Numerical and Theoretical Investigation of Compressible Boundary Layer

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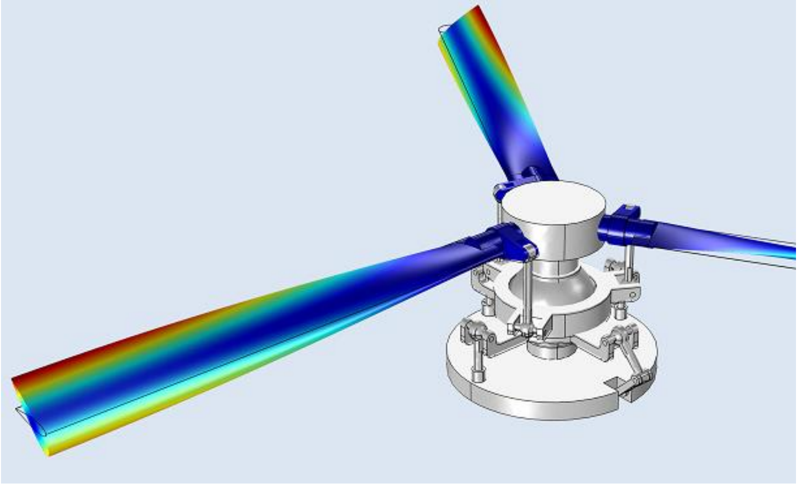
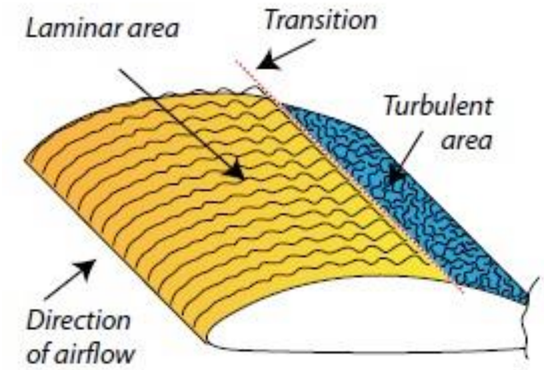
Introduction

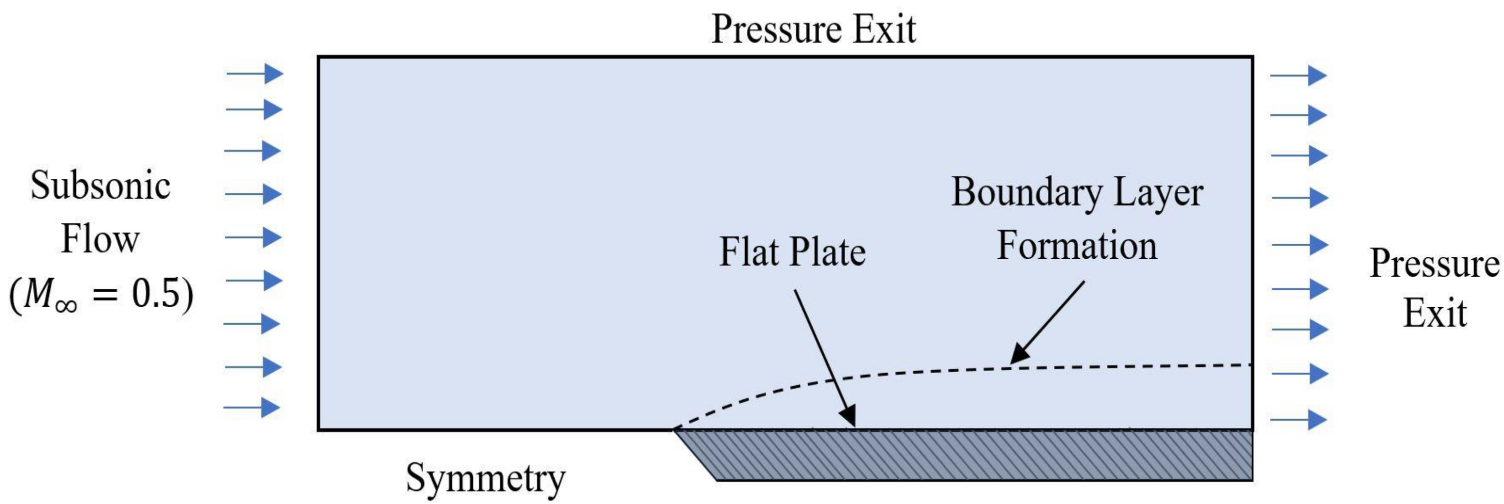
- Flow around a moving body exhibits a thin boundary layer along solid surface
- Dominated by viscous effects
- Flow can exist as a laminar or turbulent flow field or both
- Compressible boundary layers





Problem Overview

- Mach 0.5, high Reynolds number compressible flat plate boundary layer challenge
- Prandtl number $\mathrm{Pr} = 0.71$
- The Reynolds number based on the plate length, $Re_L = 1 \times 10^6$

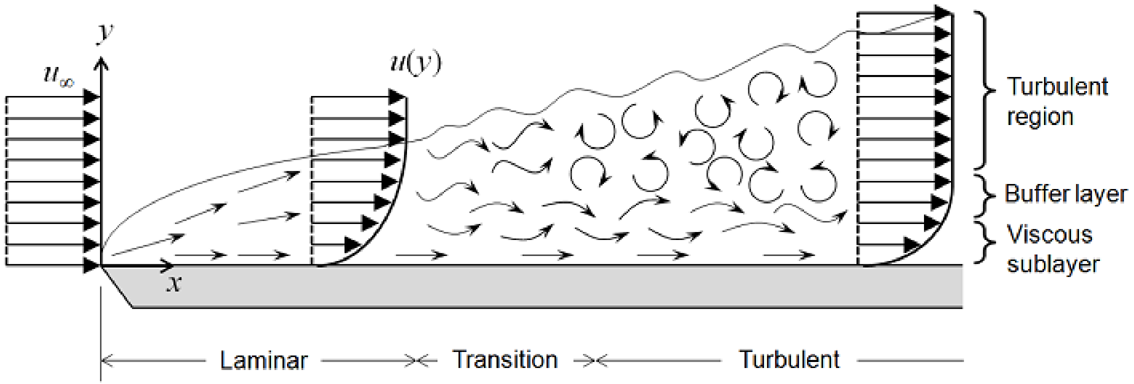


Computational Domain for the Flat Plate Boundary Layer

[Ref.] 1st International Workshop on High-Order CFD Methods

Challenges

Flow Field Expectations:



Numerical Capabilities:

- High order method
- Highly clustered mesh
- Capability to capture transition from laminar to turbulent
- Appropriate and exact boundary conditions
- Very thin boundary layers
- Capability to capture sharp velocity and temperature gradient

Navier-Stokes Equations

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

x-Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = \frac{\partial(-p + \tau_{xx})}{\partial x} + \frac{\partial(\tau_{yx})}{\partial y}$$

y-Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} = \frac{\partial(\tau_{xy})}{\partial x} + \frac{\partial(-p + \tau_{yy})}{\partial y}$$

Energy :

$$\frac{\partial}{\partial t}[\rho(ie + ke)] + \frac{\partial}{\partial x}[\rho u(ie + ke)] + \frac{\partial}{\partial y}[\rho v(ie + ke)]$$
$$= \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) - \frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} + \frac{\partial(u\tau_{xx})}{\partial x}$$
$$+ \frac{\partial(u\tau_{yx})}{\partial y} + \frac{\partial(v\tau_{xy})}{\partial x} + \frac{\partial(v\tau_{yy})}{\partial y}$$

