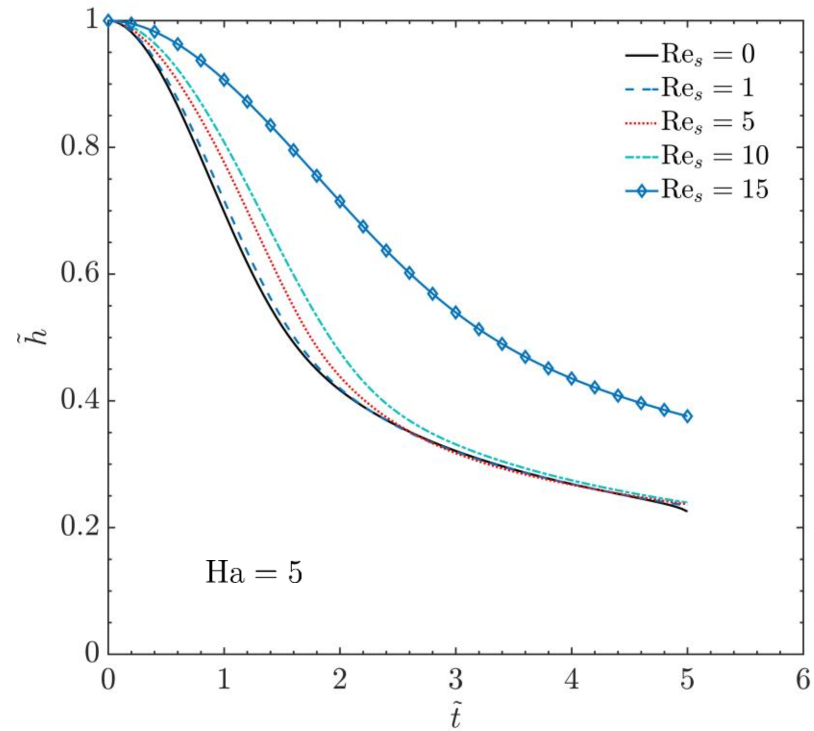
Problem 1: Constant load with temporal inertia effects

Parameters:
$\Omega = \mathbb{R}(0,1)^2$ (unit square), $\quad f(t) = -1, \quad m = 1, \quad k = 0$

Initial conditions:
$h_0 = 1, \quad \dot{h}_0 = 0$



Inertia effect, no MHD effect



Inertia effect, with MHD effect

Transient Results

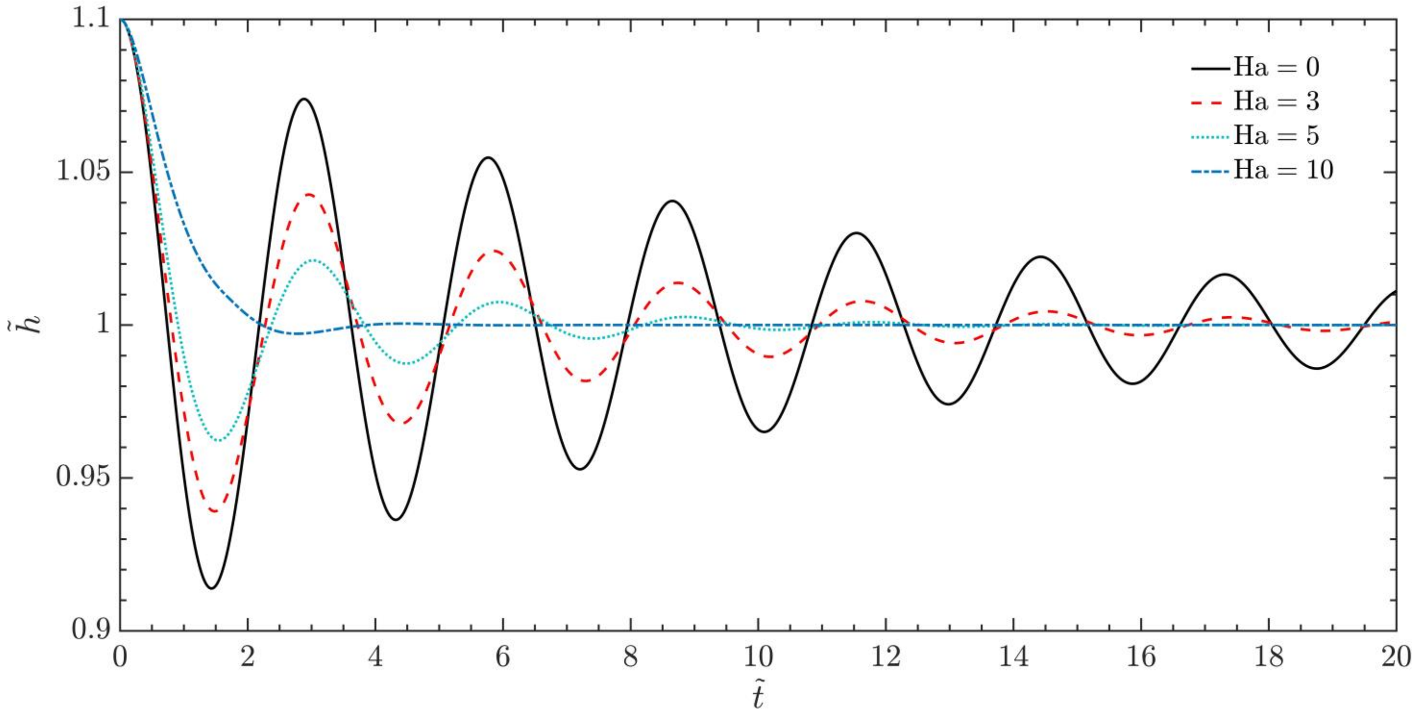
Problem 2: Free vibration with varying magnetic field strength

