BLAST: Boundary Layer Analysis & Simulation Tool for Reacting Nonequilibrium Flows

Presented by: Domenico Lanza

THERMAL & FLUIDS
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Introduction

Code Purpose:

- External solution (on boundary layer edge)
  - Enthalpy
  - Mass fractions
  - Velocity
- Wall boundary conditions
  - Enthalpy
  - Species
- Initial guess

→

Solve the whole boundary layer:
- Velocity profile
- Enthalpy profile
- Mass fractions
- Heat flux
- Skin friction coefficient

Main Assumptions:

- Steady flow
- Local Thermal Equilibrium
- Local Chemical non-Equilibrium (CNE)
- Laminar flow
- Thin boundary layer
- Axisymmetric or 2D flow
- External body forces neglected

Literature Review

The numerical solution of compressible, reactive boundary layers stands on a wide body of literature:

- **Early works:** Lees (1958), Libby (1960), Blottner (1964, 1970, 1975), and others.
- **More recent works**: DEKAF, and analysis of blowing effects (Miró Miró).

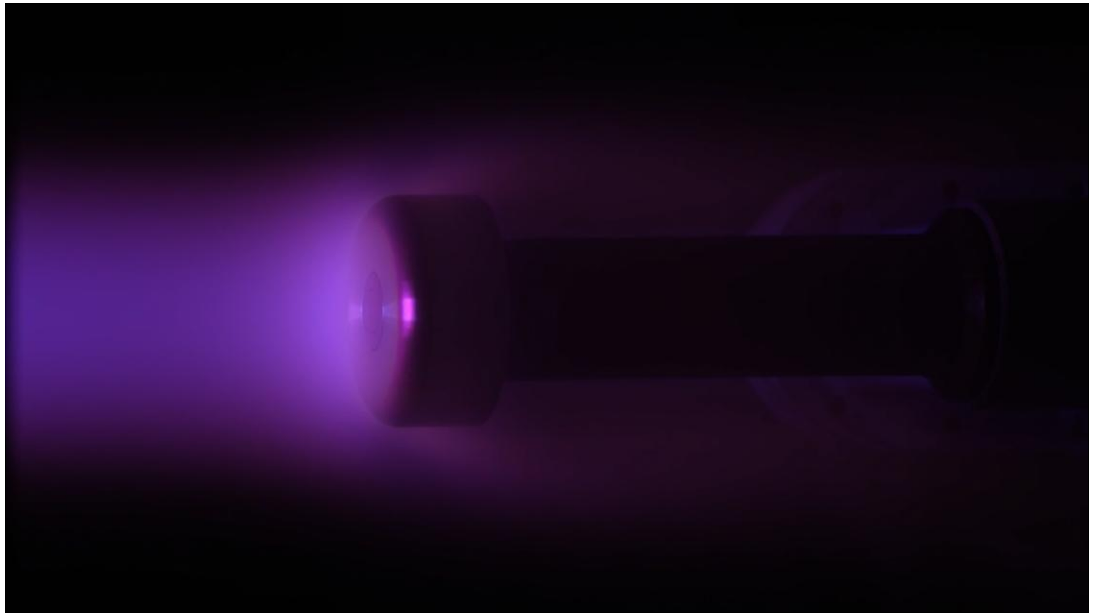
The method adopted in this work, implement in **BLAST**, was first developed by **Peters N.** and later adopted by **Barbante P.** at the Von Karman Institute for Fluid Dynamics.

Aim of this work: integrate this established approach with the modern reactive flow models offered by **Mutation++** and extend the formulation to analyze ablation phenomena.
Final goal: provide an open-source tool for supporting the experiments performed inside **high-enthalpy ground test facilities**.

