Calculation of Radiative Fin Performance Parameters for Radiator Analysis

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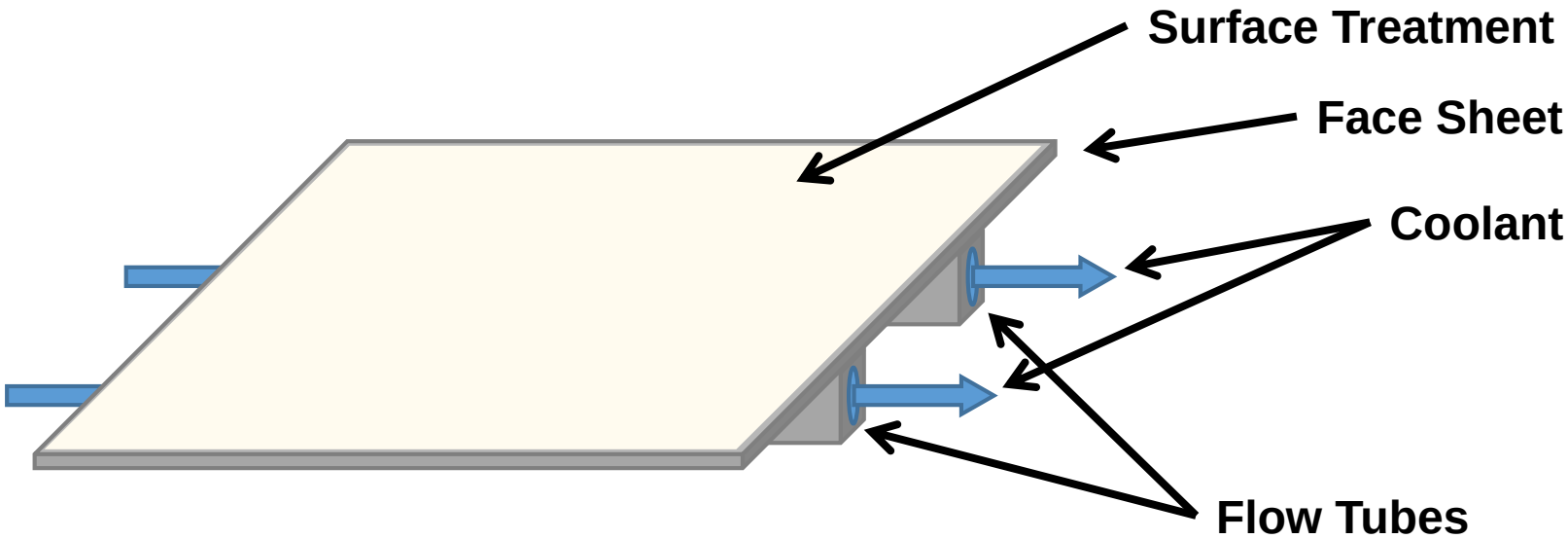
Spacecraft Radiators

- Active radiators provide the primary source of heat rejection for long duration manned spacecraft
- Radiators consist of a series of flow tubes bonded to a highly conductive face sheet with a specialized surface coating to maximize its radiative heat loss
- The face sheet effectively acts as a series of fins which extends the surface areas of the individual flow tubes



Radiator Surface Efficiency

- Temperature of the face sheet decreases as the distance from the flow tube increases due to the material's conductive resistance
- Reduces the amount of additional heat rejection gained by adding additional surface area
- Fin efficiency (η) provides a convenient measure of how effectively the face sheet extends the heated surface area
- Fin efficiency is the ratio of a fin's actual heat rejection to its theoretical maximum heat rejection that would be obtained if the entire fin was at its base temperature (infinite thermal conductivity)

