A METHOD FOR REDUCTION OF RADIATION NETWORKS FOR SIMPLIFICATION OF THERMAL MODELING AND ANALYSIS

Michael Saeger
ATA Engineering, San Diego, California

ABSTRACT

A common consideration in thermal analyses involving heat transfer by radiation is the need to keep the number of radiative conductors from becoming too large. Thermal simulations that contain large radiative conductance networks require significant computational time, effort, and resources to perform, so there is often a desire to limit the size of radiation conductance networks. Complying with such limitations may require a significant amount of effort on the part of the thermal analyst, as the number of discretized radiating surfaces must be kept small, which in turn may necessitate reworking geometry to simplify it and remove small features. In this work, a method is presented for approximating large radiation networks consisting of many conductors with much smaller radiation networks consisting of fewer conductors. The method's mathematical basis is discussed, and some examples of its use on thermal models, including comparisons between reduced and unreduced radiation results, are presented. Some potential benefits of reducing radiation networks include faster run times; fewer computer resources, such as memory and storage space, needed to run a thermal simulation; less need for geometry simplification and abstraction before meshing, and the ability to reduce to a manageable size radiation networks that would otherwise be too large or impractical to simulate.

NOMENCLATURE, ACRONYMS, ABBREVIATIONS

A	surface area
E	blackbody emissive power
F	blackbody view factor
G	radiative conductance (multiplicative inverse of R)
J	radiosity
Q	heat flow rate
R	emissive power resistance (multiplicative inverse of G)
r	length of distance vector between surface locations
ε	emissivity
θ	angle between distance vector and surface normal

INTRODUCTION

This paper presents a method for reducing large radiation conductor networks and converting them into significantly smaller radiation conductor networks that are close approximations of the original network. Using reduced radiation networks can lead to significant savings in computer resources and time required to perform a thermal simulation involving radiation heat transfer. The method presented is not specific to any particular software used for thermal simulation or radiation view factor computation, but it is presumed that a suitable thermal capacitance and nonlinear (radiation) conductance network solver and a radiation view factor calculation/ray-tracing tool are available to the analyst.

In order to simplify a radiation network within a thermal model, the necessary radiation data must be extracted from the model, converted to a simplified network, and then inserted back into the thermal model in place of the original radiation network. This process is shown in Figure 1.

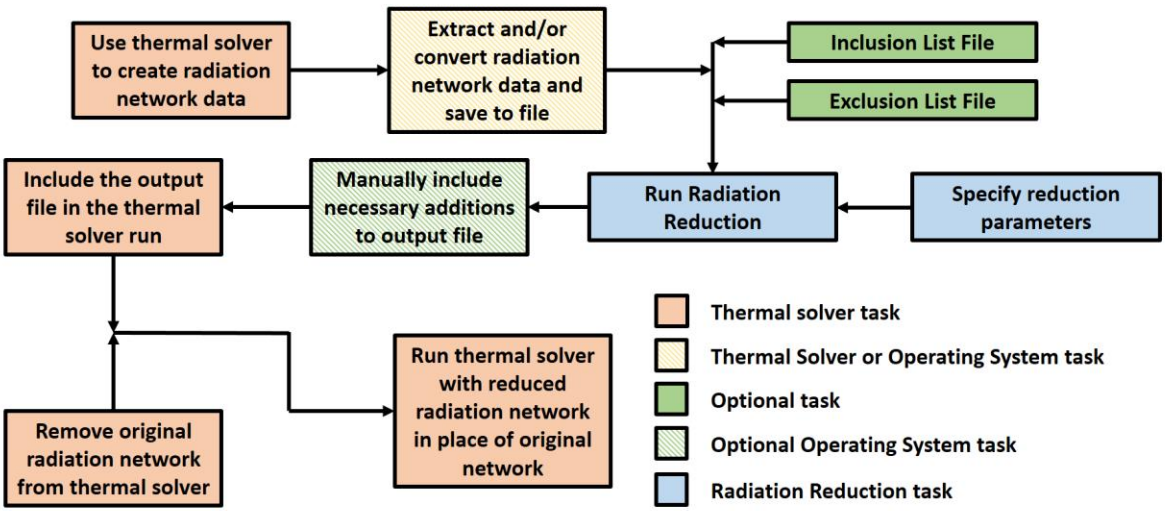


Figure 1. Thermal simulation with radiation reduction flow chart.

METHOD

Radiation networks consist of many individual conductors, each of which quantifies the radiative heat exchange between two surfaces. For an enclosure of fully diffuse greybody surfaces, the Oppenheim method may be used to compute a radiation network from blackbody view factors and surface emissivity values. Blackbody view factors can be computed from the equation

$$F_{12} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2 dA_1$$

The radiation exchange within an enclosure of fully diffuse greybody surfaces is the same as that within an enclosure of blackbody surfaces with an additional conductance added at each greybody surface to account for its emissivity being less than unity, as shown in Figure 2.