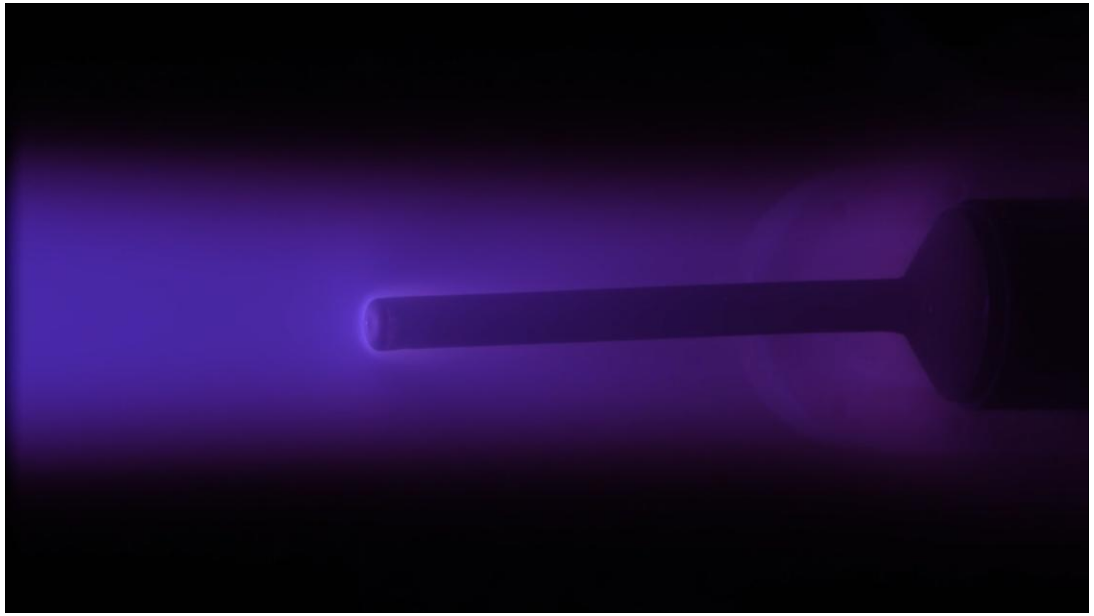
This work is an evolution of **PlasFlowSolver**, a data reduction tool built for the **Plasmatron X** wind tunnel under thermochemical equilibrium assumption.

- Peters, N., "Solution of the boundary layer equations for chemically reacting gases using a multi-point method", German Aerospace ; Research Report ; 72.58, Dept. of Scientific Reporting of the German Research and TestingInstitute for Aeronautics and Astronautics, DFVLR, Porz-Wahn, 1972.
- Barbante, P. F., Degrez, G., and Sarma, G. S. R., "Computation of Nonequilibrium High-Temperature Axisymmetric Boundary-Layer Flows," Journal of Thermophysics and Heat Transfer, Vol. 16, No. 4, 2002, pp. 490–497
- D. Lanza, M. Franco, G. Elliott, M. Panesi, and F. Panerai, "PlasFlowSolver: An Aerothermodynamic Data Reduction Model for Inductively Coupled Plasma Wind Tunnel Facilities," AIAA Paper 2025-0449, AIAA SciTech Forum, Orlando, FL, Jan. 2025.

Motivations



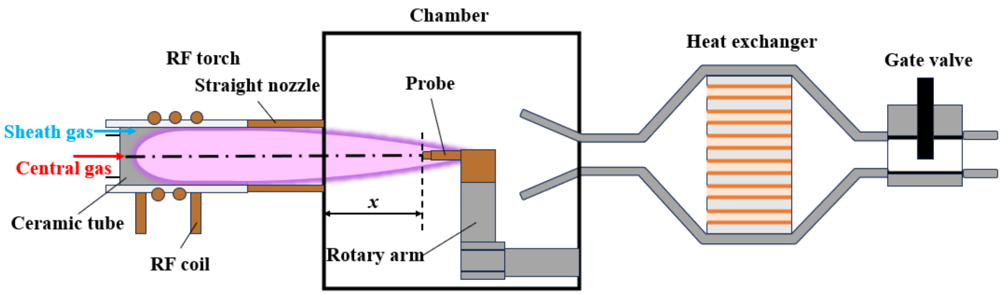
Slug calorimeter



Pitot probe

Why?

