Physics Based Validation of an Improved Numerical Technique for Solving Thermal Fluid Related Problems

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Background

Problem: There is no analytical solution for all real problems.

- Simplified Model equation; i.e: 1D Linear wave equation

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

Initial condition:

$$u(x,0) = F(x) \qquad -\infty < x < \infty$$

Analytical solution:

$$u(x,t) = F(x - ct)$$

Background

Problem: There is no analytical solution for all real problems.

- 3-D Navier Stoke Equations (NSE)

$$\iiint \frac{d\rho}{dt} dv + \oiint \rho \overline{V} \cdot d\overline{s} = 0 \qquad (1)$$

$$\iiint \frac{d(\rho \overline{V})}{dt} dv + \oiint \rho \overline{V} (\overline{V} \cdot d\overline{s}) = - \oiint P d\overline{s} + \oiint \widehat{\tau} \cdot d\overline{s} \qquad (2)$$

$$\begin{aligned} \iiint \frac{d(\rho e)}{dt} dv + \oiint \rho e (\overline{V} \cdot d\overline{s}) &= - \oiint P (\overline{V} \cdot d\overline{s}) \\ &+ \oiint \overline{V} \cdot (\widehat{\tau} \cdot d\overline{s}) + \oiint \overline{q} \cdot d\overline{s} \end{aligned} \qquad (3)$$

Background

Boundary Conditions:

$$\rho : \Omega \times [0, t_{end}] \to \mathbb{R}$$

$$\vec{V} : \Omega \times [0, t_{end}] \to \mathbb{R}$$

$$T : \Omega \times [0, t_{end}] \to \mathbb{R}$$

Initial Conditions:

$$\rho(x, y, 0) = \rho_0, \quad x, y \in \Omega$$

$$\vec{V}(x, y, 0) = \vec{V}_0, \quad x, y \in \Omega$$

$$T(x, y, 0) = T_0, \quad x, y \in \Omega$$

Background

Numerical solution= f(Δx, Δt, Numerical Scheme)

↑

Errors !

CFD Challenges

Thermal Fluid → **Navier-Stokes Equations** → **Challenges and limitations**

Challenges & limitations

1. **No general analytical solution**
2. **Different discretization techniques**
3. **Different numerical techniques**
4. **BC & IC are required**
5. **Errors in each stage**
6. **Interpretation of the numerical dataset**

Background

Ultimate Objectives

1. The NSE must be used for a wide class of problem with minimum user inputs/interactions (tweaking)

2. Solution must adequately capture the flow physics

3. Implement cutting edge parallel libraries to study complex problems fast and accurately

Generalized CFD Problem

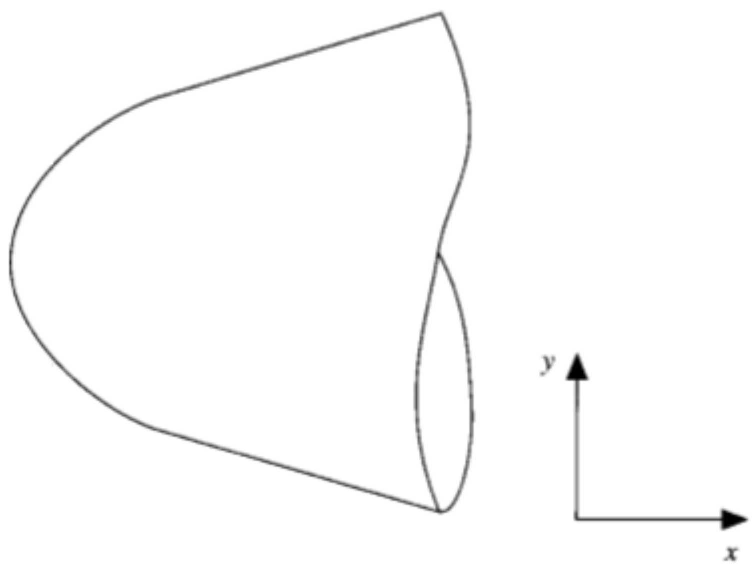


Fig.1. Schematic of the flow field over a supersonic blunt nosed body [1]

1.-John, D. Anderson JR. "Computational fluid dynamics: the basics with applications." *P. Perback, International ed., Published* (1995).