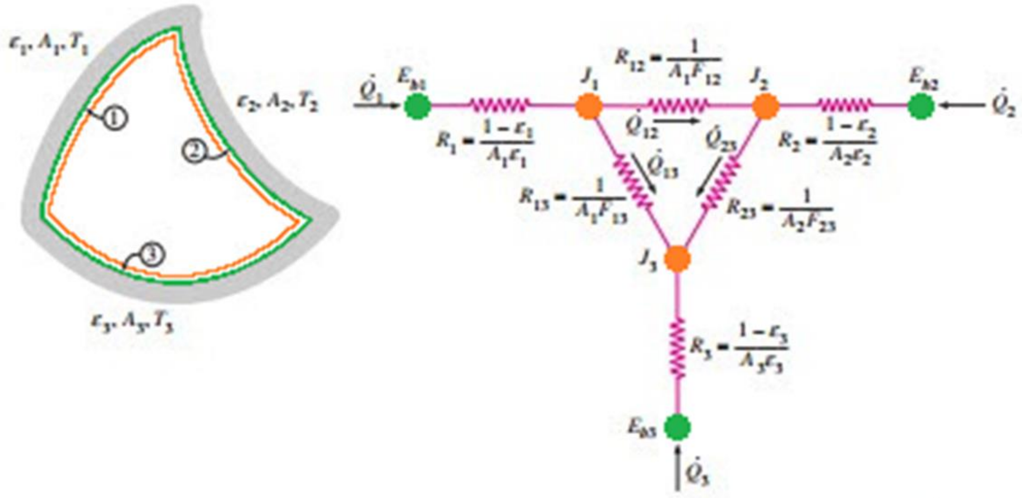


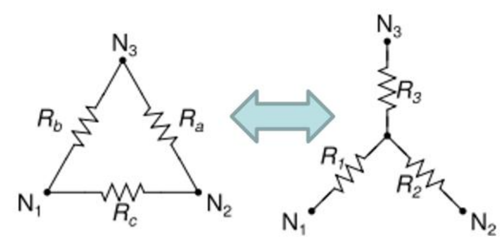
Figure 2. Oppenheim method with an Oppenheim node and conductor added to each greybody surface.

For nondiffuse (specularly reflecting or transmitting) surfaces, an alternative method such as ray tracing must be used to calculate radiative conductors; however, much of the insight that can be derived from the blackbody view factor equation still applies.

In order to model radiation inside an enclosure, a radiative conductance is needed between each surface and each other surface it sees or with which it exchanges heat, which can lead to a very large number of radiation conductors. In a typical thermal model involving radiation, the number of radiation conductors is vastly greater than the number of conductors needed to model heat transfer by conduction. A large number of radiative conductors increases the computation time as well as the memory needed to perform a thermal solution. While a large number of radiative conductors does accurately represent the physics of heat transfer in a thermal model, it is computationally expensive to model radiation in this manner. The purpose of the reduction method presented herein is to create a substitute radiation network that closely approximates a large radiation network with many fewer conductors.

The method behind reducing a radiation network is an extension of the well-known delta-wye transform, shown in Figure 3.

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$



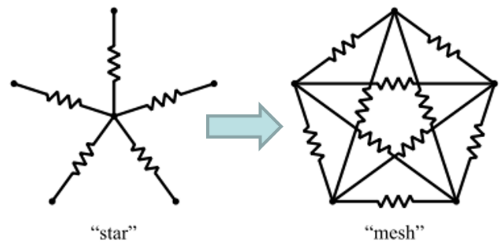
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Figure 3. Delta-wye transformation.

This transformation is a special case (N = 3) of a similar, more general transformation called a star-mesh transformation (shown in Figure 4), which involves an arbitrary number of points N.

All stars whose conductance sum is nonzero can be transformed into a mesh. This equation is always valid.

$$G_i = \frac{\sum_{n \neq i}^{N} G_{in} - \frac{G_{ik}^2}{\sum_{n \neq k}^{N} G_{kn}}}{1 - \frac{G_{ik}}{\sum_{n \neq k}^{N} G_{kn}}} \quad for \ \ k \neq i$$



$$G_{ij} = \frac{G_i G_j}{\sum_{n=1}^{N} G_n}$$

In general, meshes cannot be transformed into a star; however, for those that can, this equation is valid.

Figure 4. Star-mesh transformation and equations.

It is worth noting that the equations in Figure 4 reduce to those in Figure 3 for the case of $N = 3$. More importantly, unlike the delta-wye transform, which can be performed in either direction, the star-mesh transform, in general, can only go in one direction. Every star can be transformed into a mesh, but not every mesh can be transformed into a star. One mesh that can be transformed perfectly into a star is a mesh where every conductor has the same magnitude. As shown by the equations in Figure 4, the equivalent star will also have uniform conductors. This mesh is an important case and will be addressed later. For a typical mesh, even the most accurate possible star will be approximate.

An example problem with relevance for more general thermal analyses is two parallel plates. The radiation network for this scenario is shown in Figure 5.

