Numerical and Theoretical Investigation of Compressible Boundary Layer
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Introduction

- Flow around a moving body exhibits a thin boundary layer along solid surface
- Dominated by viscous effects
- Flow can exist as a laminar or turbulent flow field or both
- Compressible boundary layers
Problem Overview

- Mach 0.5, high Reynolds number compressible flat plate boundary layer challenge
- Prandtl number $Pr = 0.71$
- The Reynolds number based on the plate length, $Re_L = 1 \times 10^6$

Computation Domain for the Flat Plate Boundary Layer

[Ref.] 1st International Workshop on High-Order CFD Methods
Challenges

Flow Field Expectations:

Numerical Capabilities:

• High order method
• Highly clustered mesh
• Capability to capture transition from laminar to turbulent
• Appropriate and exact boundary conditions
• Very thin boundary layers
• Capability to capture sharp velocity and temperature gradient
Navier-Stokes Equations

Continuity:
\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0
\]

x-Momentum:
\[
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho uv)}{\partial y} = \frac{\partial (-p + \tau_{xx})}{\partial x} + \frac{\partial (\tau_{yx})}{\partial y}
\]

y-Momentum:
\[
\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho uv)}{\partial x} + \frac{\partial (\rho v^2)}{\partial y} = \frac{\partial (\tau_{xy})}{\partial x} + \frac{\partial (-p + \tau_{yy})}{\partial y}
\]

Energy:
\[
\frac{\partial}{\partial t} [\rho (ie + ke)] + \frac{\partial}{\partial x} [\rho u (ie + ke)] + \frac{\partial}{\partial y} [\rho v (ie + ke)] = \frac{\partial}{\partial x} (K \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (K \frac{\partial T}{\partial y}) - \frac{\partial (up)}{\partial x} - \frac{\partial (vp)}{\partial y} + \frac{\partial (u \tau_{xx})}{\partial x} + \frac{\partial (v \tau_{yx})}{\partial y} + \frac{\partial (v \tau_{xy})}{\partial x} + \frac{\partial (v \tau_{yy})}{\partial y}
\]
Non-Dimensional Parameters

**Primitive Variables:**

\[
\begin{align*}
\bar{\rho} &= \frac{\rho}{\rho_\infty} \implies \rho = \bar{\rho} \rho_\infty \\
\bar{u} &= \frac{u}{V_\infty} \implies u = \bar{u} V_\infty \\
\bar{v} &= \frac{v}{V_\infty} \implies v = \bar{v} V_\infty \\
\bar{T} &= \frac{T}{T_\infty} \implies T = \bar{T} T_\infty
\end{align*}
\]

**Spatial Coordinates:**

\[
\begin{align*}
\bar{x} &= \frac{x}{L} \implies x = \bar{x} L \\
\bar{y} &= \frac{y}{L} \implies y = \bar{y} L
\end{align*}
\]

**Directly Derived Parameters:**

\[
\begin{align*}
E &= ie + ke = C_v T + \frac{u^2 + v^2}{2} \\
\bar{\mu} &= \frac{\mu}{\mu_\infty} \implies \mu = \bar{\mu} \mu_\infty \\
\mu &= \mu_\infty \left( \frac{T}{T_\infty} \right)^{3/2} \frac{T_\infty + 110}{T + 110} \\
\tau_{xx} &= \frac{2}{3} \mu (\nabla V) + 2\mu \frac{\partial u}{\partial x} \\
\tau_{xy} &= \tau_{yx} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\
\tau_{yy} &= \frac{2}{3} \mu (\nabla V) + 2\mu \frac{\partial v}{\partial y}
\end{align*}
\]
IDS Boundary Condition Challenges:

\[ u = 1.0 \quad \rho = 1.0 \]
\[ \frac{\partial v}{\partial y} = 0 \quad \frac{\partial T}{\partial y} = 0 \]

\[ u = 1.0 \]
\[ v = 0.0 \]
\[ \rho = 1.0 \]
\[ \frac{\partial T}{\partial x} = 0 \]

Symmetry

Adiabatic Wall
IDS Scheme

• The Integral Differential Scheme (IDS) is developed based on a unique combination of the differential and integral form of the Navier-Stokes (NS) equations.

• For solving the NS equations in a 2D flow field, IDS operates with conserved quantities \((\rho, \rho u, \rho v, e)\), as opposed to the primitive variables \((\rho, u, v, T)\).