Generalized CFD Problem

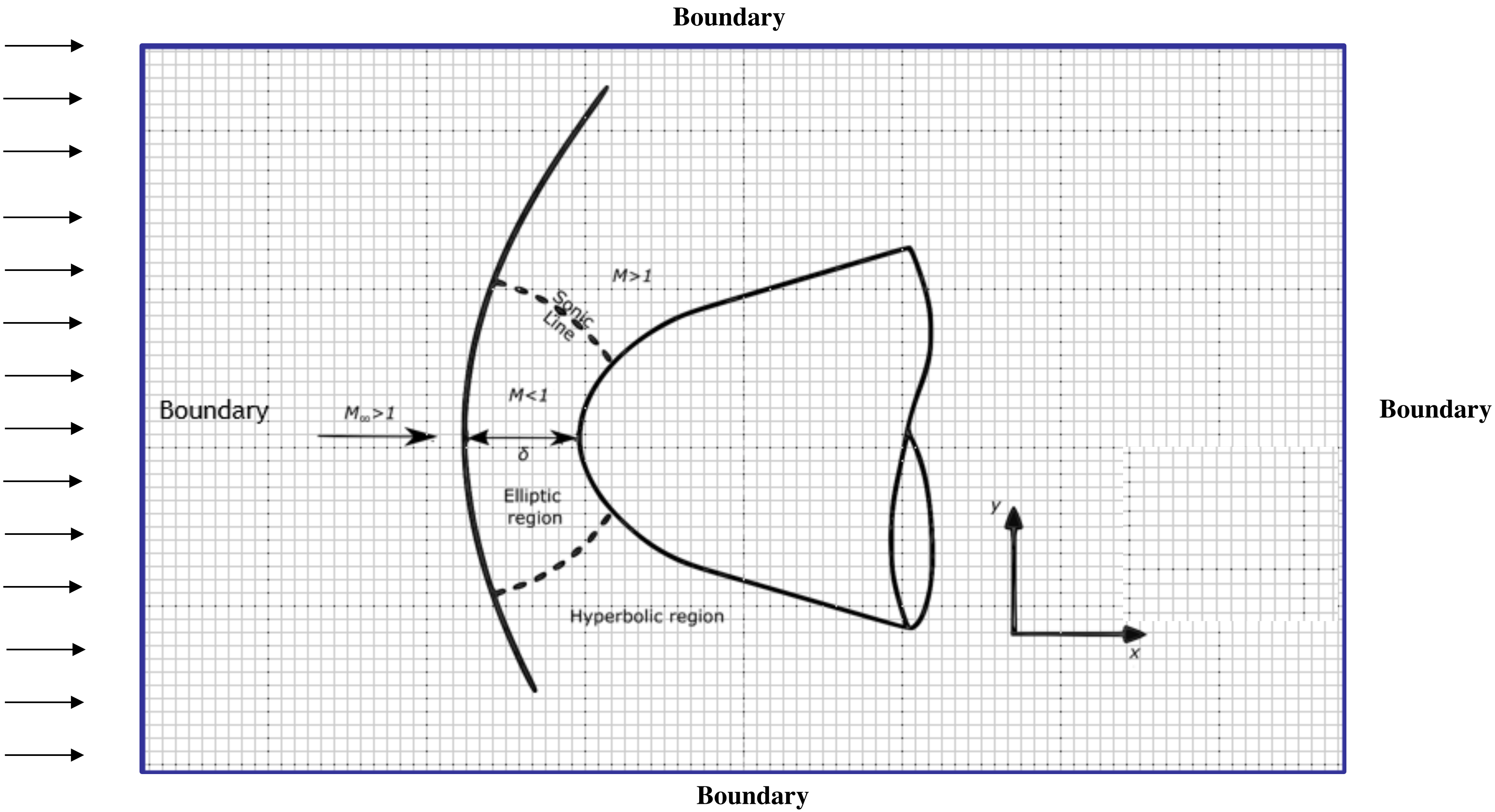


Fig.2. Computational representation of a supersonic blunt nosed body [1]

Flow Physics Extraction Functions

1. Gradient of density (normal)
2. Normal Mach number
3. Magnitude of the gradient of entropy
4. Q Criterion

Method based on flow property gradient

Pagendarm et al. 1993 proposed a shock detection method based on the gradient of density in the direction of velocity.

$$\frac{d\rho}{dn} = \nabla\rho \cdot \frac{V}{|V|} \qquad (4)$$

Positive values correspond to shock waves, while negative values correspond to expansion waves.

Flow Physics Extraction Functions

<u>**Method based on normal Mach number**</u>

Lovely et al. 1999 proposed a shock detection method based on the local pressure gradient.

$$Ma_n = \frac{Ma \cdot \nabla p}{|\nabla p|} = 1 \qquad (5)$$

This parameter captures shock waves only.

Flow Physics Extraction Functions

Ziniu et al. 2013, Lovely et al. 1999 and Ma et al. 1996 concluded that both parameters may produce false or incomplete results due to numerical errors.

Gradient of density:

$$d\rho/dn > \epsilon \quad (6)$$

Normal Mach:

$$\begin{cases} \dfrac{|\nabla p \cdot \eta|}{|\nabla p|} \leq c \\ |\nabla p| \geq |\nabla p|_{\eta} = \eta |\nabla p|_{max} \end{cases} \quad (7)$$

Flow Physics Extraction Functions

Method relating thermodynamics properties and fluid kinematics

Crocco 1937 found that an irrotational flow is isentropic and homenergic.

$$V \times (\nabla \times V) = \nabla h_0 - T\nabla s \qquad (8)$$

$$\left|\nabla S\right| = \sqrt{\left(\frac{u_j\left(\nabla \times \overline{V}\right)}{T} - \frac{\partial h_0}{T\partial x}\right)^2 \hat{i} + \left(-\frac{u_i\left(\nabla \times \overline{V}\right)}{T} - \frac{\partial h_0}{T\partial y}\right)^2 \hat{j}} \qquad (9)$$

Flow Physics Extraction Functions

<u>**Method based on fluid kinematics**</u>

Hunt *et al.* 1988 found that identifying regions in a flow can provide important method for analysis the dynamics of the flow.

$$Q = \frac{1}{2}\left(||\Omega||^2 - ||S||^2\right) \qquad (10)$$

$$||S|| = [tr(SS^t)]^{1/2}$$

$$||\Omega|| = [tr(\Omega\Omega^t)]^{1/2}$$

Hypersonic flow over a flat plate

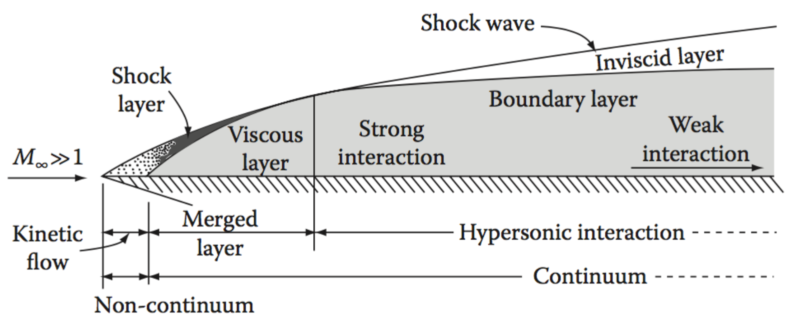


Figure 3. Illustration of the flat plate problem[2]

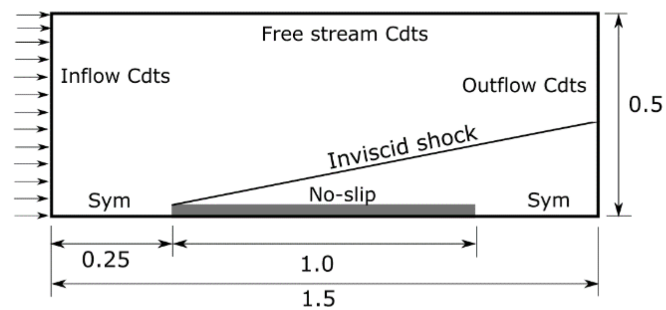


Figure 4. Computational representation

Challenges

1. Transition from laminar and turbulence
2. Viscous – Inviscid interaction
3. From Kinetic theory to continuum

2.- Pletcher, R. H., Tannehill, J. C., & Anderson, D. (2012). *Computational fluid mechanics and heat transfer*: CRC Press.