- ➢ **Large amount of data → need for a fast numerical tool**
- ➢ **Guiding the experiments → requires parametric analysis**



Plasmatron X ICP wind tunnel at the University of Illinois at Urbana-Champaign

Franco, M., Capponi, L., Oruganti, S., Elliott, G., and Panerai, F., "Investigation of Slug Calorimeter Heat Flux Measurements in the Plasmatron X Wind Tunnel," *AIAA SCITECH 2024 Forum*

Boundary Layer Equations

Unknowns: u, v, h, y_i

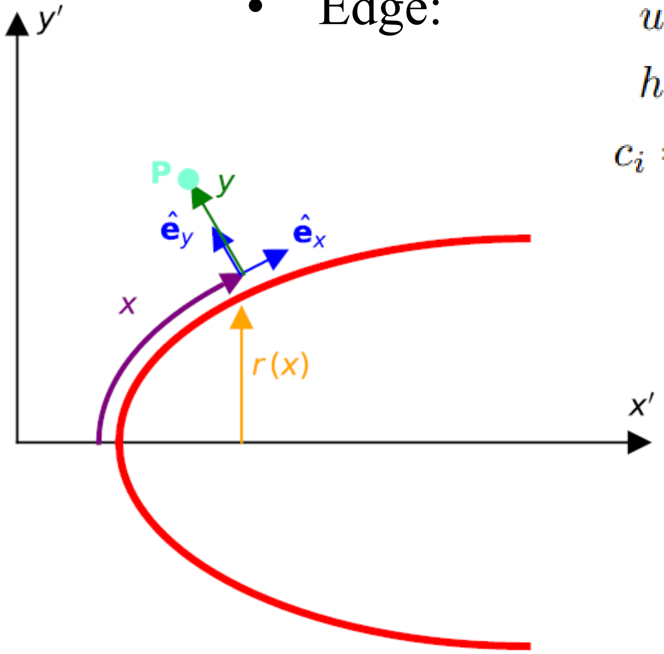
Boundary conditions:

- Wall:

$$u = 0$$
$$v = 0$$
$$h = h(T_w(x)) \quad \text{or} \quad q_w = 0$$
$$J_{i,w}^y = \dot{w}_{i,w} \quad \text{or} \quad \text{equilibrium wall}$$

- Edge:

$$u = u_e(x)$$
$$h = h_e(x)$$
$$c_i = c_{i,e}(x)$$



Continuity

$$\frac{\partial(\rho u r^\epsilon)}{\partial x} + \frac{\partial(\rho v r^\epsilon)}{\partial y} = 0$$

Species Continuity

$$\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} = -\frac{\partial J_i^y}{\partial y} + \dot{w}_i$$

x Momentum

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\mathrm{d}p_e}{\mathrm{d}x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

y Momentum

$$\frac{\partial p}{\partial y} = 0$$

Energy

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = -\frac{\partial q^y}{\partial y} + u \frac{\mathrm{d}p_e}{\mathrm{d}x} + \mu \left(\frac{\partial u}{\partial y}\right)^2$$

$$\boxed{\frac{\mathrm{d}\,p_e}{\mathrm{d}\,x} = -\rho_e u_e \frac{\mathrm{d}\,u_e}{\mathrm{d}\,x}}$$

$\varepsilon = 0$: 2D

$\varepsilon = 1$: Axisymmetric

Heat flux and Diffusion Fluxes

Heat Flux

$$\mathbf{q} = -\lambda_{\mathrm{fr}} \nabla T + \sum_{i=1}^{N_s} h_i \mathbf{J}_i + p \sum_{i=1}^{N_s} \chi_i \mathbf{U}_i$$

$$\mathbf{J}_i = \rho_i \mathbf{U}_i$$

Diffusion Fluxes: Stefan-Maxwell Equations

$$\mathbf{J}_i = -\rho_i \left(\sum_{j=1}^{N_s} D_{ij} \mathbf{d}_j + \chi_i \nabla (\log T) \right)$$