Non-Dimensional Parameters

Primitive Variables:

$$\bar{\rho} = \frac{\rho}{\rho_\infty} \Longrightarrow \rho = \bar{\rho}\rho_\infty$$

$$\bar{u} = \frac{u}{V_\infty} \Longrightarrow u = \bar{u}V_\infty$$

$$\bar{v} = \frac{v}{V_\infty} \Longrightarrow v = \bar{v}V_\infty$$

$$\bar{T} = \frac{T}{T_\infty} \Longrightarrow T = \bar{T}T_\infty$$

Spatial Coordinates:

$$\bar{x} = \frac{x}{L} \Longrightarrow x = \bar{x}L$$

$$\bar{y} = \frac{y}{L} \Longrightarrow y = \bar{y}L$$

Directly Derived Parameters:

$$E = \mathrm{ie} + \mathrm{ke} = C_v T + \frac{u^2 + v^2}{2}$$

$$\bar{\mu} = \frac{\mu}{\mu_\infty} \Longrightarrow \mu = \bar{\mu}\mu_\infty$$

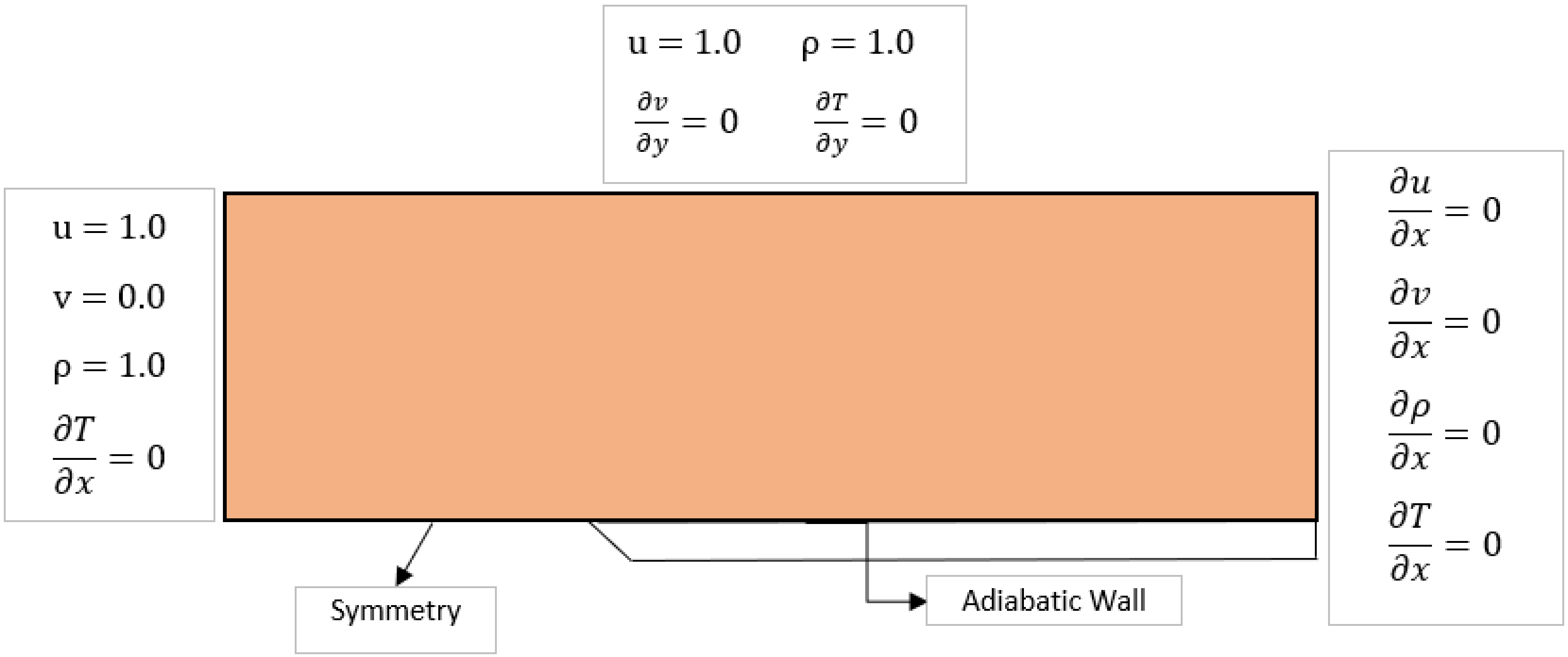
$$\mu = \mu_\infty \left(\frac{T}{T_\infty}\right)^{3/2} \frac{T_\infty + 110}{T \quad + 110}$$

$$\tau_{xx} = \frac{2}{3}\mu(\nabla V) + 2\mu\frac{\partial u}{\partial x}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\tau_{yy} = \frac{2}{3}\mu(\nabla V) + 2\mu\frac{\partial v}{\partial y}$$

IDS Boundary Condition Challenges:



IDS Scheme

- The Integral Differential Scheme (IDS) is developed based on a unique combination of the differential and integral form of the Navier-Stokes (NS) equations
- For solving the NS equations in a 2D flow field, IDS operates with conserved quantities $(\rho, \rho u, \rho v, e)$, as opposed to the primitive variables (ρ, u, v, T)
- The IDS is built upon two sets of cells: spatial and temporal cells. For 2-D flows, the IDS considers an elementary control volume as a collection of four spatial cells and a single temporal cell
- Does not rely on turbulence models or limiters

IDS Scheme

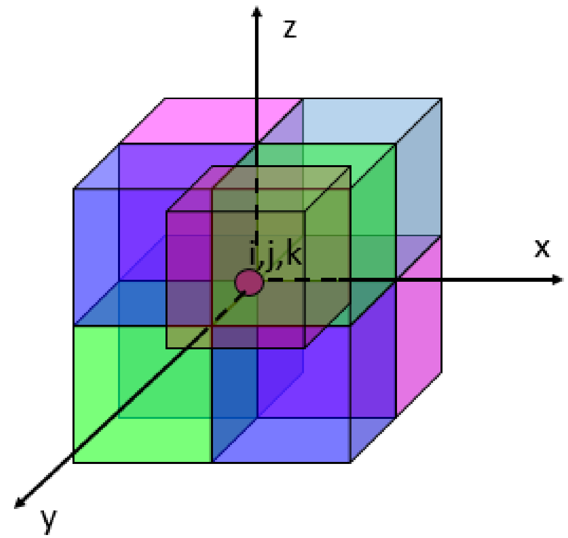
- Accounting of the mass, momentum, and energy fluxes are conducted with the aid of the mean value theorem

$$(U_m)_{i,j}^{t+\Delta t} = (U_m)_{i,j}^{t} + \frac{dU_m}{dt}_{i,j} \Delta t$$

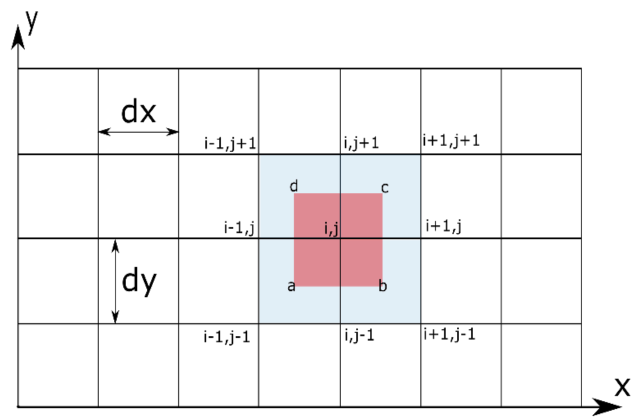
$$[U_1\ U_2\ U_3\ U_4]^t = [\rho\ \rho u\ \rho v\ e]$$

- The solution vector is computed using a consistent averaging procedure

- Does not solve for pressure



IDS computational control volume



IDS Discretization

Grid Independence Study

- Numerical solution for the high Reynolds compressible flat plate problem for 4 set of grids