CFD Outcome

Hypersonic flow over a flat plate

Property (Freestream)	Value
Mach	8.6
Gamma and Prandtl	1.4 ; 0.70
Density	0.022497 (kg/m^3)
Temperature	360 (K)
Viscosity	2.117x10^{-5} (k/ms)
Re_L	3.47577x10^6
Length	1.0 (m)
Height	0.5 (m)

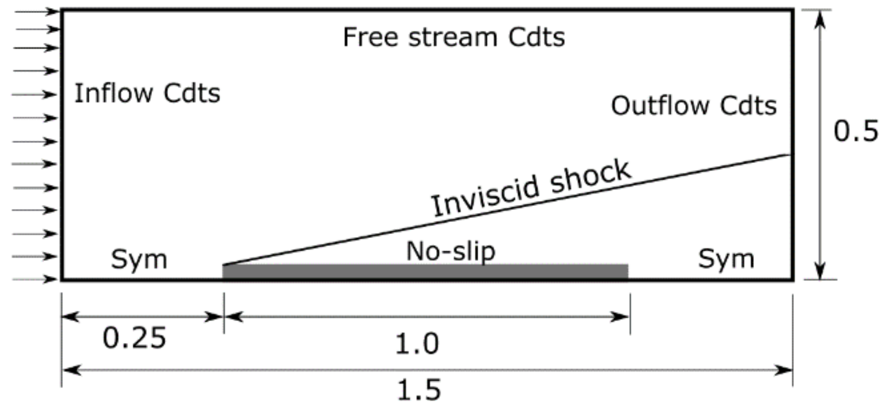
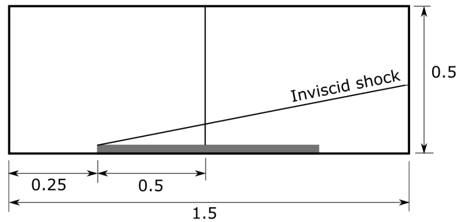