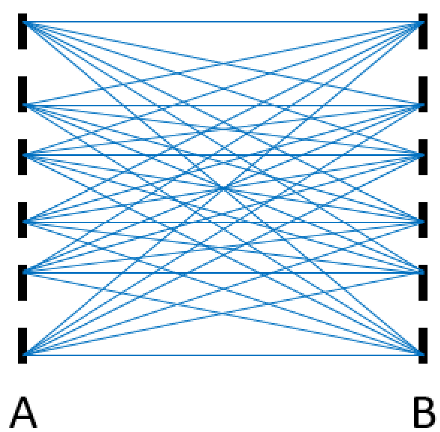


Figure 5. Conductor mesh between parallel plates.

This radiation network is similar to that shown in the mesh of Figure 4 but with one important difference: every node in Figure 4 has a conductance to every other node in the network, but every surface in Figure 5 has a view to every surface of the opposite plate but no view to any other surfaces of its own plate. If this radiation network is replaced by a star, that star will be equivalent to a mesh of conductors between plate A and plate B along with conductors between plate A and itself and conductors between plate B and itself. These conductors that connect each plate to itself will be present and cannot be ignored even if they are not desired.

While these self-viewing conductors cannot be avoided, they can be counteracted by the inclusion of two additional stars- one added to each plate. In order to counterbalance the unwanted self-viewing conductors, the two additional stars must have their sign be the opposite of the sign of values of the first star (and that of the original mesh conductors). If the additional star values are properly selected, then they will perfectly balance and cancel out the undesired self-viewing conductors. The resulting network is shown in Figure 6.

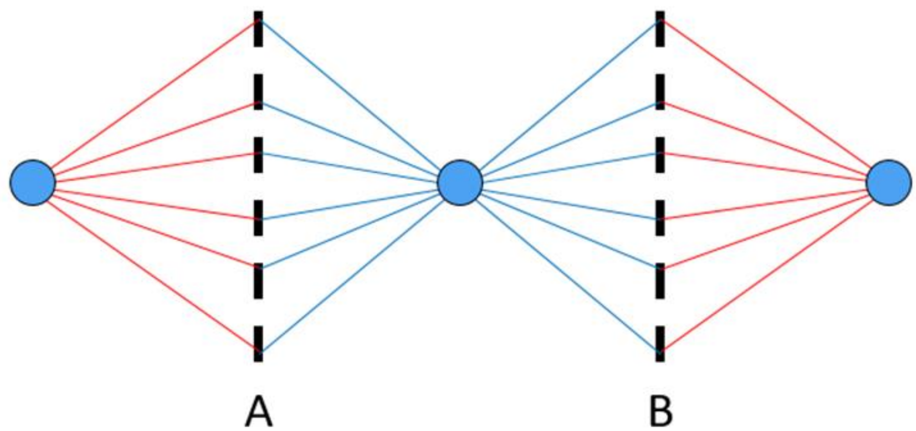


Figure 6. Star approximation, with counterbalancing, of the conductor mesh between parallel plates.

Negative-valued conductors are not physically realizable, as they would defy the second law of thermodynamics, and in the absence of other mitigating factors, a single negative-valued conductor would drive a thermal simulation to instability. For these reasons, the thermal solver will likely issue multiple warning messages about negative-valued radiation conductors; however, current versions of most thermal simulation solvers will solve the thermal model despite these concerns.

Expanding on the concept of modeling radiation between two plates with the network shown in Figure 6, the reduction of a thermal radiation network between a more general group of surfaces may be accomplished by dividing that group of surfaces into subgroups and constructing a radiation network similar to that shown in Figure 7.

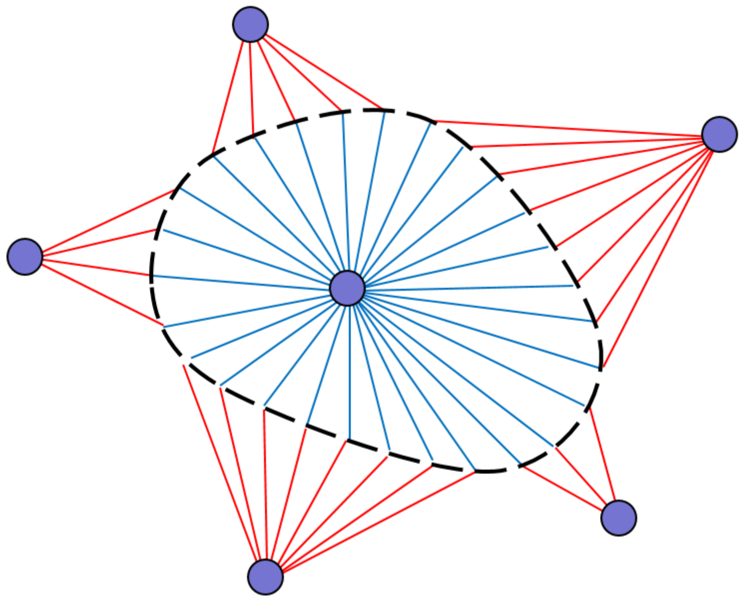


Figure 7. Conductor star approximation of the radiation network mesh for a generic enclosure.