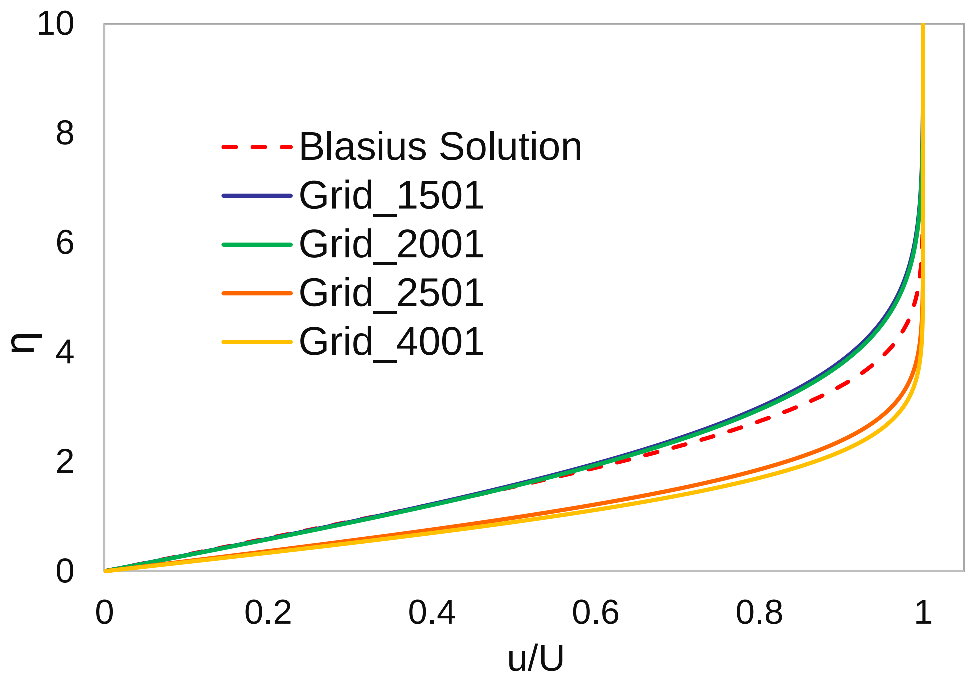
- In each case, the solution set was iterated until the criteria for the error norm reached a value of less than 10^{-7}

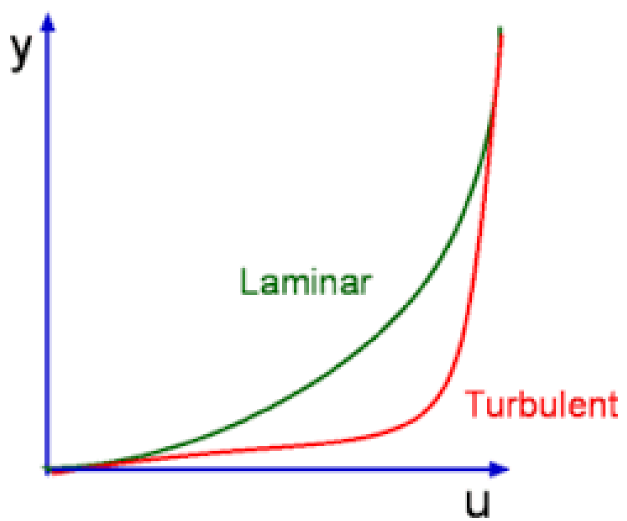
- Recorded and analyzed u-velocity and friction coefficient, Cf_x

Grid Size	No. of Points Inside B.Layer	Behavior
1501X1501	32	Laminar
2001X2001	42	Laminar
2501X2501	53	Turbulent
4001X4001	85	Turbulent

Grid Independence Study

- IDS predicts laminar profile for coarser grids
- Finer grids show turbulent features
- Minimum no. of grid points to capture turbulent boundary layer

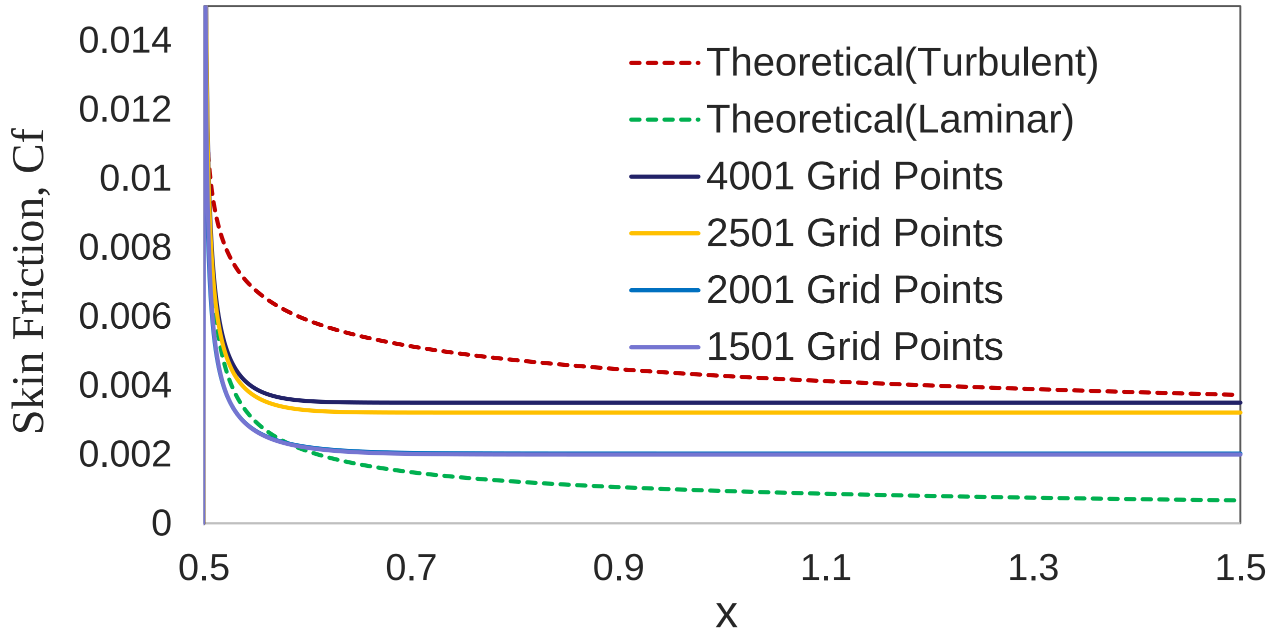




Similarity variable,

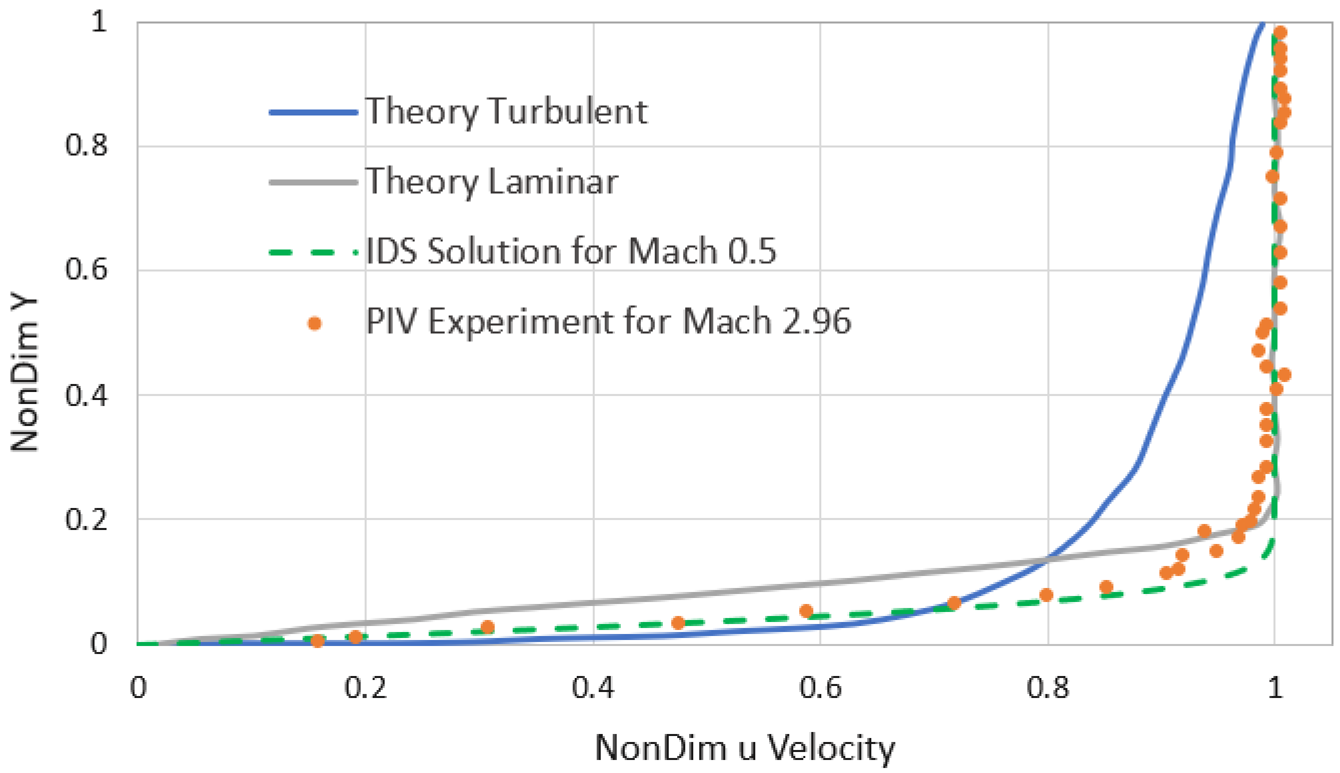
$$\eta = y\sqrt{\frac{U}{\vartheta x}}$$