Figure 4. Computational representation

CFD Outcome

Hypersonic flow over a flat plate



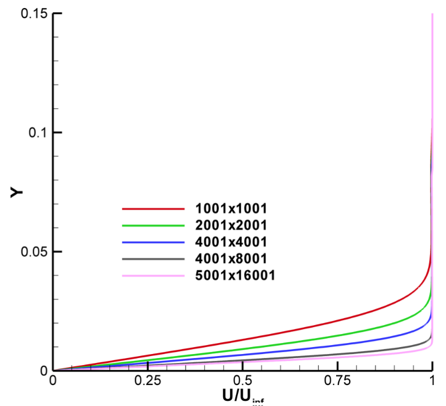


Figure 5. U-Velocity distribution at 0.5*L

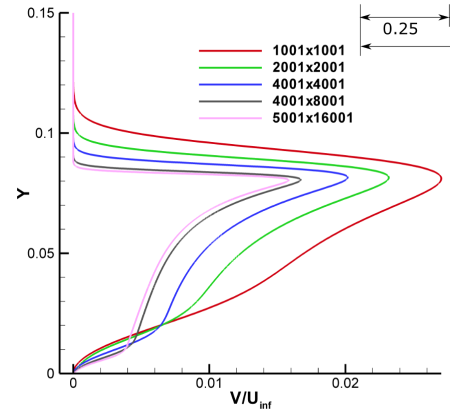


Figure 6. V-Velocity distribution at 0.5*L

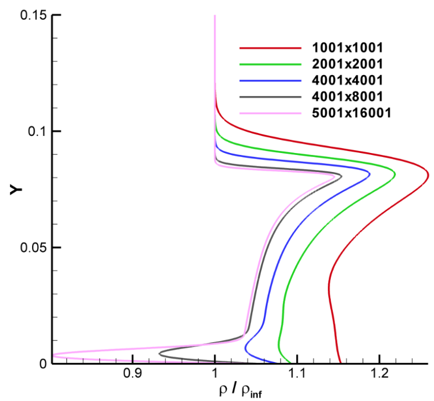


Figure 7. Density distribution at 0.5*L

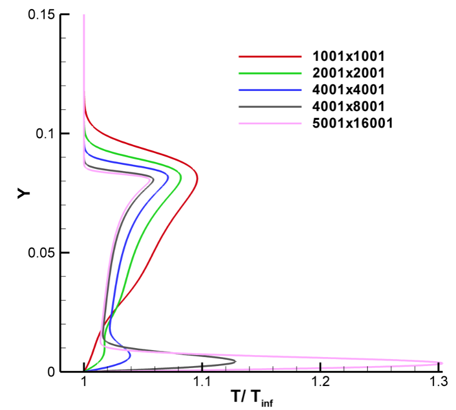


Figure 8. Temperature distribution at 0.5*L

Hypersonic flow over a flat plate

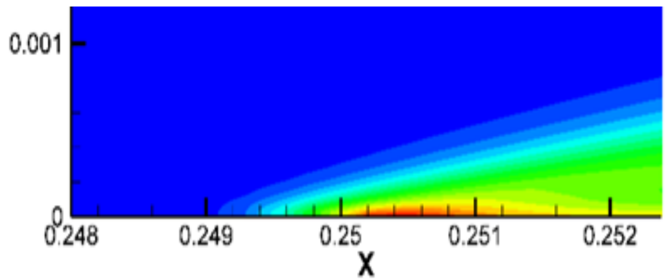


Figure 9. Density Contour

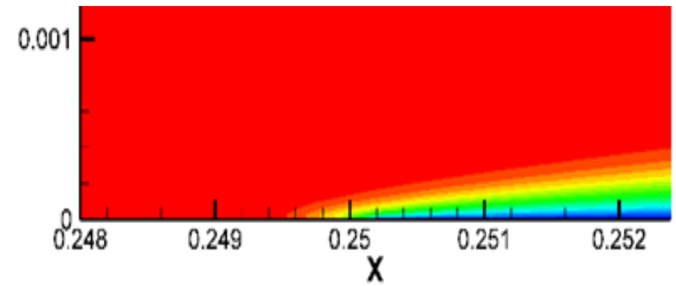


Figure 10. “U” Velocity Contour

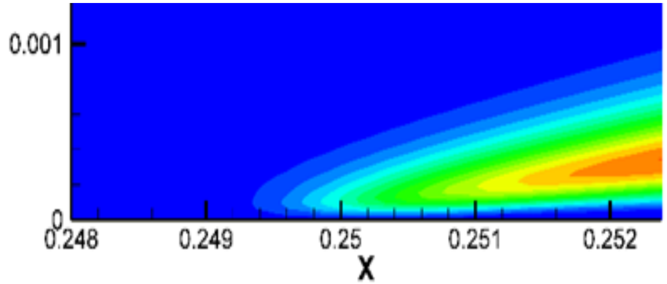


Figure 11. “V” Velocity contour

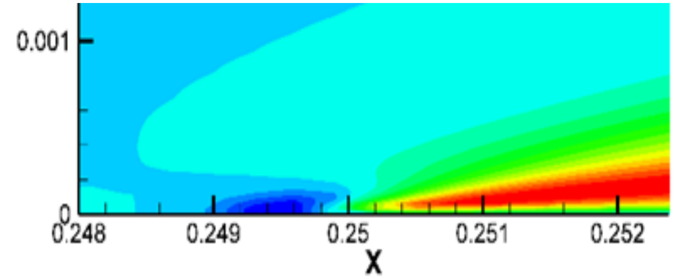


Figure 12. Temperature distribution at 0.5*L

CFD Outcome

Hypersonic flow over a flat plate

$$Ma_n = \frac{Ma \cdot \nabla p}{|\nabla p|} = 1$$

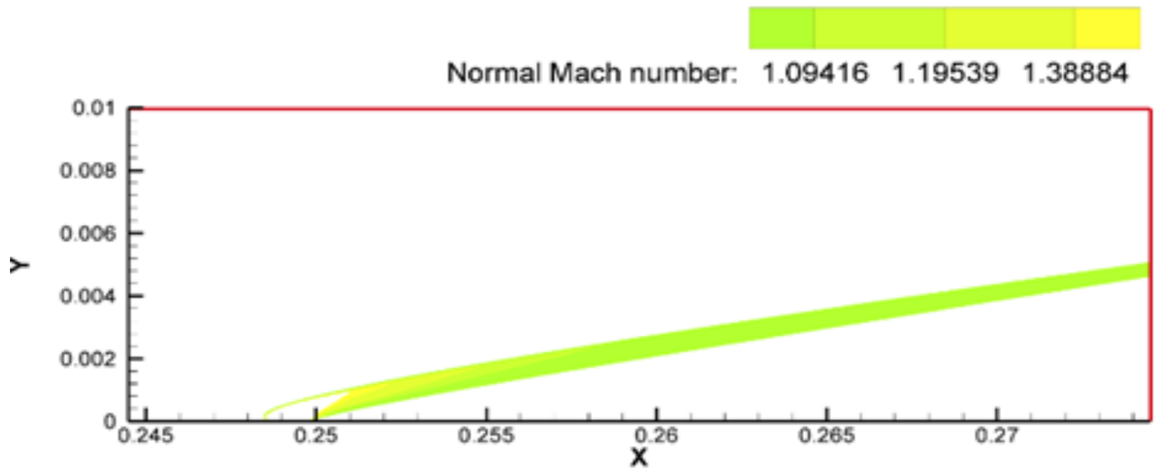


Figure 13. Normal Mach number contour

$$Q = \frac{1}{2}\left(||\Omega||^2 - ||S||^2\right)$$

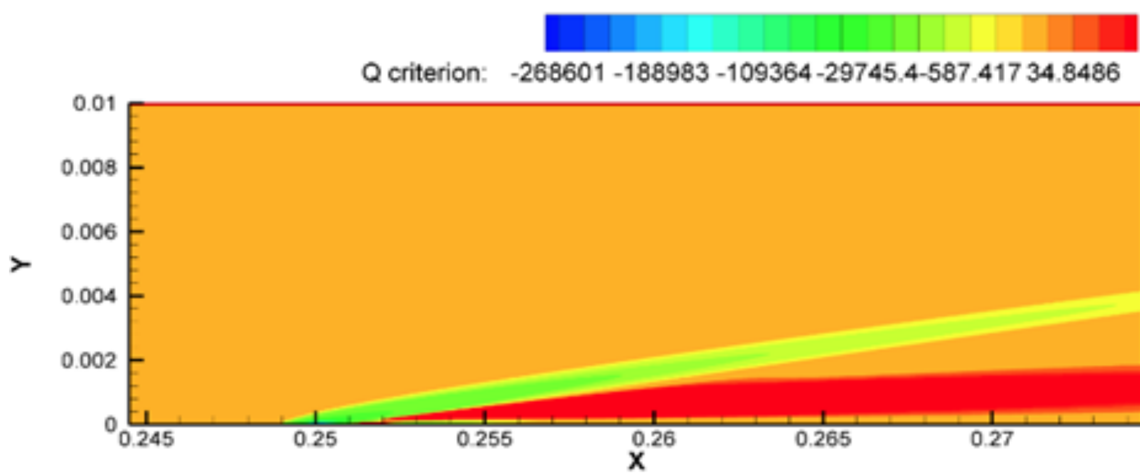
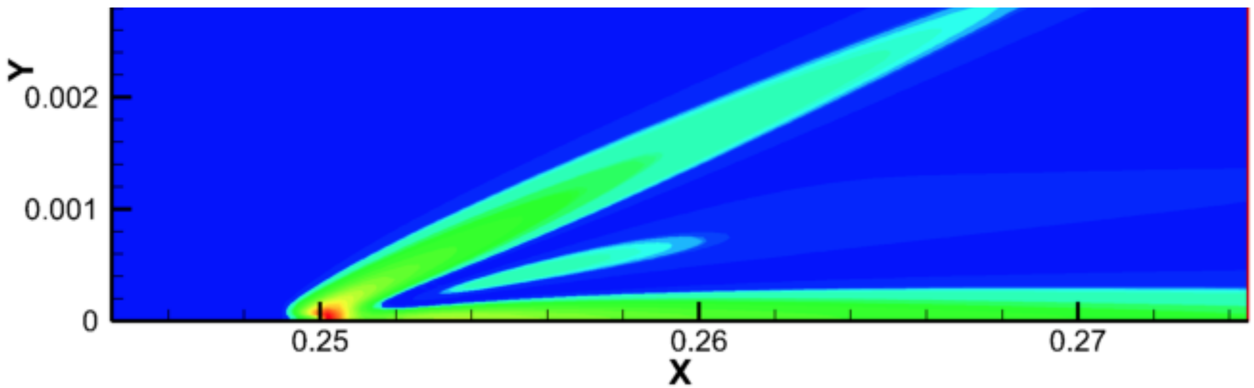