• The IDS is built upon two sets of cells: spatial and temporal cells. For 2-D flows, the IDS considers an elementary control volume as a collection of four spatial cells and a single temporal cell.

• Does not rely on turbulence models or limiters.
• Accounting of the mass, momentum, and energy fluxes are conducted with the aid of the mean value theorem

\[
(U_m)^{t+\Delta t}_{i,j} = (U_m)^t_{i,j} + \frac{dU_m}{dt}_{i,j} \Delta t
\]

\[
[U_1 \ U_2 \ U_3 \ U_4]^t = [\rho \ \rho u \ \rho v \ e]
\]

• The solution vector is computed using a consistent averaging procedure

• Does not solve for pressure
Grid Independence Study

• Numerical solution for the high Reynolds compressible flat plate problem for 4 set of grids

• In each case, the solution set was iterated until the criteria for the error norm reached a value of less than $10^{-7}$

• Recorded and analyzed $u$-velocity and friction coefficient, $C_{fx}$

<table>
<thead>
<tr>
<th>Grid Size</th>
<th>No. of Points Inside B.Layer</th>
<th>Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>1501X1501</td>
<td>32</td>
<td>Laminar</td>
</tr>
<tr>
<td>2001X2001</td>
<td>42</td>
<td>Laminar</td>
</tr>
<tr>
<td>2501X2501</td>
<td>53</td>
<td>Turbulent</td>
</tr>
<tr>
<td>4001X4001</td>
<td>85</td>
<td>Turbulent</td>
</tr>
</tbody>
</table>
Grid Independence Study

- IDS predicts laminar profile for coarser grids
- Finer grids show turbulent features
- Minimum no. of grid points to capture turbulent boundary layer

![Graph showing laminar and turbulent profiles](image)

**Blasius Solution**

![Graph comparing different grids](image)

Similarity variable,

\[ \eta = y \frac{U}{\sqrt{\frac{\partial U}{\partial x}}} \]
• Theoretical, Turbulent: $Cf_x = \frac{0.059}{Re_x^{\frac{1}{5}}}$  
Laminar: $Cf_x = \frac{0.664}{\sqrt{Re_x}}$

• For fine grids, the skin friction coefficient aligns closely with the well-established turbulent quantities close to trailing edge

• Aligns with laminar quantities close to leading edge
Validation

[Ref.] H.Lee, Y.J.Kim, “Mach 3 Boundary Layer Measurement Over a Flat Plate Using the PIV and IR Thermography Technique”, AIAA 2017
Illustration of behavior of the pressure profile
• Evidence of both the transient and the unsteady nature of the IDS solution

• Used the converged restart file to observe error norm

• Transient behavior of the flow field is identical

• \( \varepsilon = e^{-\lambda t} \sin(\omega t) \)
• Unsteady behavior has the same frequency for all runs
• Quasi unsteady flow field gives strong evidence of turbulence
Illustration of unsteady behavior of the v-velocity profile
Illustration of unsteady behavior of the temperature profile
The velocity profile for turbulent flow consists of an inner and outer layer, plus an intermediate overlap between the two.

For, $y^+ \leq 5$, the velocity profile is linear i.e. $y^+ = u^+$, which is called the viscous sublayer.

Between $5 \leq y^+ \leq 30$, there is the buffer layer which is a smooth merge between the two.

In the log-law region, $u^+ = \frac{1}{k} ln(y^+) + B$.

The coefficients, $k$ and $B$ are defined by their respective turbulent correlations.
Law of The Wall

\[ u_+ = \frac{1}{\kappa} \ln y_+ + B \]

viscous sublayer

buffer layer

inertial sublayer

defect layer

integral scales

inertial scales

dissipation scales
• Transformation of u-velocity profile using the turbulent variables

• Comparison with previously conducted efforts

• Prediction is 100% on target in the viscous sublayer

• A little bit off in the inertial layer
Conclusion

• IDS solver is capable of solving very complex flow fields
• Conditions of high Reynolds number
• Compressible flow field
• Features of subsonic flow field
• IDS delivered turbulence-like solution
• Evidence of quasi-unsteady behavior for fine grids
• To fully confirm the nature of the boundary layer, further IDS evaluations are needed
Future Work

- Capability of handling clustered grid
- Capability of handling arbitrary grid shapes
- Comparison with a true solution
- Problem of flow separation and reattachment
Thank you for your attention!
Any questions?