Grid Independence Study



- Theoretical, Turbulent: $Cf_x = \frac{0.059}{Re_x^{\frac{1}{5}}}$ Laminar: $Cf_x = \frac{0.664}{\sqrt{Re_x}}$
- For fine grids, the skin friction coefficient aligns closely with the well-established turbulent quantities close to trailing edge
- Aligns with laminar quantities close to leading edge

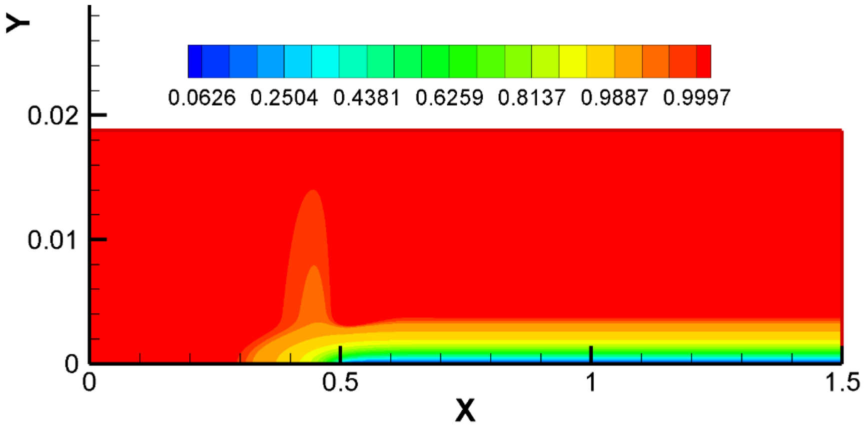
Validation



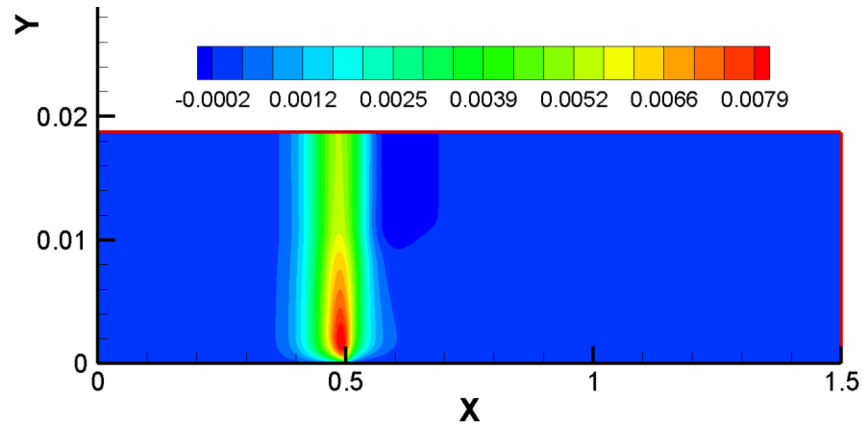
[Ref.] H.Lee, Y.J.Kim, "Mach 3 Boundary Layer Measurement Over a Flat Plate Using the PIV and IR Thermography Technique", AIAA 2017