Figure 14. Q-Criterion Contour

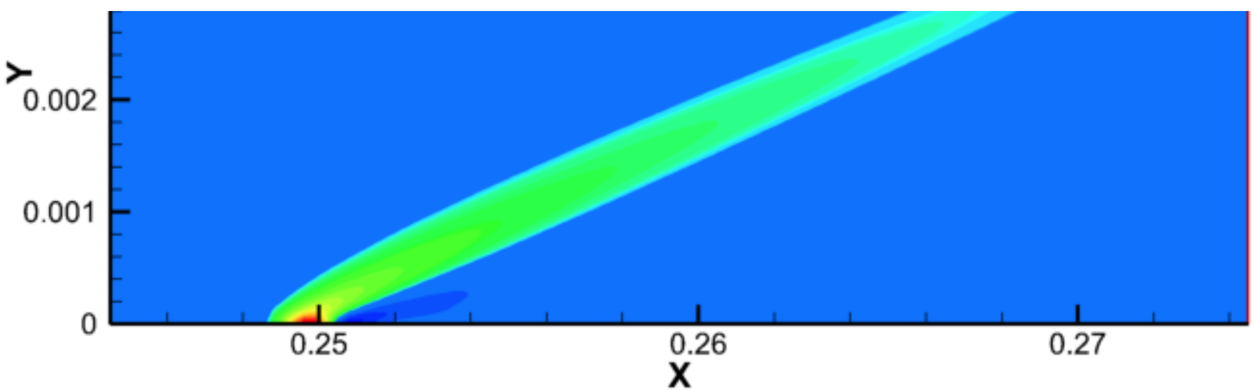
CFD Outcome

Hypersonic flow over a flat plate



$$\left|\frac{d\rho}{dn}\right| = \nabla\rho$$

Figure 15. Density Gradient magnitude

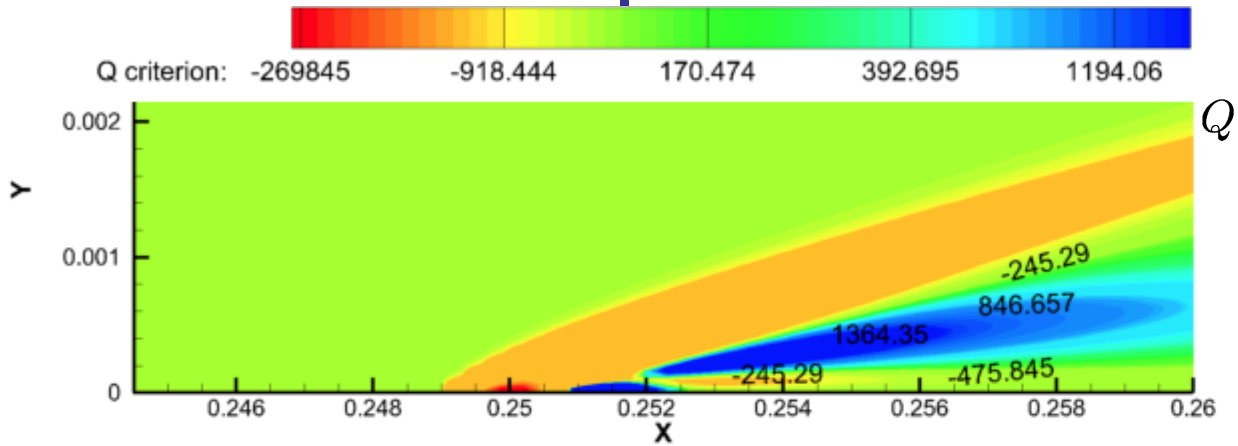


$$\frac{d\rho}{dn} = \nabla\rho \cdot \frac{V}{|V|}$$

Figure 16. Normal density gradient

CFD Outcome

Hypersonic flow over a flat plate



$$Q = \frac{1}{2}\left(||\Omega||^2 - ||S||^2\right)$$

Figure 17. Q criterion (Leading-edge tip)

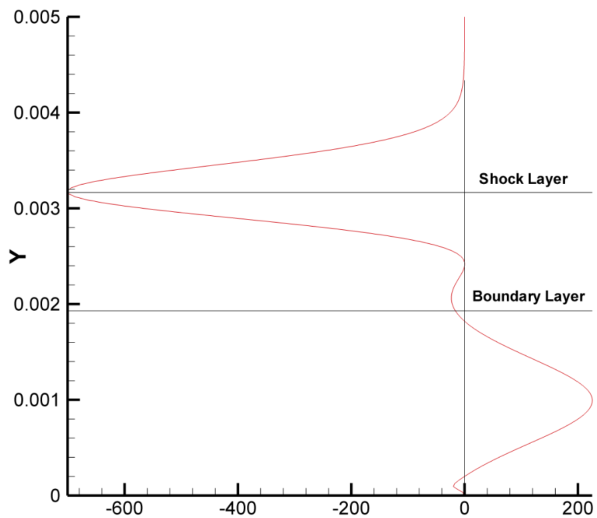


Figure 18. Q criterion (X=0.27)

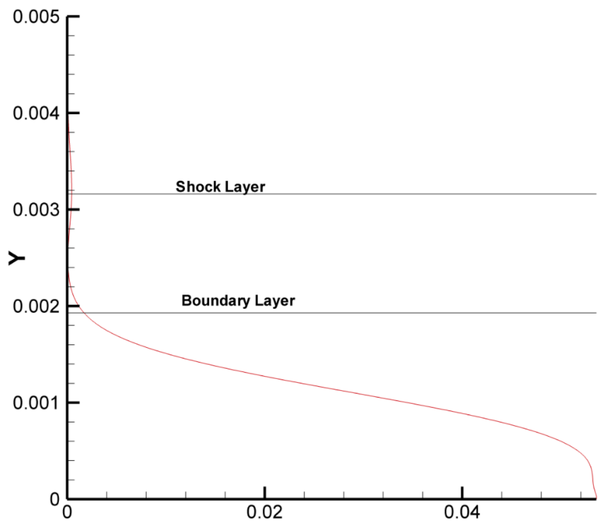


Figure 19. Vorticity (X=0.27)

Hypersonic flow cross jet interaction

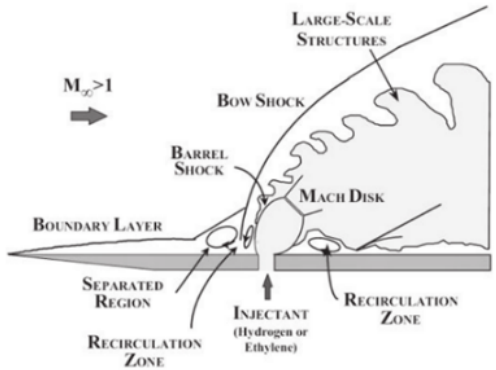


Figure 20. Illustration of the flat plate problem[3]