This newly constructed network consists of a central star and a counterbalancing star for each subgroup. The conductance values of the central star and counterbalancing stars are assigned such that the corresponding mesh is an approximation of the original mesh. The choice of which surfaces to include and which to exclude is critical in determining how closely the star network will approximate the original mesh. As mentioned earlier, a uniform-conductor mesh can be perfectly represented by a star, so selecting element surfaces that have near-uniform conductance distributions with each other will be well approximated. At the same time, it is beneficial to target larger-valued conductors over smaller-valued ones, as they represent a larger fraction of the overall conductance of the network. For this reason, selecting surface elements to include in a reduction iteration is something of a balancing act between choosing surfaces with large conductors and uniform conductance distribution.

A typical radiation network (or even a subset of that network) is not well represented by a handful of stars as shown in Figure 7. This is because of the one-way nature of the star-mesh transformation. Even the best possible choice of star conductance values generally has a rather poor agreement with the original mesh of conductors. However, with multiple reduction iterations, it is possible to get a closer and closer approximation to the original conductor mesh—often with many fewer conductors.

EXAMPLE 1

The first example is a thermal model of a simple antenna reflector. The model was created in NX, and the thermal simulation was created in NX Space Systems Thermal. The model consists entirely of thin shell elements. It is shown in Figure 8.

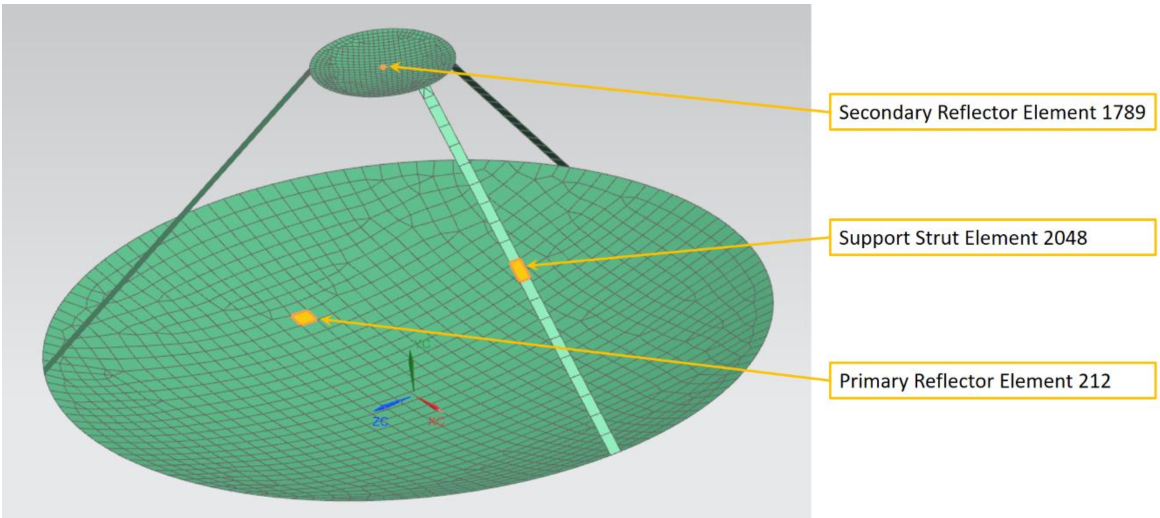


Figure 8. Example one: antenna reflector model.

The antenna is placed in a space environment and placed in Low Earth orbit with an Earth-facing orientation with large eccentricity to induce temperature fluctuations.

The original radiation network consisted of 2,093,124 radiation conductors, and the reduced radiation network consisted of 50,148 radiation conductors—a decrease in conductor count of about 97.6%. Three simulations were performed to assess the agreement between the original radiation network and the reduced radiation network: a simulation using the original radiation network, a simulation using the reduced radiation network, and a simulation in which only the radiative conductors to space are included (i.e., all radiation exchange from the antenna to itself is excluded). The purpose of the third simulation was to determine the error that would occur if the internal radiation exchange were to be ignored entirely. The discrepancy between the first (original radiation) and third (no internal radiation) simulations provides a reference point against which the discrepancy between the first (original radiation) and second (reduced radiation) simulations may be compared.

Results for three sample elements are compared in Figure 9 through Figure 11.

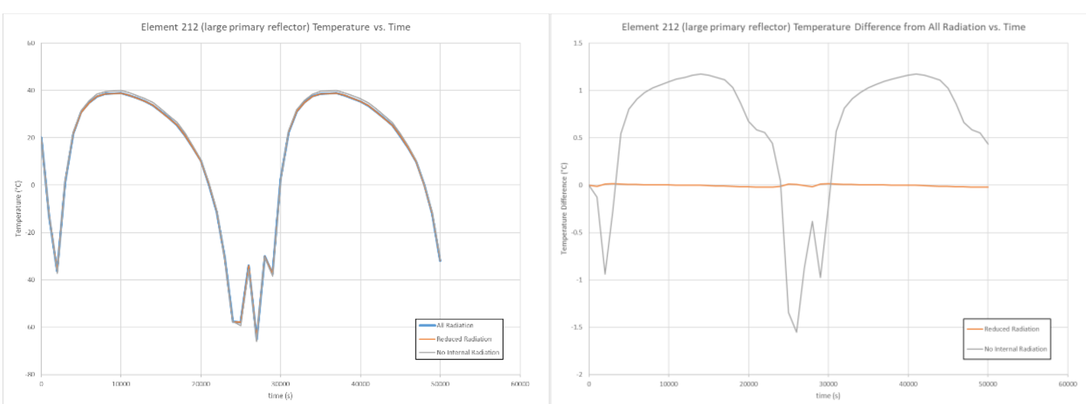


Figure 9. Temperature comparisons on the primary reflector element 212.