IDS Flat Plate Solution

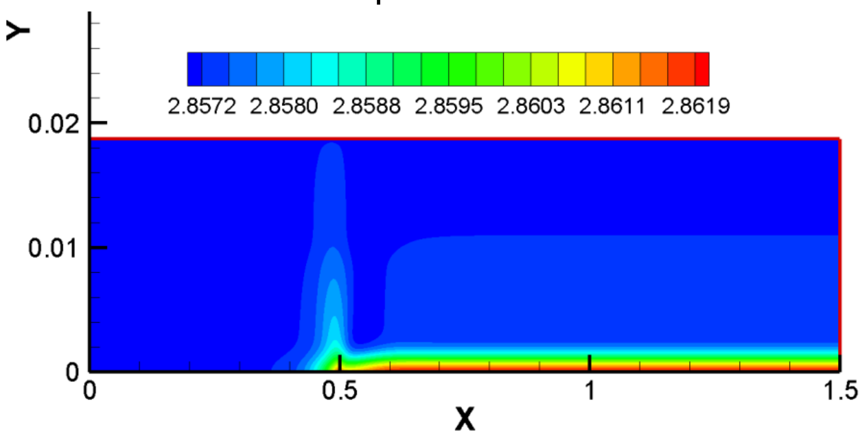
u Velocity



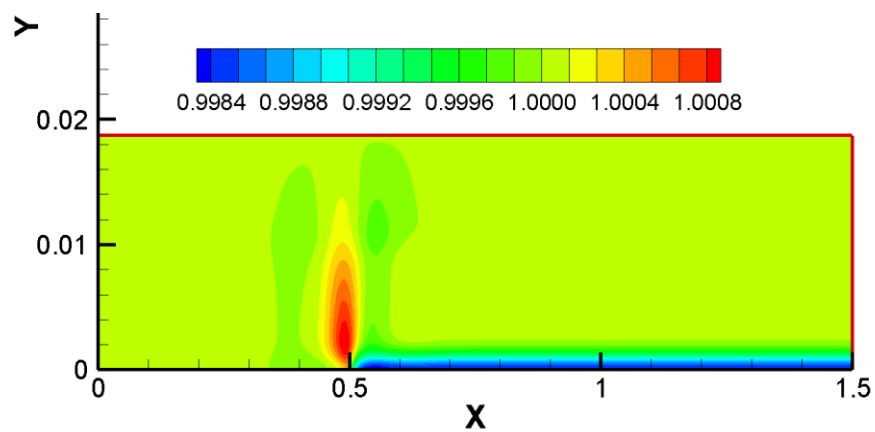
v Velocity



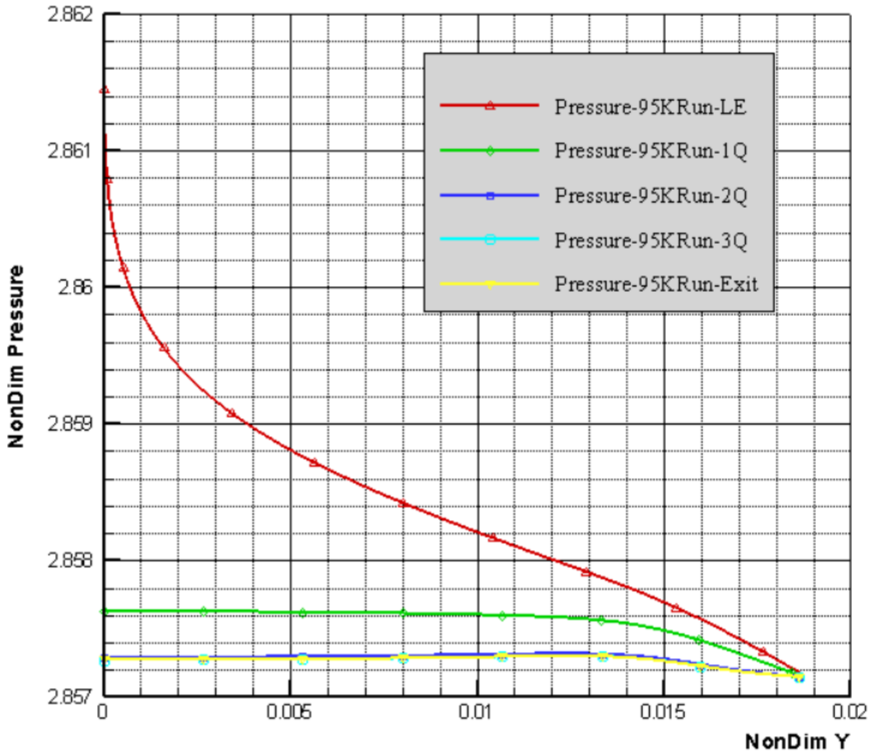
Temperature



Density



IDS Flat Plate Solution



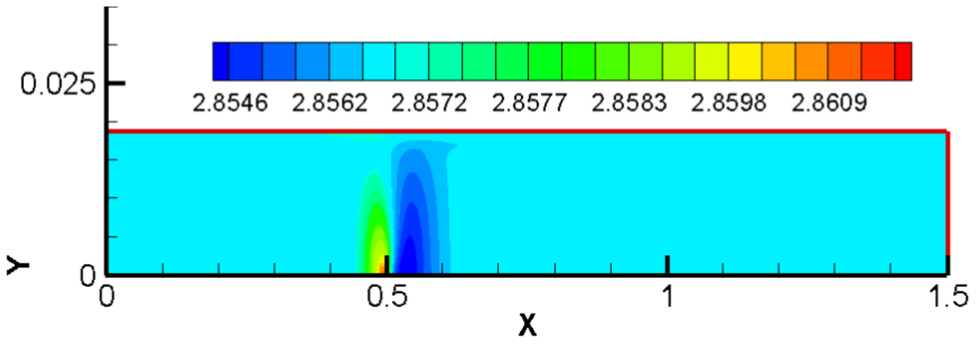
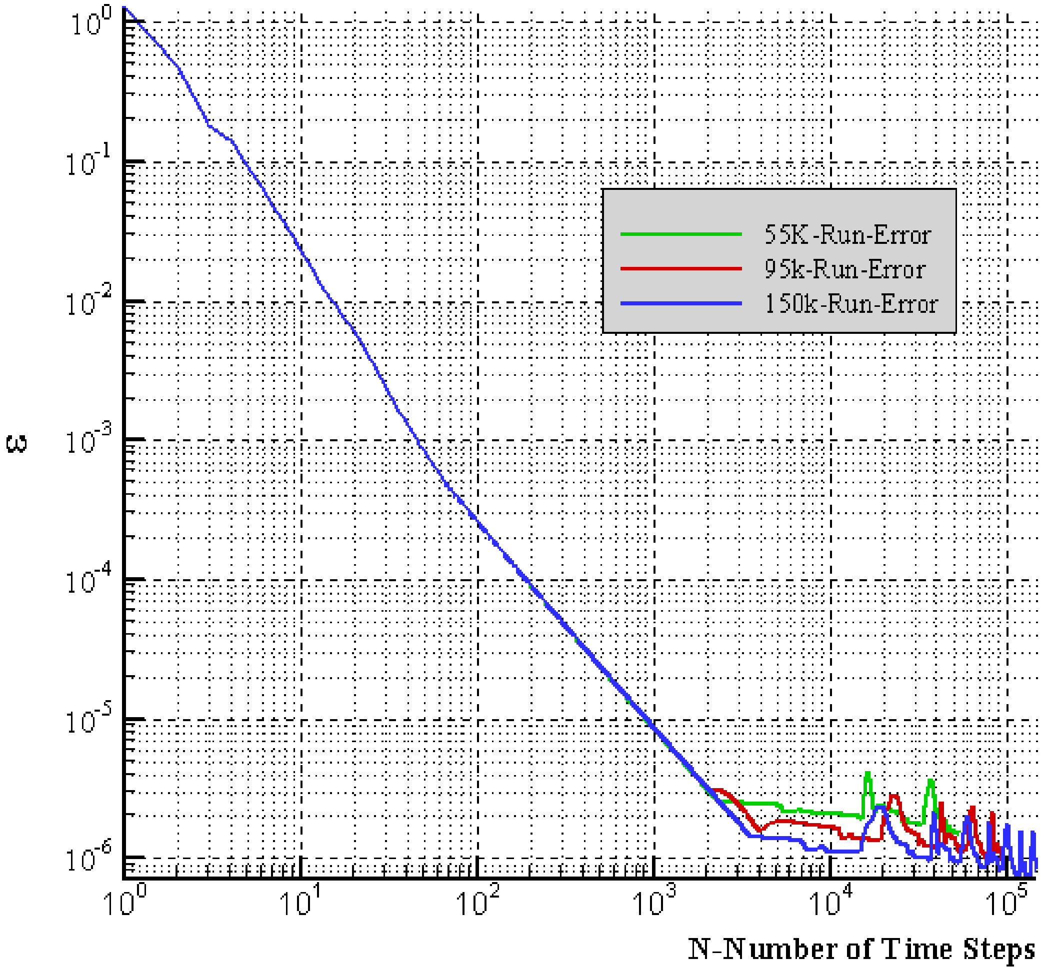


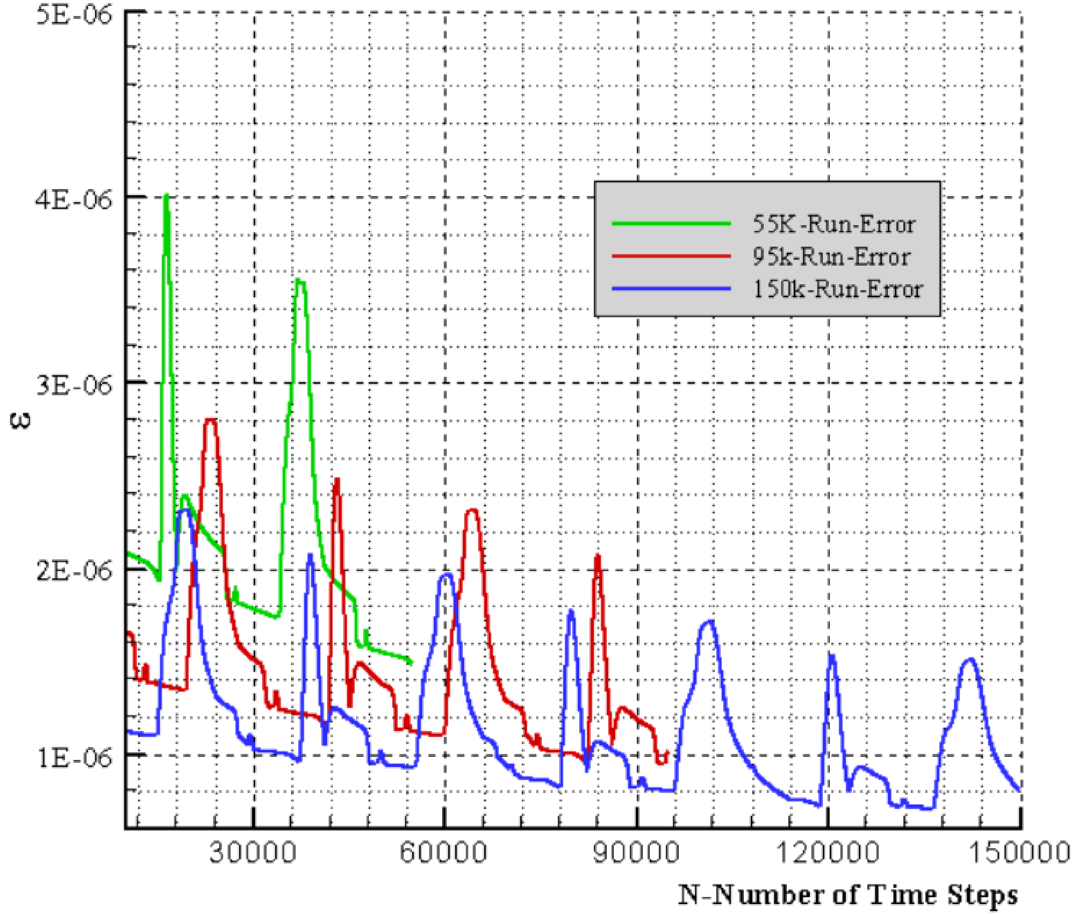
Illustration of behavior of the pressure profile