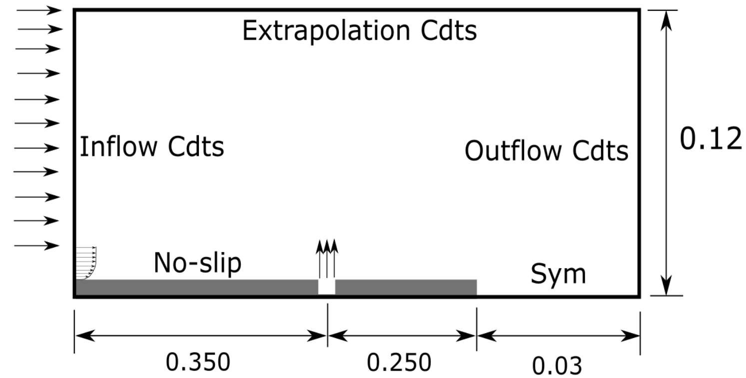


Figure 21. Computational representation

Challenges

1. Separation and reattachment of the boundary layer
2. Different vortical structures that enhance mixing
3. Complex shock structure that interacts with the flow field

3.- M. Gruber, A. Nejad, T. Chen and J. Dutton, *Journal of Propulsion and Power 11 (2), 315-323 (1995)*

Hypersonic flow cross jet interaction

Property (Freestream)	Value
Mach	6.0
Gamma and Prandtl	1.4 ; 0.789
Density	0.090 (kg/m^3)
Temperature	57.23 (K)
Viscosity	3.7655x10^{-5} (k/ms)
Re_L	1.3047x10^7
Length	0.6 (m)
Height	0.12 (m)

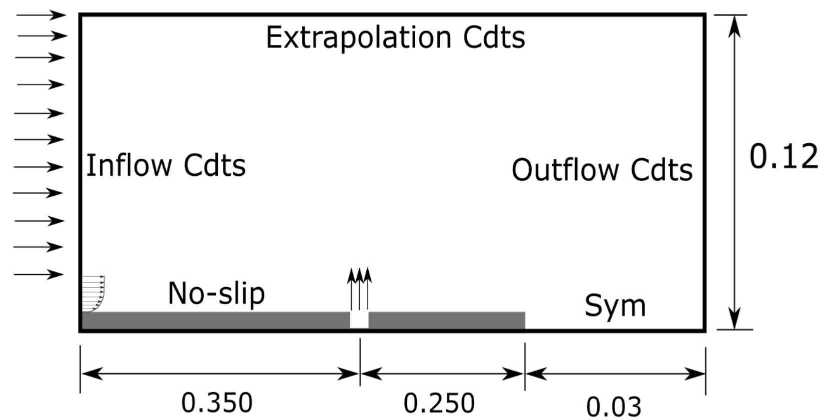
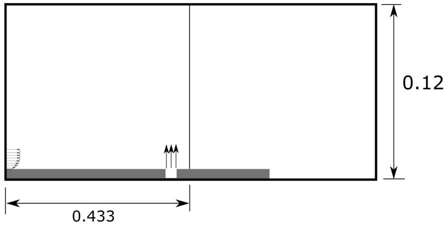


Figure 21. Computational representation

Hypersonic flow cross jet interaction



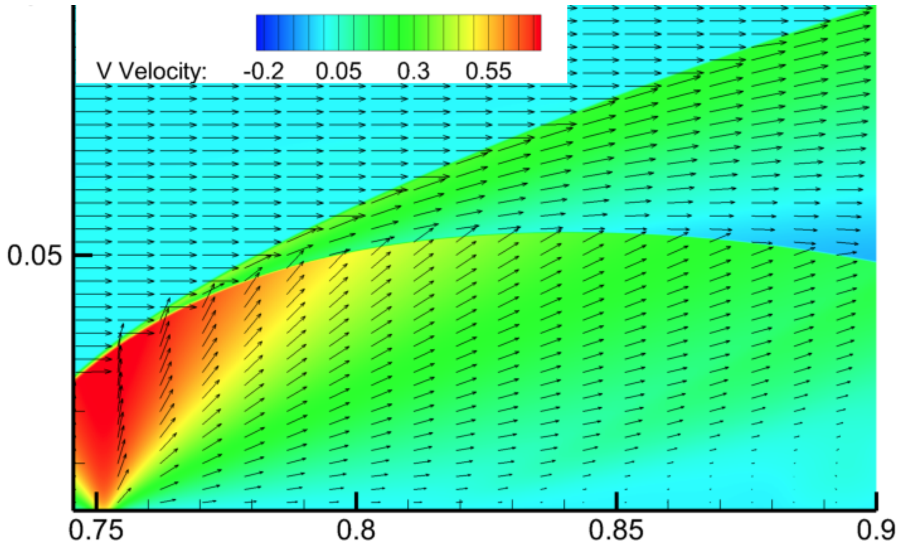


Figure 22. "V" Velocity Contour with vectors

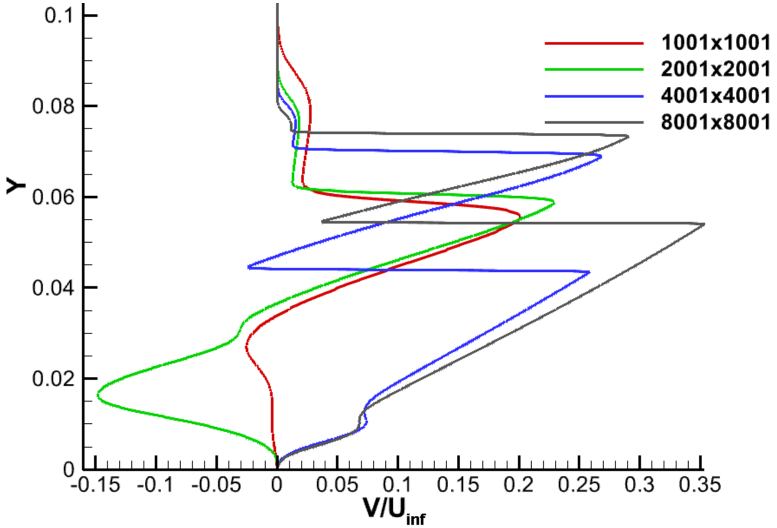
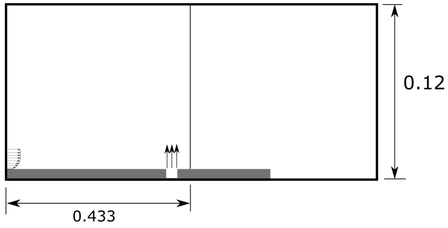