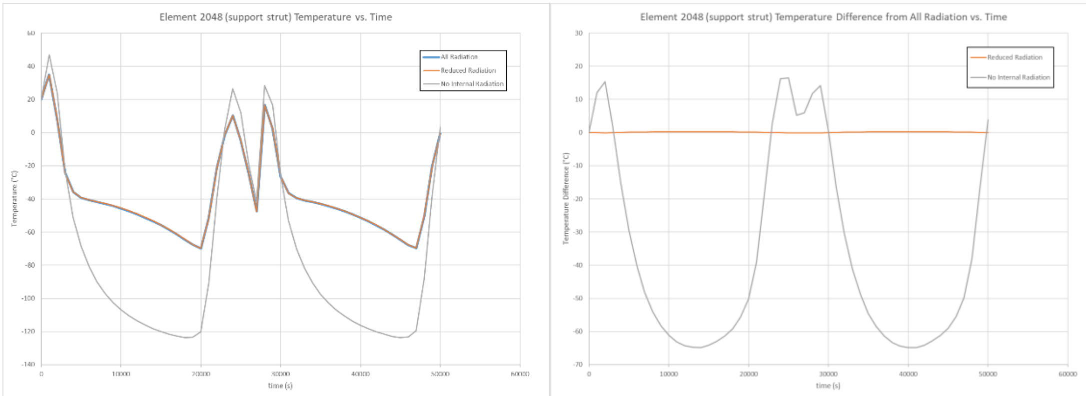


Figure 10. Temperature comparisons on the support strut element 2048.

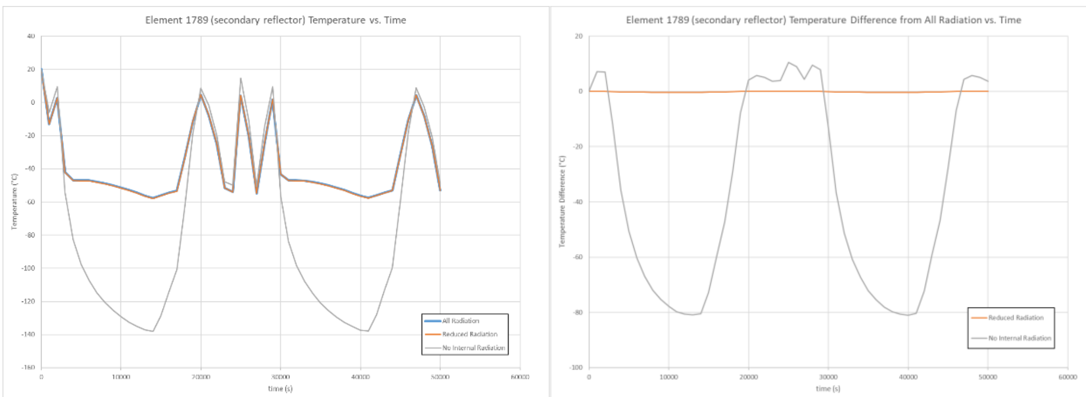


Figure 11. Temperature comparisons on the secondary reflector element 1789.

Figure 9 through Figure 11 show results from original and reduced radiation networks with and relative to the run with no internal radiation heat exchange. These results show that the reduced radiation network, despite having only about 2.4% as many conductors, is indeed a good approximation of the original network evidenced by how much better it agrees with the original results than the results with no internal radiation.

The maximum temperature difference between reduced and original thermal simulation predictions for each element is illustrated in Figure 12.