Quasi Unsteady Flow Field

- Evidence of both the transient and the unsteady nature of the IDS solution
- Used the converged restart file to observe error norm
- Transient behavior of the flow field is identical
- $\varepsilon = e^{-\lambda t}\sin(\omega t)$

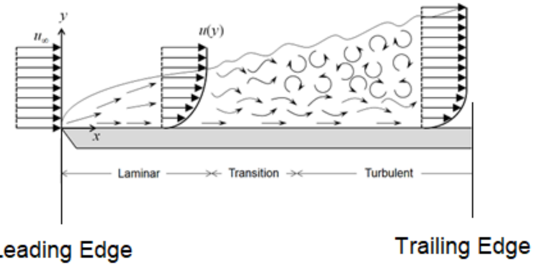


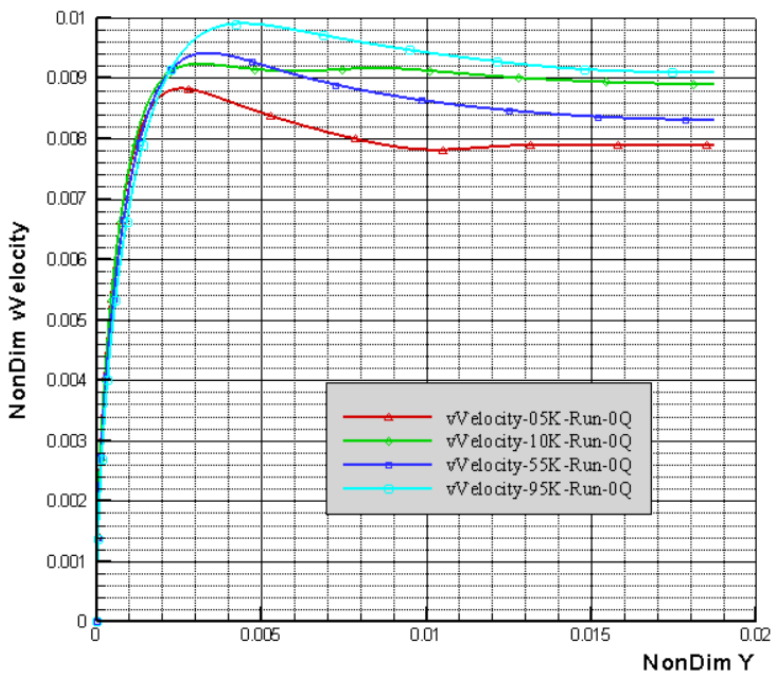
Quasi Unsteady Flow Field



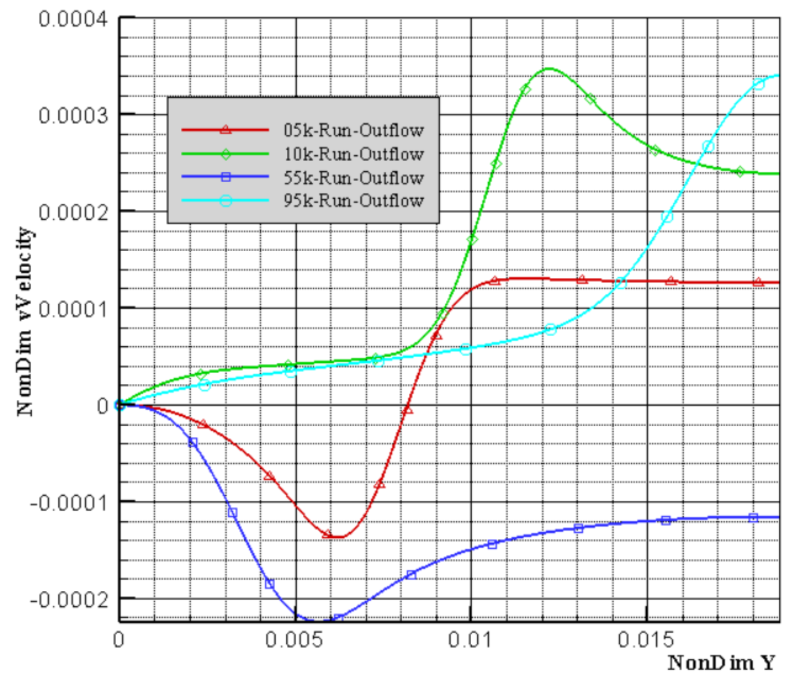
- Unsteady behavior has the same frequency for all runs
- Quasi unsteady flow field gives strong evidence of turbulence

Unsteady Boundary Layer Profile





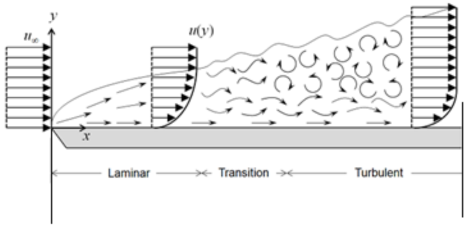
At the Leading Edge



At the Trailing Edge

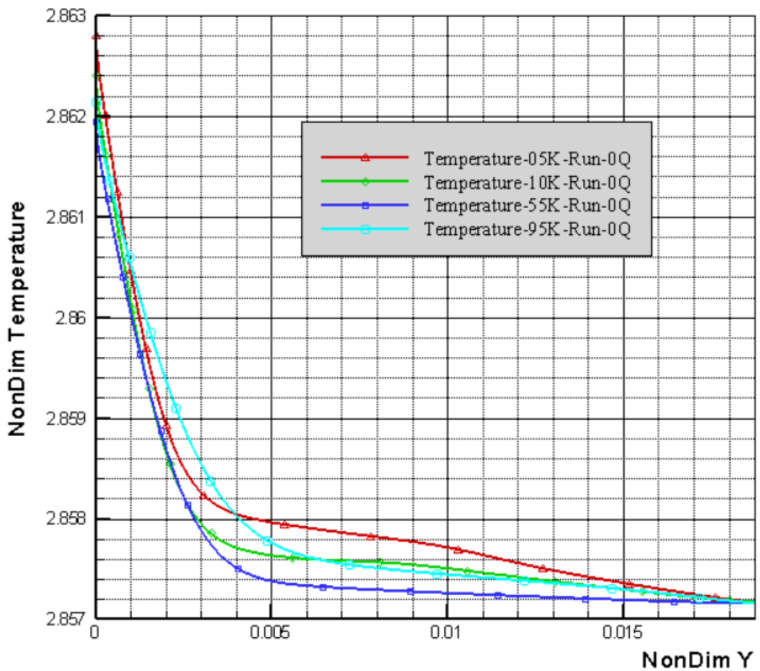
Illustration of unsteady behavior of the v-velocity profile