$$\mathbf{d}_i = \nabla X_i + (X_i - \frac{\rho_i}{\rho}) \nabla (\log p) - \frac{\rho_i}{\rho p} \left(\rho \, \mathbf{F}_i - \sum_{k=1}^{N_s} \rho_k \mathbf{F}_k \right)$$

$$\boxed{\frac{M}{\rho} \sum_{j=1}^{Ns} \left(\frac{x_i \vec{J}_j}{M_j \mathcal{D}_{ij}} - \frac{x_j \vec{J}_i}{M_i \mathcal{D}_{ij}} \right) = \vec{d}_i - \frac{y_i}{p} \left(\rho q_i - \sum_{k=1}^{Ns} \rho_k q_k \right) \vec{E}_{amb}}$$

Catalytic Fluxes: gamma model

$$\dot{w}_{i,cat} = \frac{2\gamma_i}{2 - \gamma_i} \rho_i \sqrt{\frac{RT}{2\pi \mathcal{M}_i}}$$

Heat flux and Diffusion Fluxes

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$$\mathbf{d}_i = \nabla X_i + \cancel{\left(X_i - \frac{\rho_i}{\rho}\right) \nabla (\log p)} -$$

Ambipolar constraint

$$\boxed{\frac{M}{\rho} \sum_{j=1}^{Ns} \left(\frac{x_i \vec{J}_j}{M_j \mathcal{D}_{ij}} - \frac{x_j \vec{J}_i}{M_i \mathcal{D}_{ij}} \right) = \vec{d}_i - \frac{y_i}{p} \left(\rho q_i - \sum_{k=1}^{Ns} \rho_k q_k \right) \vec{E}_{amb}}$$

Catalytic Fluxes: gamma model

$$\dot{w}_{i,cat} = \frac{2\gamma_i}{2-\gamma_i} \rho_i \sqrt{\frac{RT}{2\pi \mathcal{M}_i}}$$

Transformed Equations: Levy-Lees Transformation

$$f' = \frac{u}{u_e}$$

$$V = \frac{2\xi}{\frac{d\xi}{dx}}\left(f'\frac{\partial\eta}{\partial x} + \frac{\rho v r^{\epsilon}}{\sqrt{2\xi}}\right)$$

$$\xi(x) = \int_0^x (\rho\mu)_r u_e r^{2\epsilon}\, dx$$

$$\eta(x,y) = \frac{u_e r^{\epsilon}}{(2\xi)^{1/2}}\int_0^y \rho\, dy$$

$$g = \frac{h}{h_e}$$

Boundary conditions:

- Wall:

$$f'(\xi,0) = 0$$

$$V(\xi,0) = 0$$

$$g(\xi,0) = g_w(\xi) = h_w/h_e \quad \text{or} \quad \text{adiabatic}$$

$$J_{i,w}^y = \dot{w}_{i,w}$$

- Edge:

$$f' \to 1$$

$$g \to 1$$

$$c_i \to c_{i,e}$$

Continuity

$$2\xi\frac{\partial f'}{\partial \xi} + \frac{\partial V}{\partial \eta} + f' = 0$$

Species Continuity

$$2\xi f'\frac{\partial c_i}{\partial \xi} + V\frac{\partial c_i}{\partial \eta} = -\frac{\partial \mathcal{J}_i^{\eta}}{\partial \eta} + \dot{\mathcal{W}}_i$$

x Momentum

$$2\xi f'\frac{\partial f'}{\partial \xi} + V\frac{\partial f'}{\partial \eta} = \beta\left(\frac{\rho_e}{\rho} - (f')^2\right) + \frac{\partial}{\partial \eta}\left(\mathcal{C}\frac{\partial f'}{\partial \eta}\right)$$