Figure 23. V-Velocity distribution at 0.433

Hypersonic flow cross jet interaction



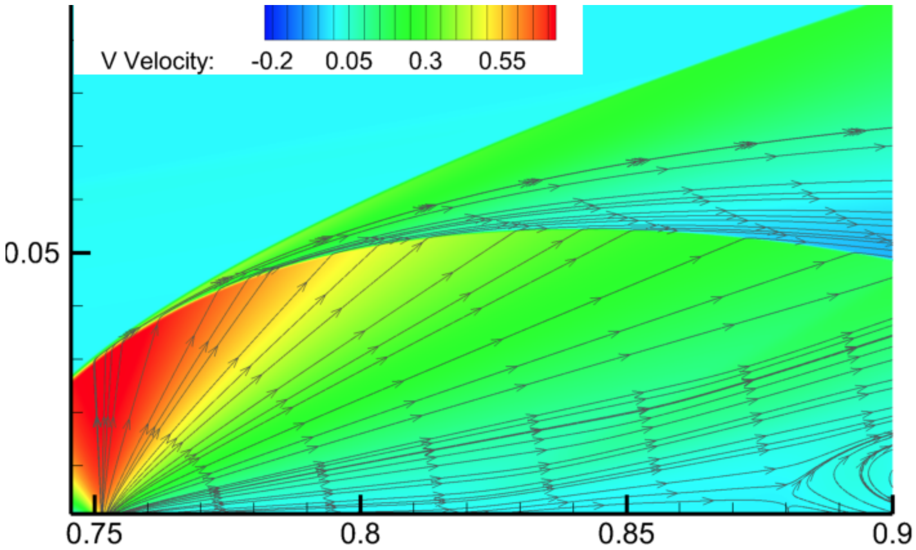


Figure 24. "V" Velocity Contour with stream tracers

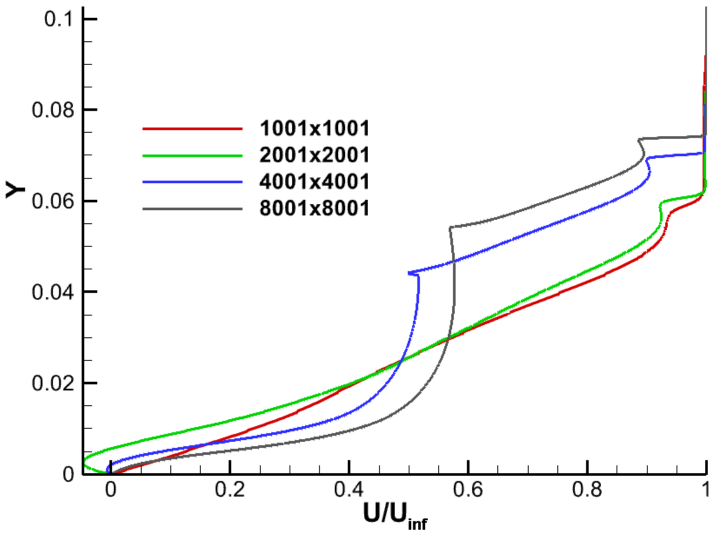


Figure 25. U-Velocity distribution at 0.433

CFD Outcome

Hypersonic flow cross jet interaction

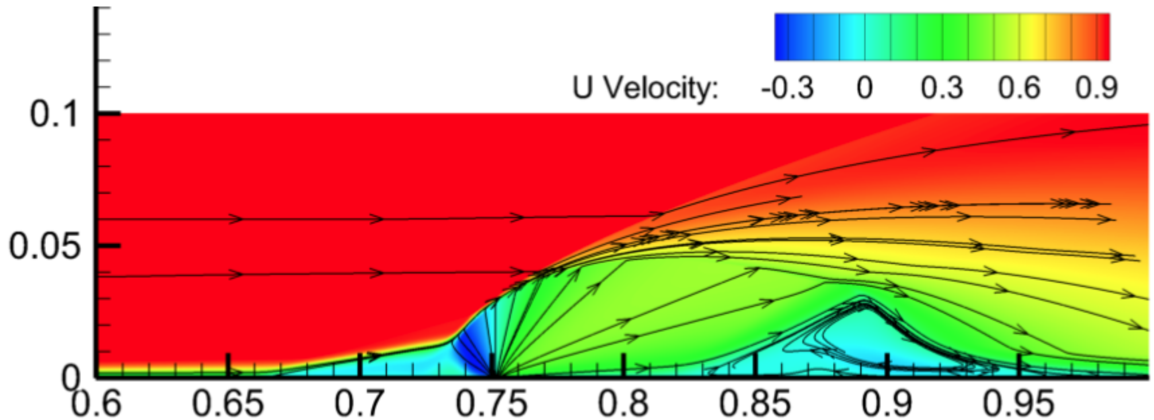


Figure 26. “U” Velocity Contour with stream tracers

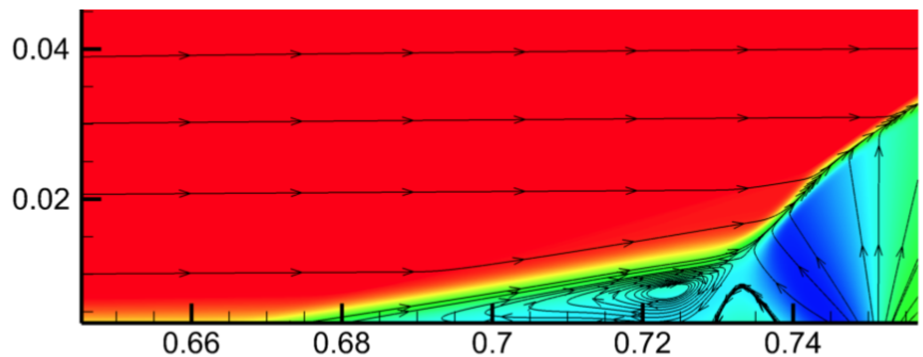


Figure 27. U-Velocity Contour ahead the injection

Hypersonic flow cross jet interaction

$$Q = \frac{1}{2}\left(||\Omega||^2 - ||S||^2\right)$$

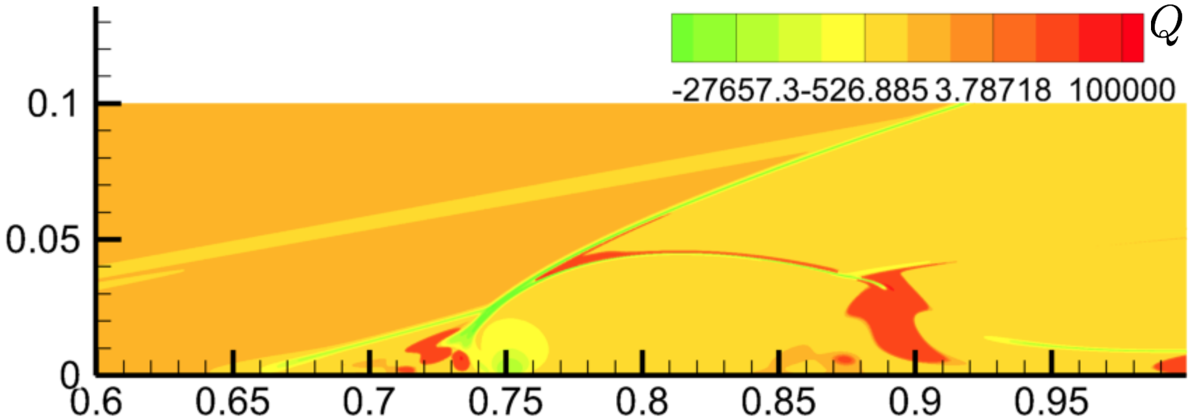


Figure 28. Q-criterion

$$Ma_n = \frac{Ma \cdot \nabla p}{|\nabla p|} = 1$$

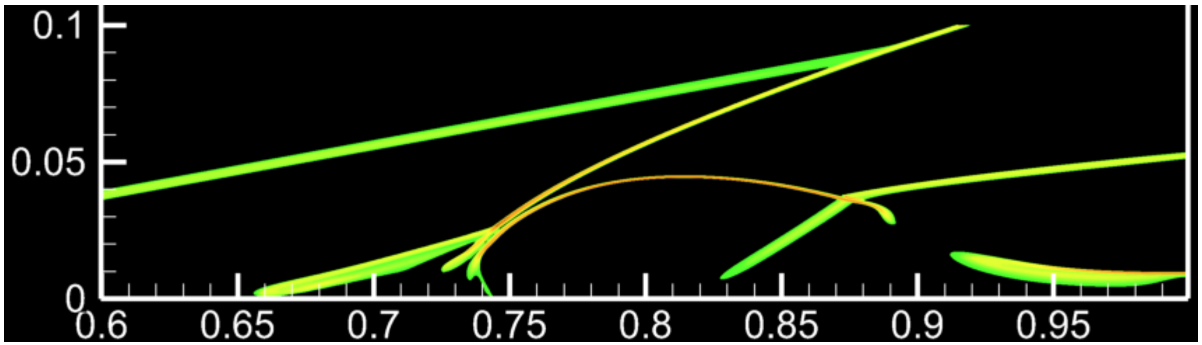


Figure 29. Normal Mach number Contour

Hypersonic flow cross jet interaction

$$\frac{d\rho}{dn} = \nabla\rho \cdot \frac{V}{|V|}$$

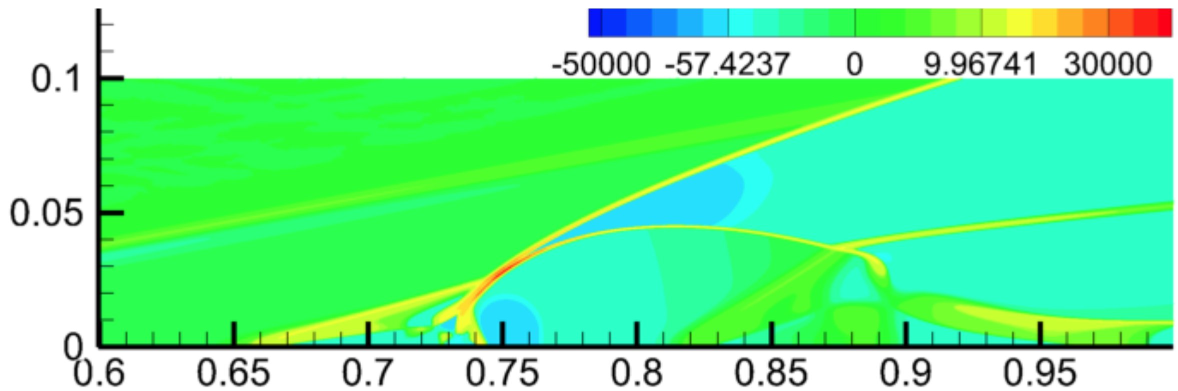


Figure 30. Q-criterion