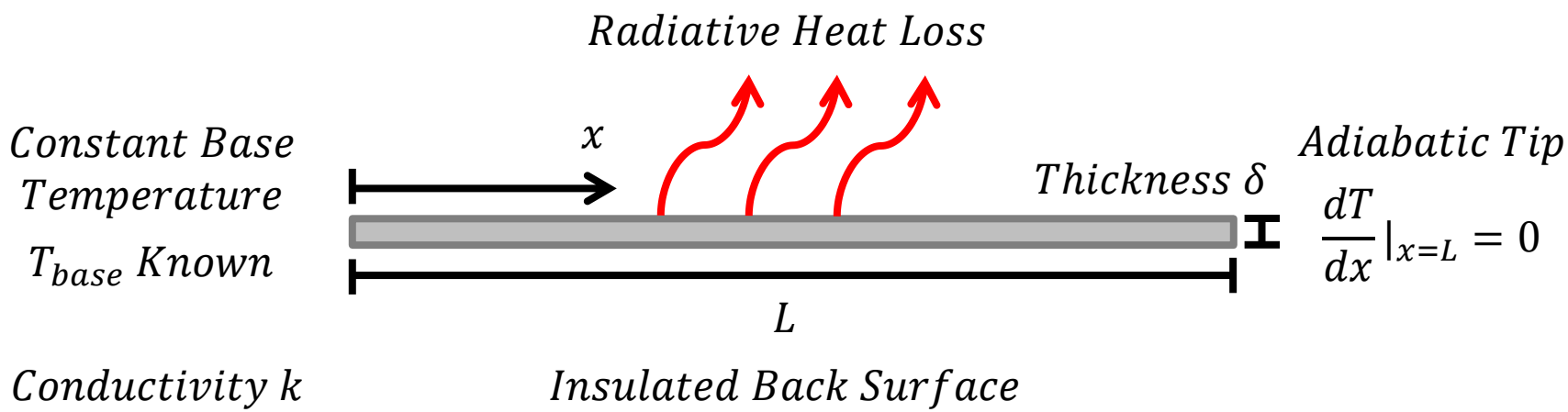
$$\eta = \frac{q_{Actual}}{q_{Max}}$$

- Maximum theoretical heat rejection (q_{max}) can be calculated directly from the net radiative heat transfer relationship:

$$q_{Max} = \sigma \varepsilon L \Delta y \left(T_{base}^4 - T_{sink}^4\right)$$

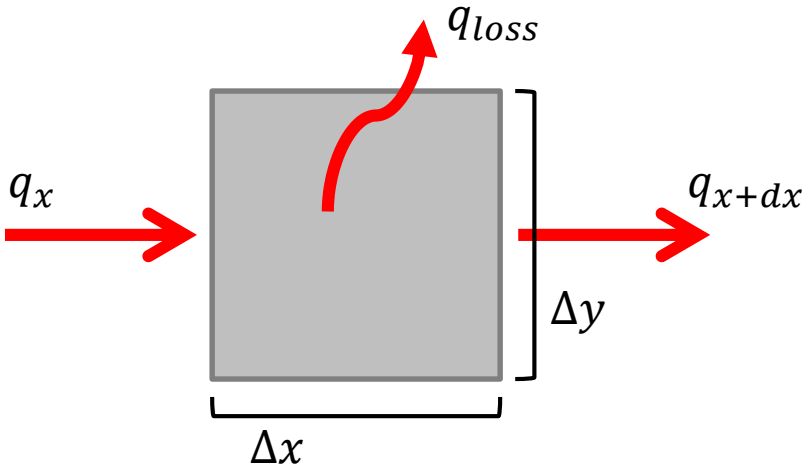
Actual Heat Rejection

- Actual heat rejection is equal to the total amount of heat conducted through the fin's base
- Heat conduced through the base can be calculated from the temperature gradient at the base using Fourier's Law
- Requires solving a differential equation for the surface temperature distribution



Fin Model

Differential Equation Development



Fin Differential Element

$$q_x = -k\delta\Delta y\frac{dT}{dx} \quad (Fourier's\ Law)$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx \quad (Taylor\ Expansion)$$

$$q_{x+dx} = -k\delta\Delta y\frac{dT}{dx} - k\delta\Delta y\frac{d^2T}{dx^2}dx$$

$$q_{loss} = \sigma\varepsilon\Delta y\Delta x\left(T^4 - T_{sink}^4\right)$$

$$q_x = q_{x+dx} + q_{loss} \quad (Conservation\ of\ Energy)$$

$$Note\text{:}\ \Delta x \approx dx$$

$$-k\delta\Delta y\frac{dT}{dx} = -k\delta\Delta y\frac{dT}{dx} - k\delta\Delta y\frac{d^2T}{dx^2}dx + \sigma\Delta y\Delta x\varepsilon\left(T^4 - T_{sink}^4\right)$$

$$\frac{d^2T}{dx^2} = \frac{\sigma\varepsilon}{k\delta}\left(T^4 - T_{sink}^4\right)$$