Energy

$$2\xi f'\frac{\partial g}{\partial \xi} + V\frac{\partial g}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{\mathcal{C}}{\Pr}\frac{\partial g}{\partial \eta}\right) + \frac{u_e^2}{h_e}\left[\mathcal{C}\left(\frac{\partial f'}{\partial \eta}\right)^2 - \beta\frac{\rho_e}{\rho}f'\right] - \frac{\partial}{\partial \eta}\left(\frac{\mathcal{C}}{\Pr}\sum_{i=1}^{N_s}\frac{\partial c_i}{\partial \eta}\frac{h_i}{h_e}\right)$$

$$- \frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\mathcal{J}_i^{\eta}\frac{h_i}{h_e}\right) - \frac{p}{h_e}\frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\frac{\chi_i\mathcal{J}_i^{\eta}}{\rho_i}\right) - 2\xi f'\frac{g}{h_e}\frac{\partial h_e}{\partial \xi}$$

$$\dot{\mathcal{W}}_i = \frac{2\xi}{\rho u_e (d\xi/dx)}\dot{w}_i$$

$$\mathcal{J}_i^{\eta} = \frac{r^{\epsilon}(2\xi)^{1/2}}{d\xi/dx}J_i^y$$

$$\beta = \frac{2\xi}{u_e}\frac{du_e}{d\xi}$$

$$\mathcal{C} = \frac{\rho\mu}{(\rho\mu)_r}$$

$$\Pr = \frac{\mu c_{p,\mathrm{fr}}}{\lambda_{\mathrm{fr}}}$$

Transformed Equations: Levy-Lees Transformation

$$\boxed{f' = \frac{u}{u_e}}$$

$$\boxed{V = \frac{2\xi}{\frac{\mathrm{d}\xi}{\mathrm{d}x}}\left(f'\frac{\partial \eta}{\partial x} + \frac{\rho v r^{\epsilon}}{\sqrt{2\xi}}\right)}$$

$$\xi(x) = \int_0^x (\rho\mu)_r u_e r^{2\epsilon}\,\mathrm{d}x$$

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$$\boxed{g = \frac{h}{h_e}}$$

Continuity

$$\cancel{2\xi\frac{\partial f'}{\partial \xi}} + \frac{\partial V}{\partial \eta} + f' = 0$$

Stagnation Point

Species Continuity

$$\cancel{2\xi f'\frac{\partial c_i}{\partial \xi}} + V\frac{\partial c_i}{\partial \eta} = -\frac{\partial \mathcal{J}_i^{\eta}}{\partial \eta} + \dot{\mathcal{W}}_i$$

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Boundary conditions:

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$$\boxed{\begin{aligned}\xi(x) &= \int_0^x (\rho\mu)_r u_e r^{2\epsilon}\,\mathrm{d}x \\ \eta(x,y) &= \frac{u_e r^{\epsilon}}{(2\xi)^{1/2}}\int_0^y \rho\,\mathrm{d}y\end{aligned}}$$

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$$- \frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\mathcal{J}_i^{\eta}\frac{h_i}{h_e}\right) - \frac{p}{h_e}\frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\frac{\chi_i \mathcal{J}_i^{\eta}}{\rho_i}\right) - \cancel{2\xi f'\frac{g}{h_e}\frac{\partial h_e}{\partial \xi}}$$

Stagnation Point

Homann's Flow

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$$J_{i,w}^{y} = \dot{w}_{i,w}$$

- Edge:

$$f' \to 1$$
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$$\boxed{\beta = \frac{2\xi}{u_e}\frac{\mathrm{d}u_e}{\mathrm{d}\xi}}$$

$$\boxed{\mathcal{C} = \frac{\rho\mu}{(\rho\mu)_r}}$$

$$\boxed{\mathrm{Pr} = \frac{\mu c_{p,\mathrm{fr}}}{\lambda_{\mathrm{fr}}}}$$

Numerical Solution: Overview

$$a(w)\,\frac{\partial^2 w}{\partial \eta^2} + b(w)\,\frac{\partial w}{\partial \eta} + c(w)\,w + e\,\frac{\partial w}{\partial \xi} = d(w)$$

$$\boxed{t = \frac{\eta - \eta_n}{\Delta \eta}}$$

- Discretize the domain in the ξ and η directions
- Discretize the unknown along xi with a 2nd order Lagrangian polynomial