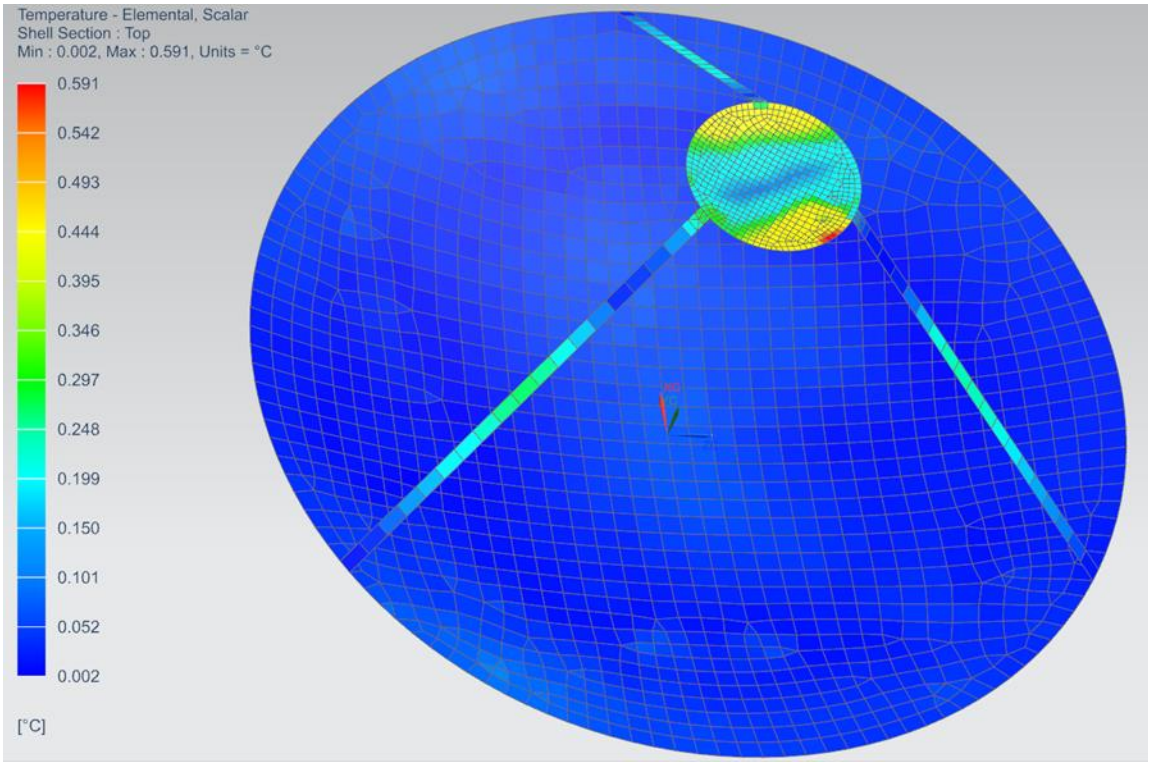


Figure 12. Maximum difference, at any time, between reduced and original models for each element.

Figure 12 shows that the maximum difference between reduced and original model is less than 0.6 °C and occurs at the edge of the secondary reflector. Overall, the maximum difference between reduced and original model has an average across all elements of about 0.113 °C and a standard deviation across all elements of about 0.136 °C.

EXAMPLE 2

The second example is a Thermal Desktop model of a simple telescope enclosure. This model consists of 4,096 surface elements. The telescope is placed inside a space enclosure with no orbital heat loads. A heater patch on the back of the telescope provides a steady 1000 W of continuous heat that keeps the telescope enclosure warm inside the cold space enclosure. Heat exchange by radiation is modeled inside and outside the telescope, and conduction exists within the enclosure walls. The initial temperature of the entire structure is 300 K. This model is shown in Figure 13.

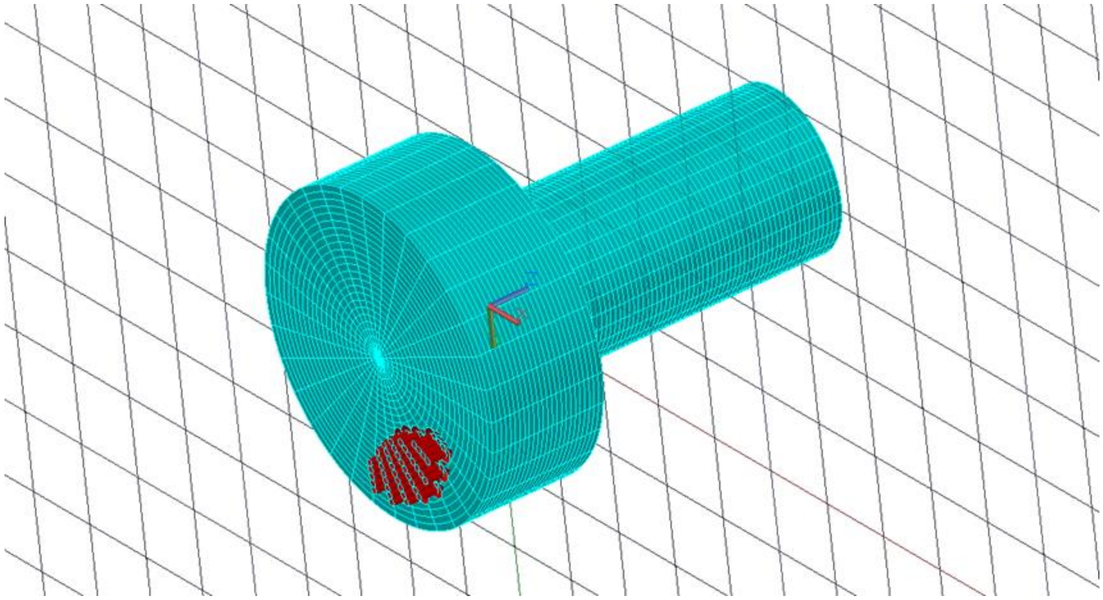


Figure 13. Example two: Thermal Desktop telescope enclosure model.