Equation Solution

$$\frac{d^2T}{dx^2} = \frac{\sigma\varepsilon}{k\delta}\left(T^4 - T_{sink}^4\right) \qquad set \quad K_1 = \frac{\sigma\varepsilon}{k\delta} \quad and \quad K_2 = \frac{\sigma\varepsilon T_{sink}^4}{k\delta}$$

$$\frac{d^2T}{dx^2} = K_1T^4 - K_2$$

$$Boundary\ Conditions: \quad T(x=0) = T_{Base} \qquad \frac{dT}{dx}\Big|_{x=L} = 0$$

$$\frac{dT}{dx} = -\left(\frac{2K_1T^5}{5} - 2K_2T + C\right)^{1/2} \qquad (from\ integrating\ once)$$

$$C = \frac{-2K_1T_{tip}^5}{5} + 2K_2T_{tip} \qquad (from\ applying\ second\ boundary\ condition)$$

$$\frac{dT}{dx} = -\left(\frac{2K_1}{5}\left(T^5 - T_{tip}^5\right) - 2K_2\left(T - T_{tip}\right)\right)^{1/2} \qquad where\ T_{tip} = T|_{x=L}$$

- T_{tip} is unknown so this expression for dT/dx is not usable until T_{tip} is found
- An additional integration must be performed so that the first boundary condition can be applied and used to solve for T_{tip}

Equation Solution

$$\int \left(\frac{2K_1}{5}\left(T^5 - T_{tip}^5\right) - 2K_2\left(T - T_{tip}\right)\right)^{-1/2} dT = -\int dx$$

- Left hand integral cannot be solved analytically
- Numerical integration methods cannot be applied because the integrand is asymptotic at T = T_{tip}
- Change of variable is required (proposed by Mackay and Leventhal)

$$set \quad v^2 = \frac{T}{T_{tip}} - 1 \quad dT = 2T_{tip} v dv$$

$$2T_{tip} v \frac{dv}{dx} = -v\left(\frac{2K_1 T_{tip}^5}{5}\right)^{1/2}\left((v^2+1)^4 + (v^2+1)^3 + (v^2+1)^2 + (v^2+1) + 1 - \frac{5K_2}{K_1 T_{tip}^4}\right)^{1/2}$$

$$\int_{v_{min}}^{v_{max}} \frac{dv}{\left((v^2+1)^4 + (v^2+1)^3 + (v^2+1)^2 + (v^2+1) + 1 - \frac{5K_2}{K_1 T_{tip}^4}\right)^{1/2}} = \int_0^L \left(\frac{K_1 T_{tip}^3}{10}\right)^{1/2} dx$$

$$where \quad v_{max} = v(T = T_B) = \left(T_B/T_{tip} - 1\right)^{1/2} \quad v_{min} = v\left(T = T_{tip}\right) = 0$$

Equation Solution

$$set \quad F(v) = \int_{v_{min}}^{v_{max}} \frac{dv}{\left((v^2+1)^4 + (v^2+1)^3 + (v^2+1)^2 + (v^2+1) \; + 1 - \frac{5K_2}{K_1 T_{tip}^4}\right)^{1/2}}$$

$$T_{tip} = \left[\frac{10}{K_1}\left(\frac{F(v)}{L}\right)^2\right]^{1/3} \qquad or \qquad 0 = T_{tip} - \left[\frac{10}{K_1}\left(\frac{F(v)}{L}\right)^2\right]^{1/3}$$

- F(v) can be evaluated using standard numerical integration methods if a value for T_{tip} is assumed
- Maximum theoretical range for T_{tip}:

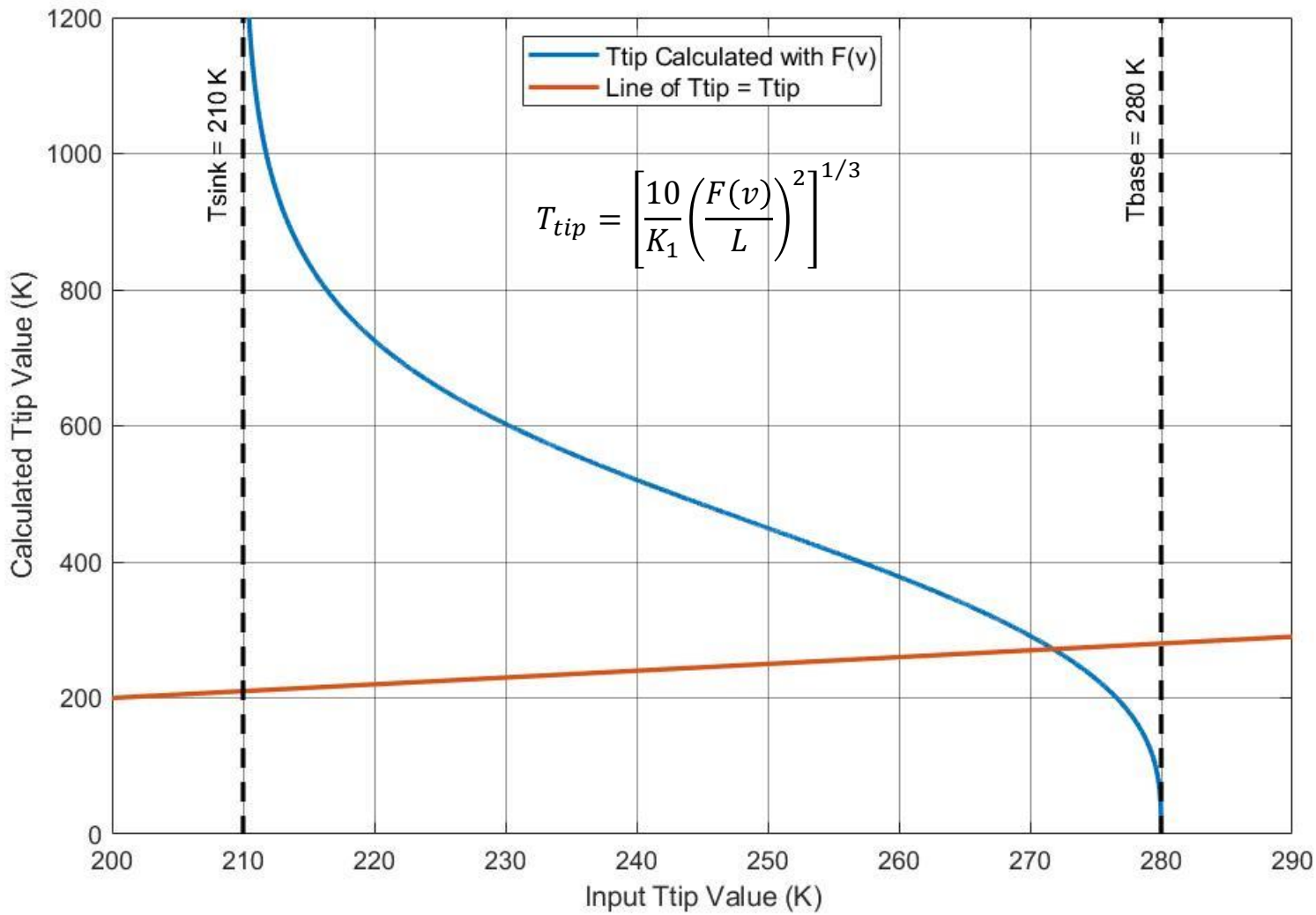
  $$T_{sink} < T_{tip} < T_{base}$$

- F(v) complex or undefined for:

  $$T_{tip} \leq T_{sink} \quad and \quad T_{tip} > T_{base} \quad (F_v \; asymptotic \; at \; T_{base})$$

- Solving for T_{tip} reduces the problem to solving a system of two equations or finding a root
- Solvable by some root finding algorithms in scientific computing software packages