Unsteady Boundary Layer Profile

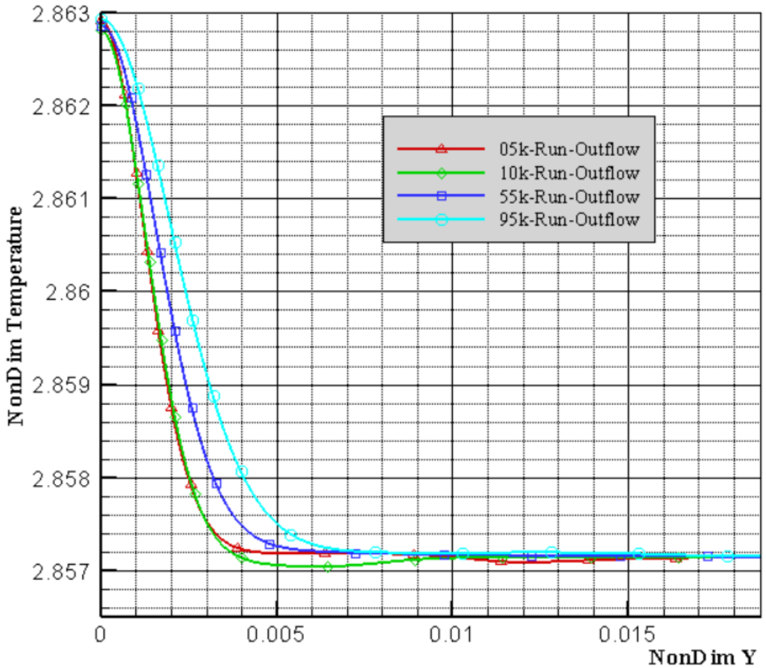


Leading Edge

Trailing Edge



At the Leading Edge



At the Trailing Edge

Illustration of unsteady behavior of the temperature profile

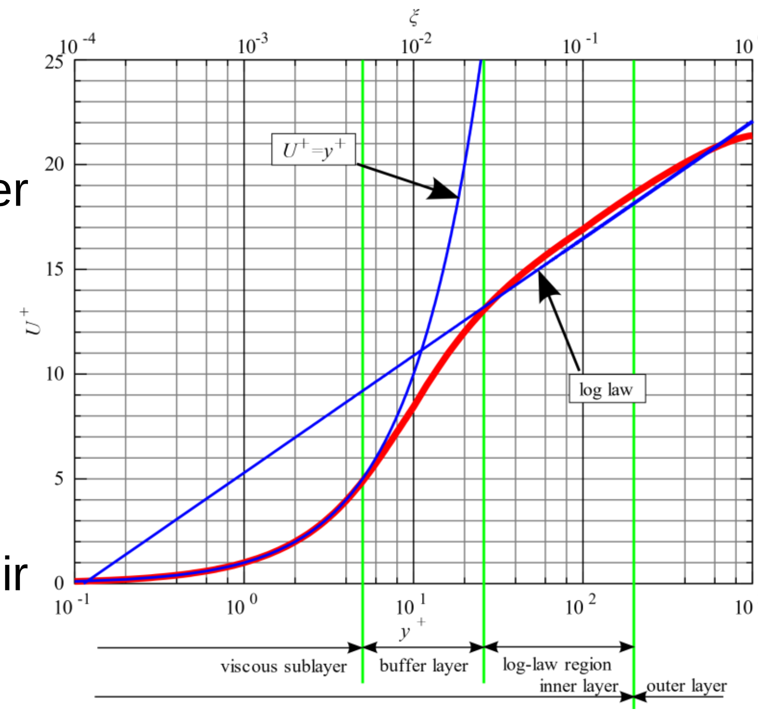
Turbulent Boundary Layer Physics

$$u^+ = \frac{u}{u_t}$$

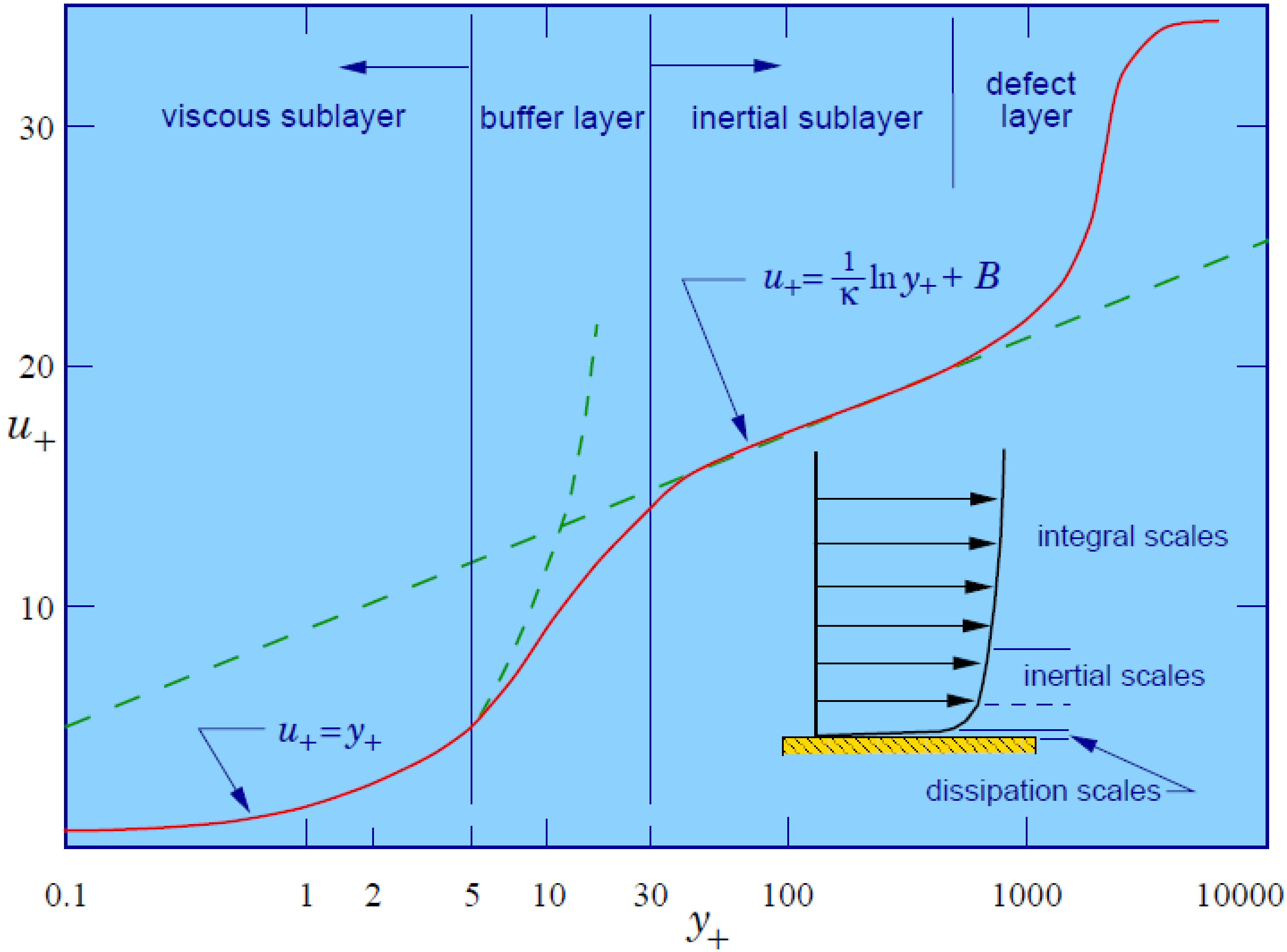
$$u_t = \sqrt{\frac{\tau_w}{\rho}}$$

$$y^+ = \frac{y u_t}{\vartheta}$$

- The velocity profile for turbulent flow consists of an inner and outer layer, plus an intermediate overlap between the two
- For, $y^+ \leq 5$, the velocity profile is linear i.e. $y^+ = u^+$, which is called the viscous sublayer
- Between $5 \leq y^+ \leq 30$, there is the buffer layer which is a smooth merge between the two
- In the log-law region, $u^+ = \frac{1}{k} ln(y^+) + B$
- The coefficients, k and B are defined by their respective turbulent correlations