The telescope model was reduced similarly to the antenna reflector model. The original telescope radiation network contained 165,950 conductors, and the reduced radiation network contained 37,842 conductors.

Results from the original telescope radiation network are shown in Figure 14.

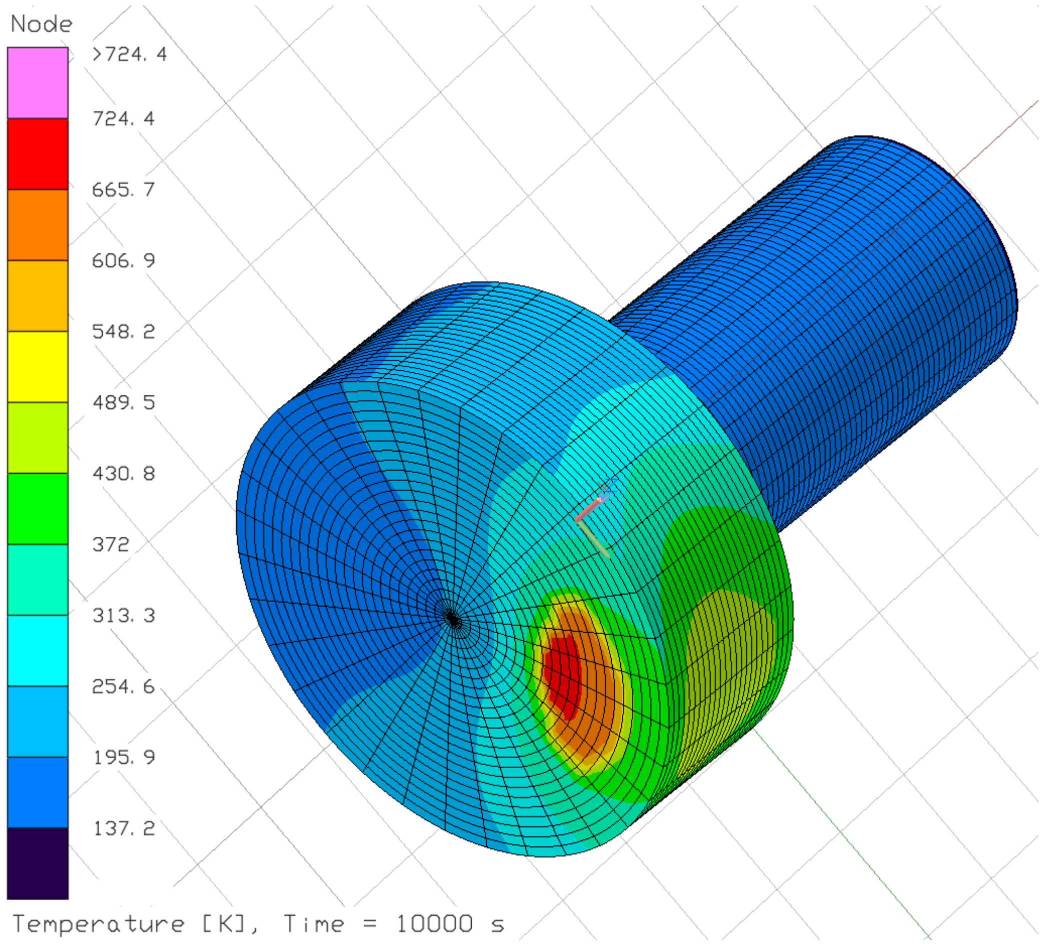


Figure 14. Telescope enclosure model results for the original radiation network.

The difference in temperature between the original radiation simulation and the reduced radiation simulation is shown in Figure 15.