Tip Temperature Function



Root Solving - Fixed Point Iteration

- A fixed point is a value for which a function returns the same value
- By starting with an initial guess and substituting the resultant value back into a fixed point function, an accurate approximation for the fixed point can be obtained
- Procedure:

$$Pick\ T_{tip0}$$

$$T_{tip} = \left[\frac{10}{K_1}\left(\frac{F(v)}{L}\right)^2\right]^{1/3} \quad with\ v(T_{tip0})$$

$$While\ \left|T_{tip} - T_{tip0}\right| < tolerance$$

$$T_{tip} = T_{tip0}$$

$$T_{tip} = \left[\frac{10}{K_1}\left(\frac{F(v)}{L}\right)^2\right]^{1/3} \quad with\ v(T_{tip0})$$

- Rate of convergence and ability of the iteration to converge is highly dependent on the accuracy of the initial guess

Root Solving - Bisection Method

- Repetitively halve the possible range based on sign change until within the desired tolerance
- Procedure:

$$Set \quad T_{min} = T_{sink} \quad T_{max} = T_{base} \quad T_{tip} = 0.5(T_{max} + T_{min})$$

$$\mathbb{F}_{max} = T_{max} - \left[\frac{10}{K_1}\left(\frac{F(v)}{L}\right)^2\right]^{1/3} \qquad with\ v(T_{max})$$

$$While \quad 0.5(T_{max} - T_{min}) < tolerance$$

$$\mathbb{F}_{tip} = T_{tip} - \left[\frac{10}{K_1}\left(\frac{F(v)}{L}\right)^2\right]^{1/3} \qquad with\ v(T_{tip})$$

$$If \quad \mathbb{F}_{tip}\mathbb{F}_{max} > 0 \quad \Rightarrow \quad T_{max} = T_{tip} \quad \mathbb{F}_{max} = \mathbb{F}_{tip}$$

$$Else \quad T_{min} = T_{tip}$$

$$T_{tip} = 0.5(T_{max} + T_{min})$$

- Iteration will always converge if a root exists in the initial range

Efficiency Calculation

- With T_{tip} known, the actual heat rejection can be calculated using Fourier's Law and the base temperature gradient

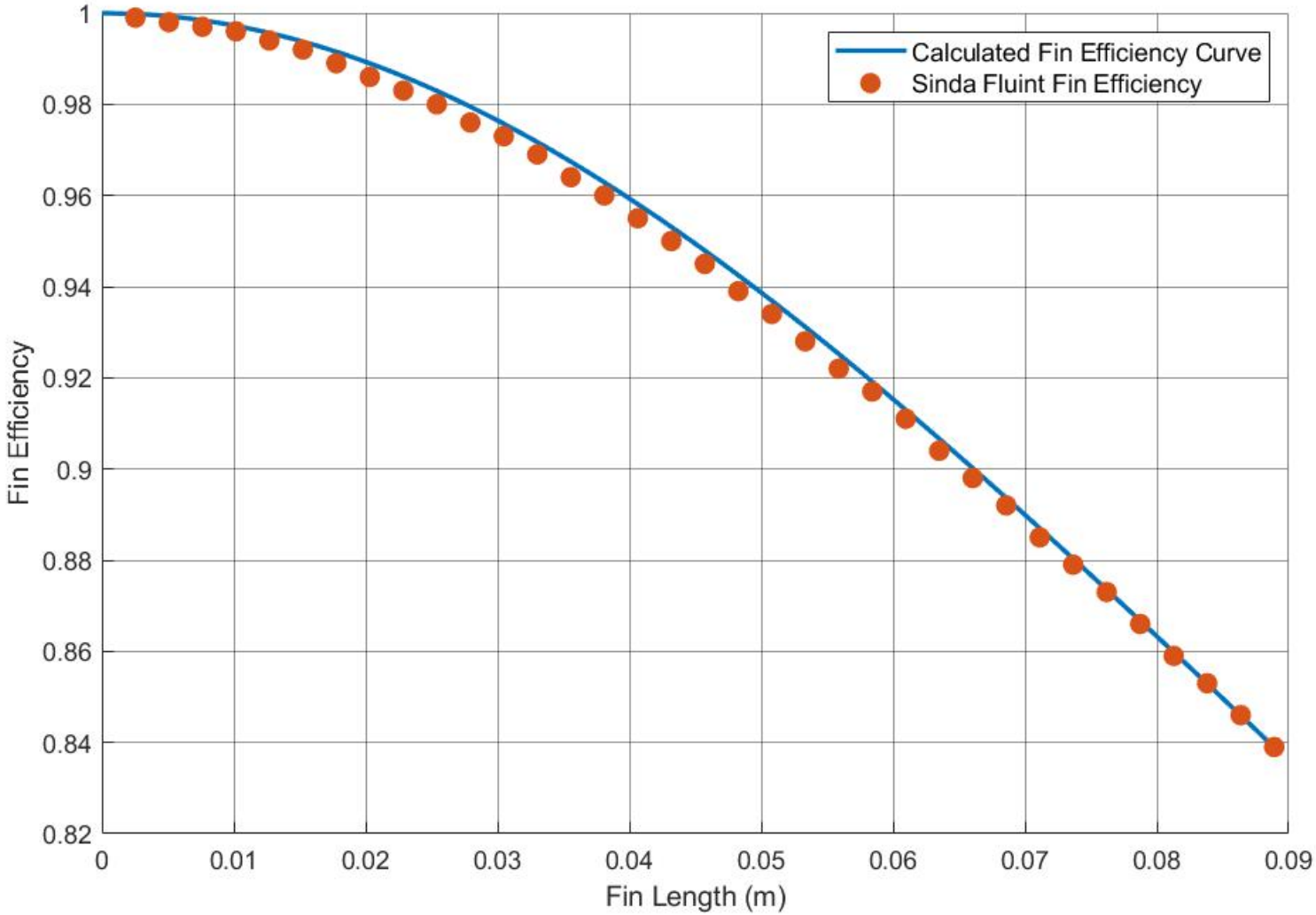
$$q_{Actual} = -k\delta\Delta y \frac{dT}{dx}\Big|_{x=0} \quad (Fourier's\ Law)$$