Parameters:

$\Omega = \mathbb{R}(0,1)^2$ (unit square), $\quad f(t) = 0, \quad m = 1, \quad k = 5, \quad \mathrm{Re}_s = 1$

Initial conditions:

$h_0 = 1.1, \quad \delta h_0 = .1, \quad \dot{h}_0 = 0$



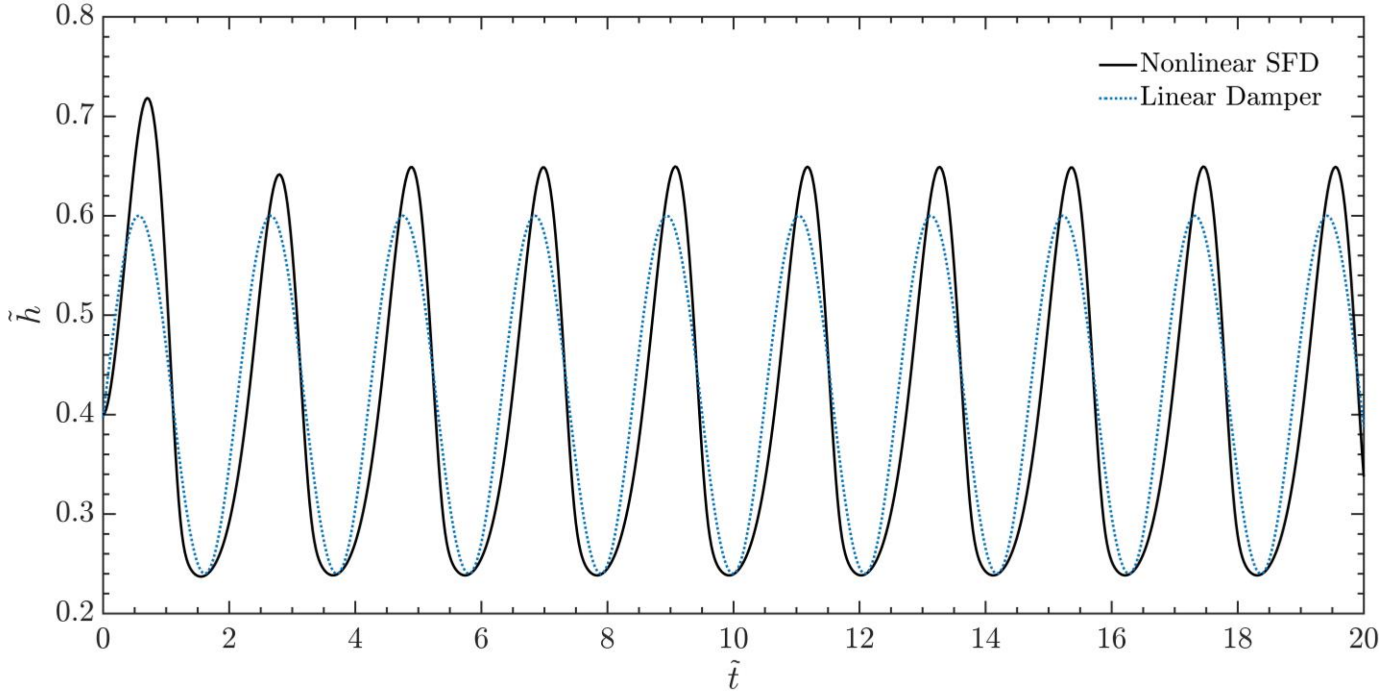
Free vibration with inertia and MHD effects