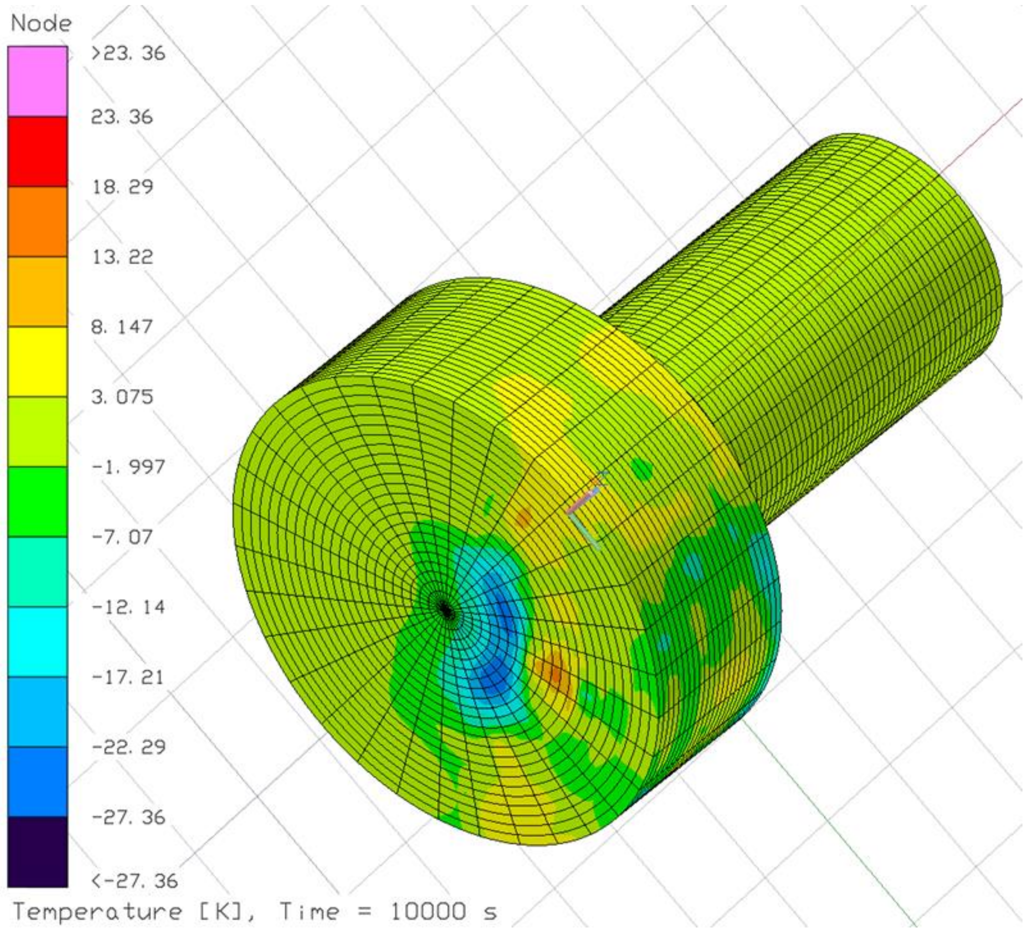


Figure 15. Telescope enclosure model results difference between original and reduced radiation.

Figure 15 shows that some temperatures are overestimated by the reduced radiation network while others are underestimated, but these discrepancies tend to be rather localized. The average temperature difference over the entire model is −0.56 °C, and the standard deviation of the difference is 3.94 °C. To look at the distribution of differences in a more detailed and quantitative manner, a scatter plot of all temperature differences on the base of the telescope enclosure is shown in Figure 16.

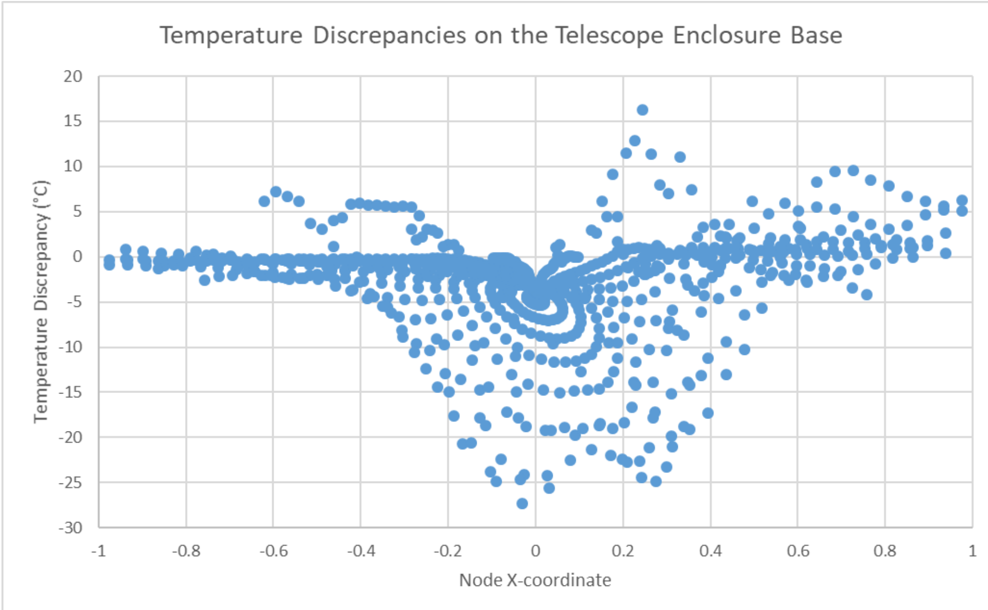


Figure 16. Temperature discrepancies between model results for full and reduced radiation on the telescope base.

Figure 16 shows that there are only a few locations with substantial discrepancies, and almost all of the other locations have small discrepancies.

CONCLUSIONS

A method for approximating large radiation conductor networks with significantly smaller radiation conductor networks has been presented. Use of this method can significantly reduce computational effort and time needed to simulate thermal models. Other potential benefits include facilitating the creation of simplified system-level component models from detailed component-level models and less time and effort reworking and meshing geometry for thermal analysis. Two examples were given in which large radiation networks were reduced significantly in size, and reduced models showed good agreement with the original models from which they were derived.