$$q_{Actual} = k\delta\Delta y \left(\frac{2K_1}{5}\left(T_{base}^5 - T_{tip}^5\right) - 2K_2\left(T_{base} - T_{tip}\right)\right)^{1/2}$$

- The normalized heat rejection $q_{Actual}/\Delta y$ is itself a valuable parameter
- With a valid expression for q_{Actual}, the fin efficiency can be calculated

$$\eta = \frac{k\delta\left(\frac{2K_1}{5}\left(T_{base}^5 - T_{tip}^5\right) - 2K_2\left(T_{base} - T_{tip}\right)\right)^{1/2}}{\sigma\varepsilon L\left(T_{base}^4 - T_{sink}^4\right)}$$

Efficiency Curve Comparison



Surface Temperature Solution

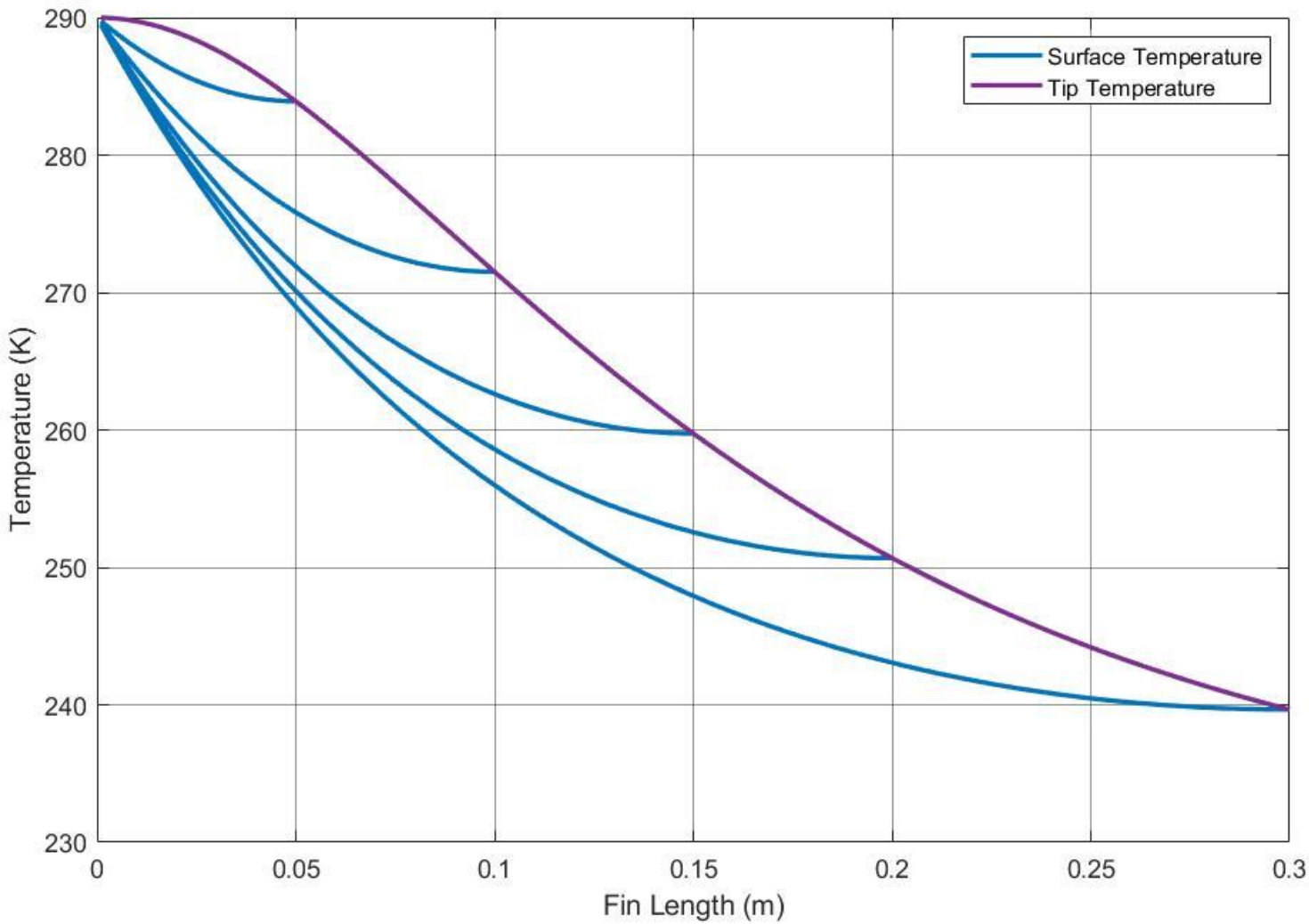
- Once T_{tip} is known, the temperature (T_i) at points (x_i) along the fin can also be solved for using a similar methodology

$$T_{tip} = \left[\frac{10}{K_1}\left(\frac{F(v)}{L_i}\right)^2\right]^{1/3} \qquad or \qquad 0 = T_{tip} - \left[\frac{10}{K_1}\left(\frac{F(v)}{L_i}\right)^2\right]^{1/3}$$

$$F(v) = \int_{v_{min}}^{v_{max}} \frac{dv}{\left((v^2+1)^4 + (v^2+1)^3 + (v^2+1)^2 + (v^2+1) + 1 - \frac{5K_2}{K_1 T_{tip}^4}\right)^{1/2}}$$

$$v_{max} = v(T = T_B) = \left(\frac{T_{base}}{T_{tip}} - 1\right)^{1/2} \qquad v_{min} = v(T = T_i) = \left(\frac{T_i}{T_{tip}} - 1\right)^{1/2}$$