Transient Results

Problem 3: Forced vibration highlighting nonlinearity

Parameters:
$\Omega = \mathbb{R}(0,1)^2$ (unit square), $f(t) = 5\cos(3t)$, $m = 1$, $k = 5$, $\mathrm{Re}_s = 1$, $\mathrm{Ha} = 0$

Initial conditions:
$h_0 = 0.4$, $\delta h_0 = 0$, $\dot{h}_0 = 0$



Forced vibration with inertia and no MHD effects

Problem 4: Modal analysis via white-noise excitation

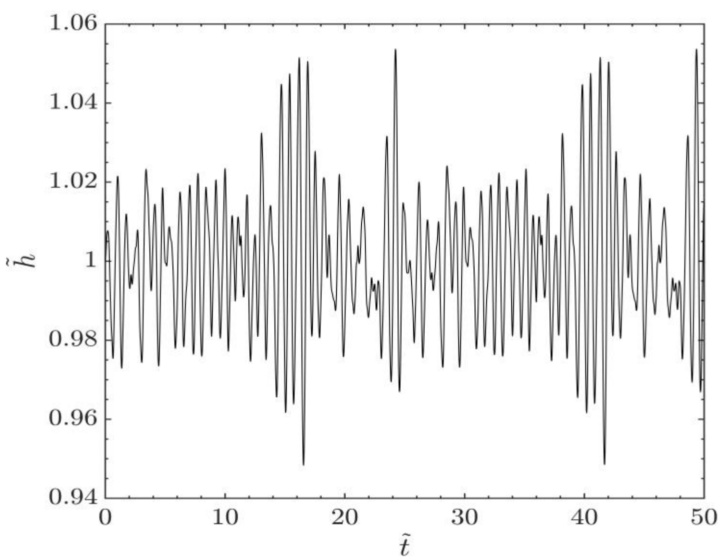
- Excite with a white-noise forcing signal

Parameters:
$\Omega = \mathbb{R}(0,1)^2$ (unit square), $\quad m = 1, \quad k = 75, \quad \mathrm{Re}_s = 0$

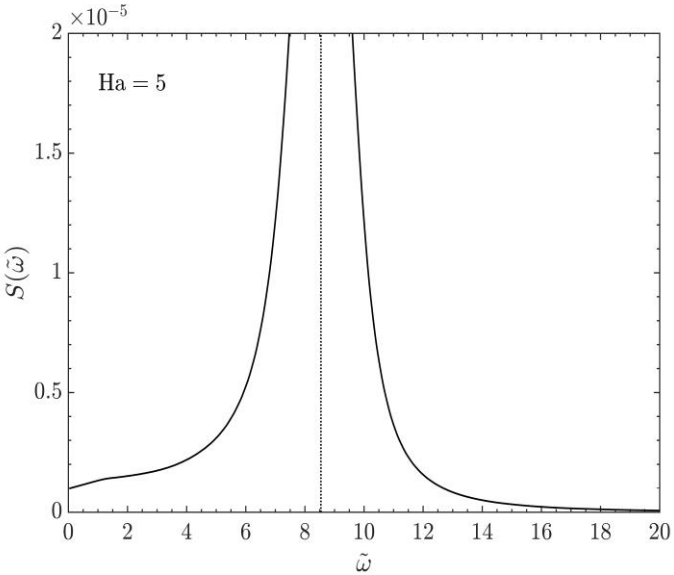
Excitation:
$f(t) = A\sum_{n=0}^{n_c} \cos(n\omega t + \phi_n), \quad A = 0.1, \quad n_c = 200, \quad \omega = 0.25$