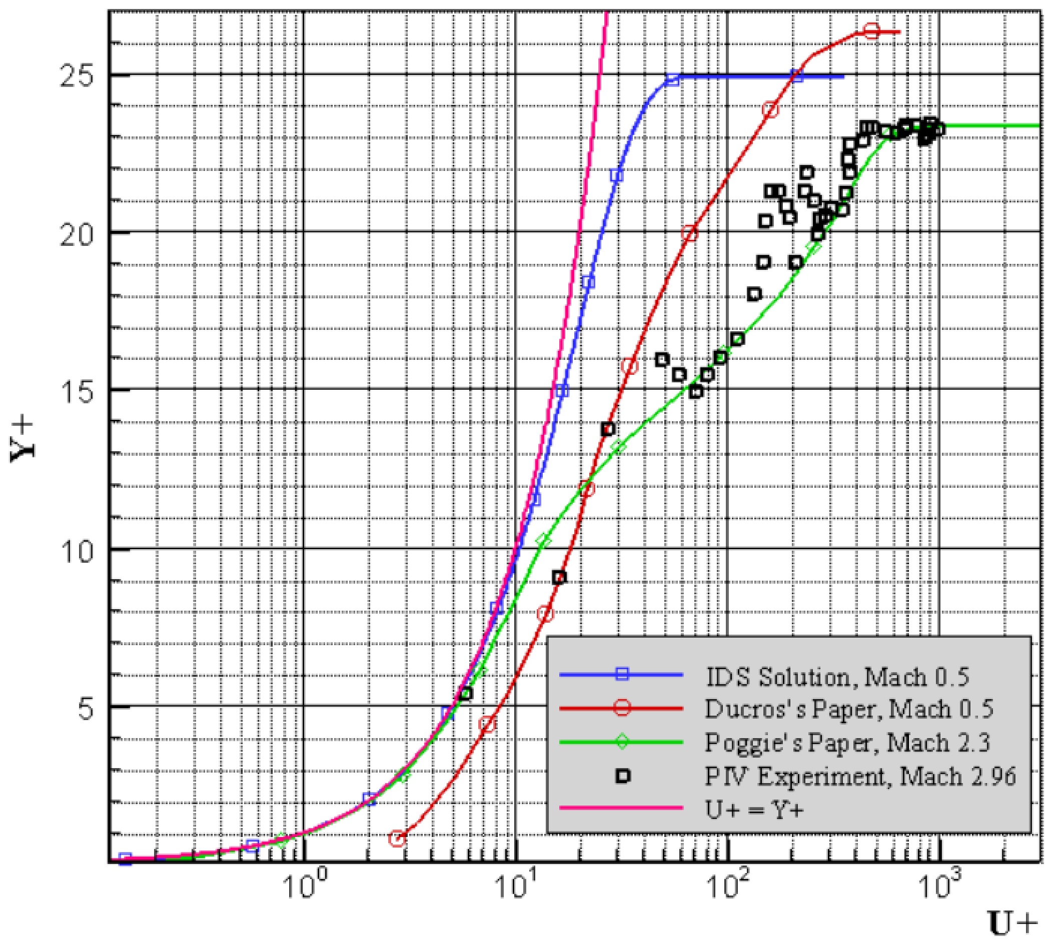


Law of The Wall



IDS Turbulent Solution

- Transformation of u-velocity profile using the turbulent variables
- Comparison with previously conducted efforts
- Prediction is 100% on target in the viscous sublayer
- A little bit off in the inertial layer

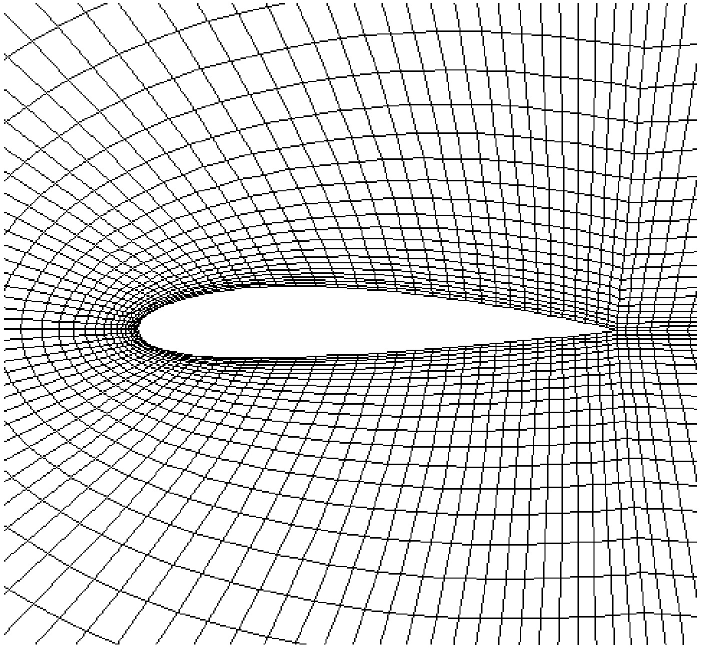


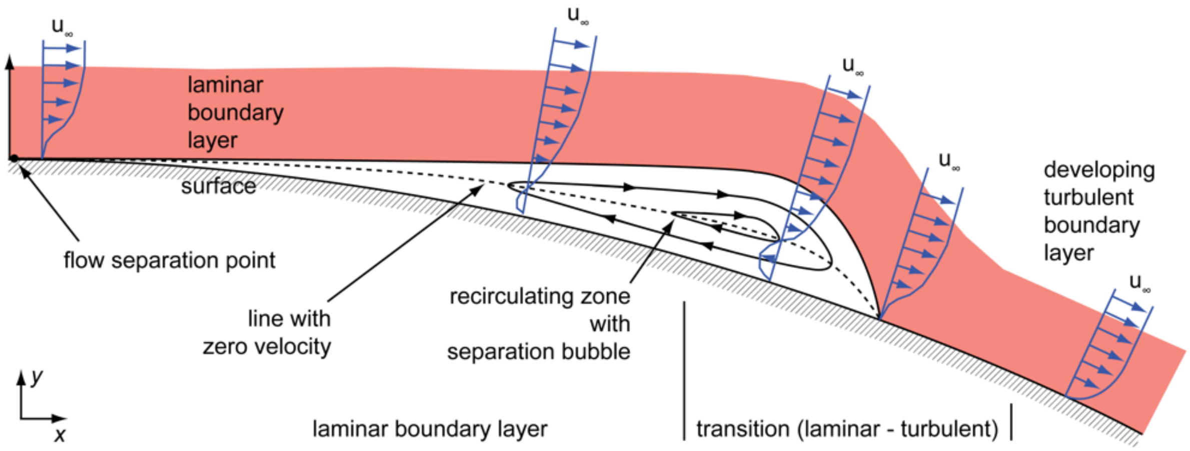
Conclusion

- IDS solver is capable of solving very complex flow fields
- Conditions of high Reynolds number
- Compressible flow field
- Features of subsonic flow field
- IDS delivered turbulence-like solution
- Evidence of quasi-unsteady behavior for fine grids
- To fully confirm the nature of the boundary layer, further IDS evaluations are needed

Future Work

- Capability of handling clustered grid
- Capability of handling arbitrary grid shapes
- Comparison with a true solution
- Problem of flow separation and reattachment





Thank you for your attention!

Any questions?