Surface Temperature Distribution

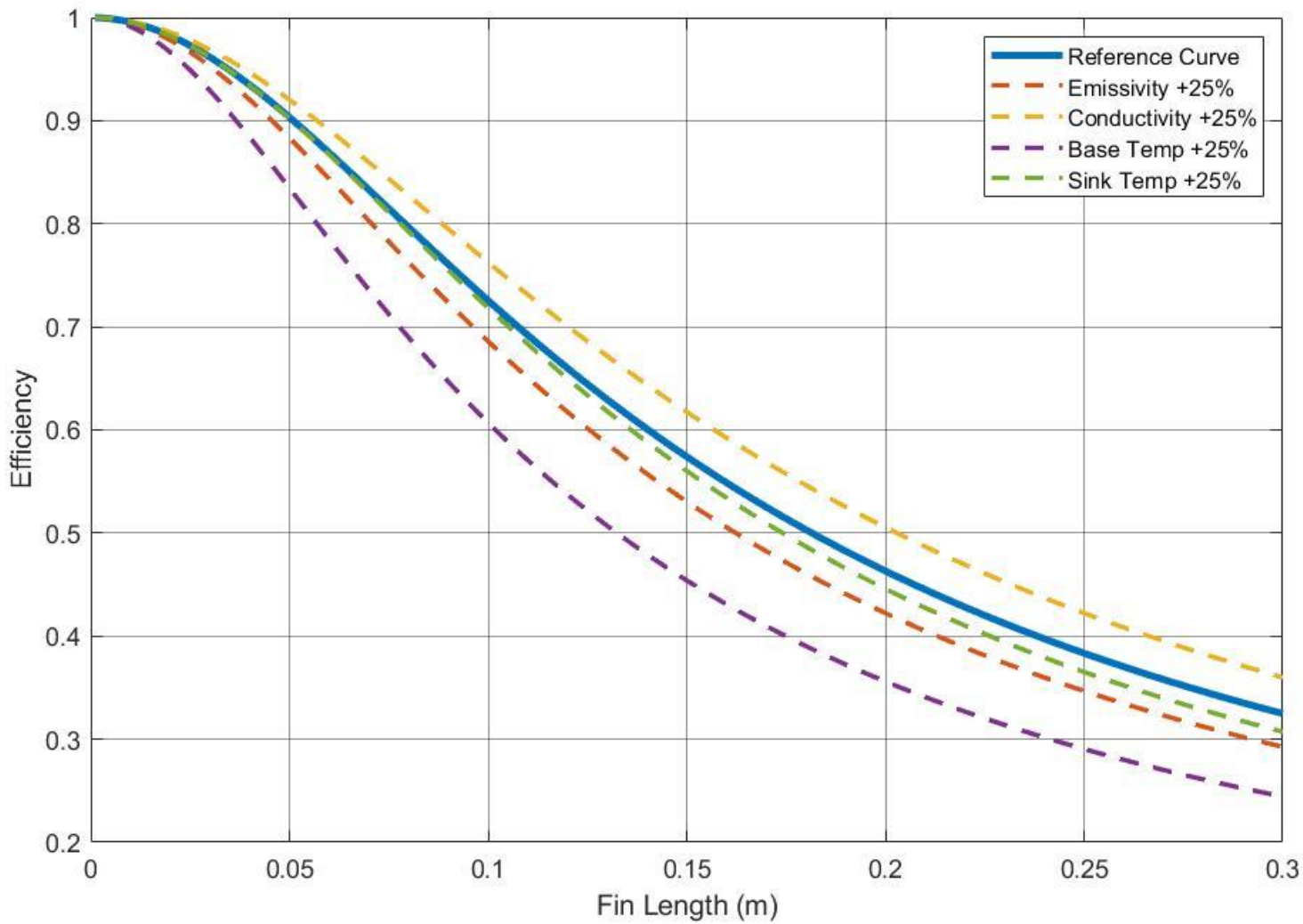


Efficiency Curve Sensitivity

- Efficiency curve is a function of ε, k, δ, T_{base}, and T_{sink}
- For a given design, ε, k, and δ will be constant but T_{base} and T_{sink} will vary depending on system operation
  - k and δ have identical effects on fin performance
  - T_{sink} has very little impact on efficiency for high efficiency fins (η > ~0.75)
- Decreased efficiency does not necessarily mean decreased heat rejection

Increase	Efficiency	Heat Rejection		Decrease	Efficiency	Heat Rejection
Emissivity	Decrease	Increase		Emissivity	Increase	Decrease
Conductivity	Increase	Increase		Conductivity	Decrease	Decrease
Thickness	Increase	Increase		Thickness	Decrease	Decrease
T_{base}	Decrease	Increase		T_{base}	Increase	Decrease
T_{sink}	Decrease	Decrease		T_{sink}	Increase	Increase

Efficiency Curve Sensitivity



Summary

- Spacecraft radiator surfaces effectively function as a series of fins
- Fin efficiency is a useful measure of radiator surface performance
- Actual heat rejection and fin efficiency can be calculated analytically if the fin tip temperature is known
- Fin tip temperature can be found using iterative numerical methods
- Surface temperatures can be calculated using similar numerical methods once the tip temperature is known
- Fin performance is a function of ε, k, δ, T_{base}, and T_{sink}

Questions

References

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- Kraus, Allan D. Aziz, Abdul. Welty, James. *Extend Surface Heat Transfer*, 2001