Initial conditions:
$h_0 = 1, \quad \delta h_0 = 0, \quad \dot{h}_0 = 0$

- Determine the resonant mode by computing the power spectrum of the response



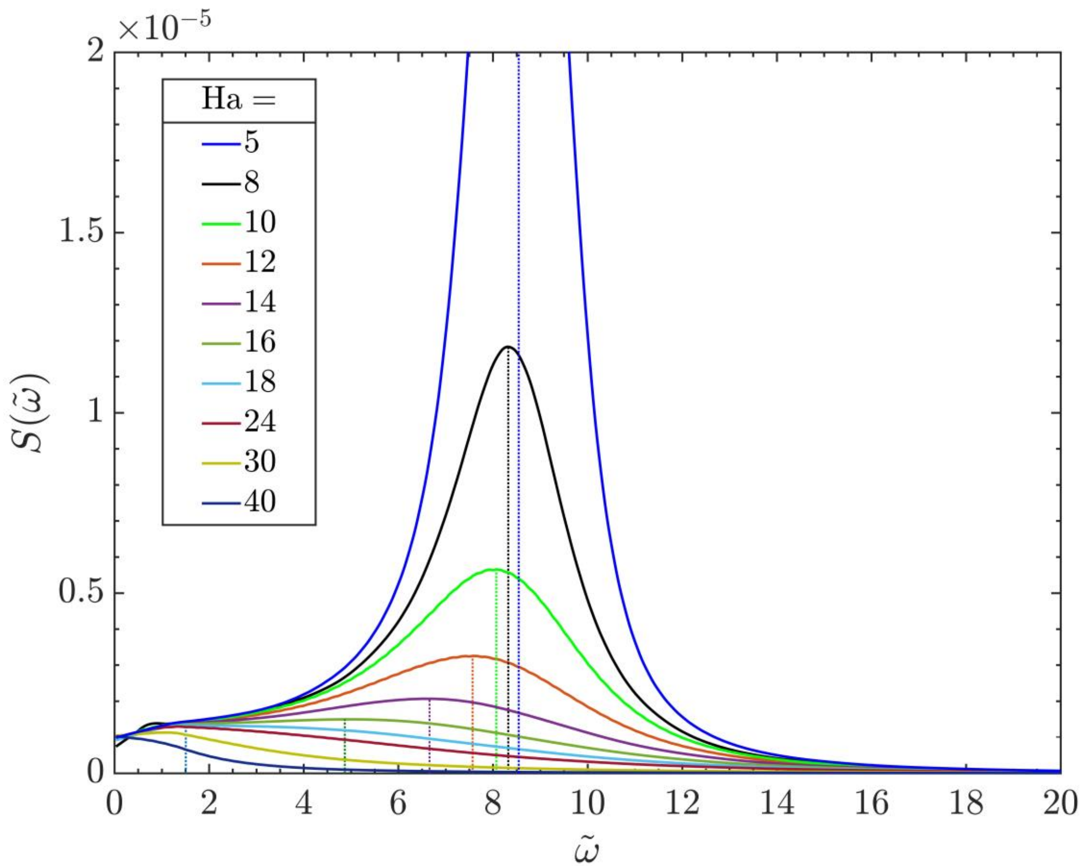
Time series response for Ha = 5



Power spectrum response

Problem 4: Modal analysis via white-noise excitation

- Repeat the process with varying magnetic field strength



Power spectra of the responses for a range of Hartmann numbers

Conclusions

Part I – quasi-steady analysis

- Novel derivation of an extension to the Reynolds equation permits modelling of MHD effects and temporal inertia (added mass) in SFDs
- MHD forces may augment the role of viscous damping
- Surface roughness can significantly influence the flow
- FEM provides an optimal solution and relaxes regularity requirements of flow conductivities

Part II – transient analysis

- Incorporating the SFD force in a dynamic system yields a nonlinear integro-differential equation
- Problem integrated in time with a Newmark/Newton-Raphson approach
- Damping properties of the system can be controlled with varying magnetic field strength

Thank you!

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Questions?