$$|\nabla S| = \sqrt{\left(\frac{u_j\left(\nabla\times\bar{V}\right)}{T} - \frac{\partial h_0}{T\partial x}\right)^2 \hat{i} + \left(-\frac{u_i\left(\nabla\times\bar{V}\right)}{T} - \frac{\partial h_0}{T\partial y}\right)^2 \hat{j}}$$

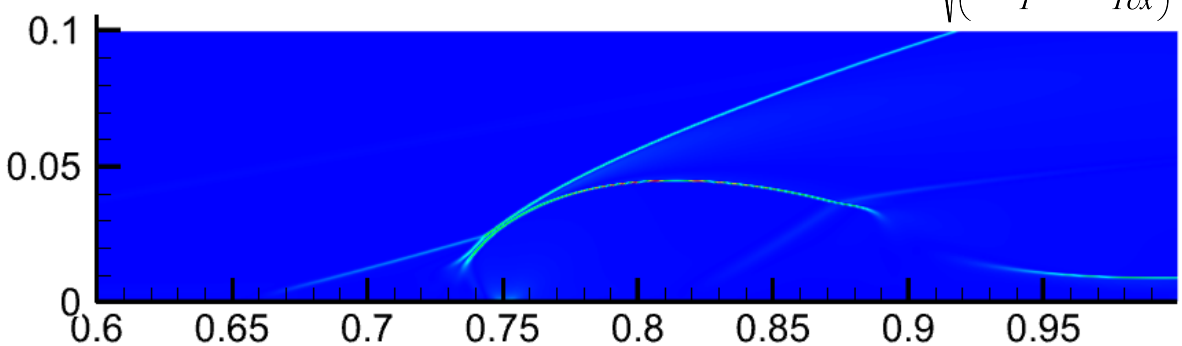


Figure 31. Magnitude of Entropy gradient

CFD Outcome

Conclusions and future works

- ✓ A new scheme for solving the 2D Navier Stokes Equations was validated using FPEF and cutting edge parallel libraries
- ✓ The IDS has the capabilities of predicting the detailed physics within complex flow fields
- ✓ In all cases the results showed very good agreement with the physical expectations of the flow interactions
- ✓ A set of FPEF functions is required evaluate a given flow field as some FPEF may require Filtering
- ✓ Future Effort: Extension of the IDS to arbitrary geometries is recommend (Under current development)
- ✓ Future Effort: Extend the Parallel version to 3D