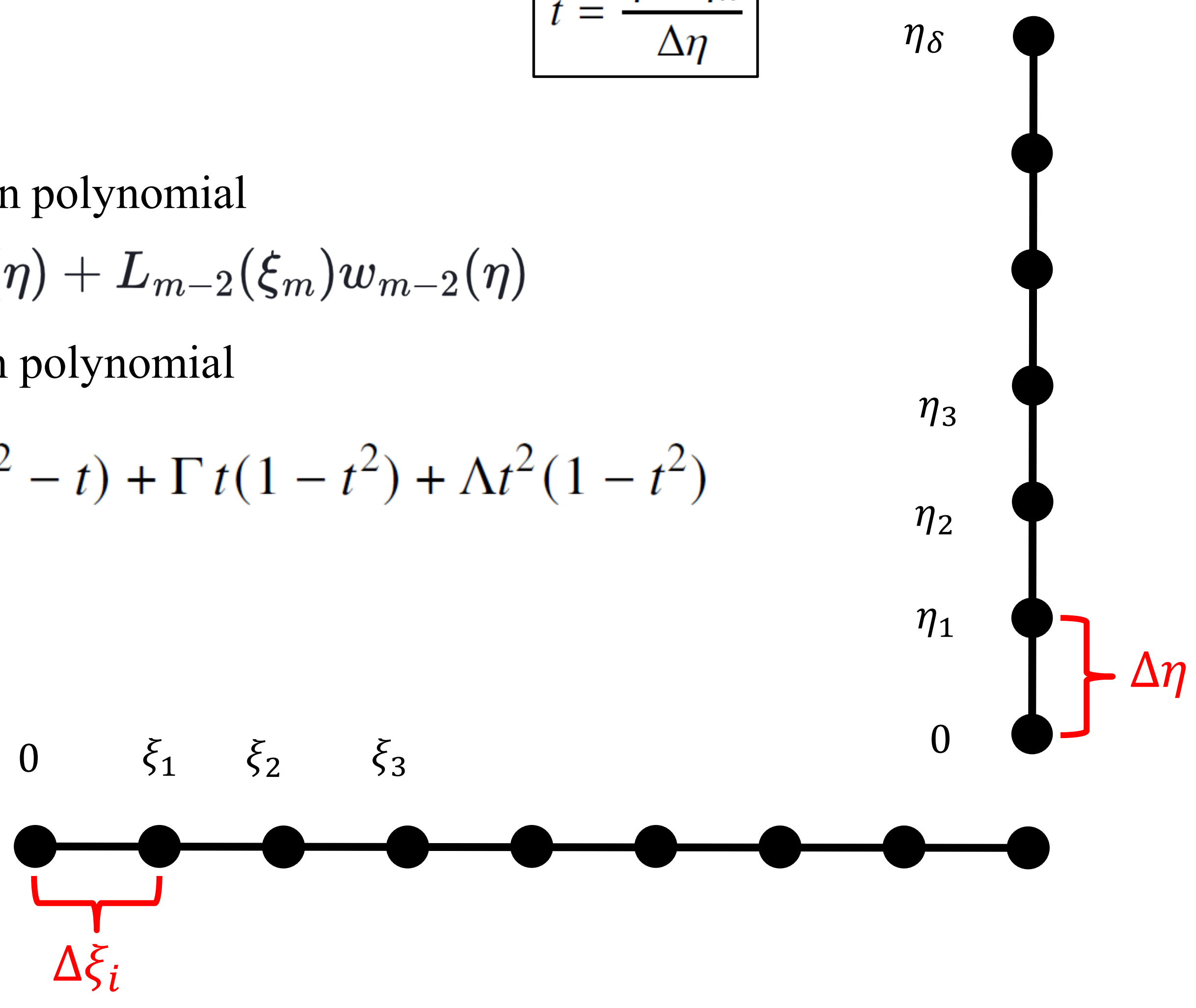
$$w(\xi_m, \eta) = L_m(\xi_m) w_m(\eta) + L_{m-1}(\xi_m) w_{m-1}(\eta) + L_{m-2}(\xi_m) w_{m-2}(\eta)$$

- Discretize the unknown along eta with a 4th order Hermitian polynomial

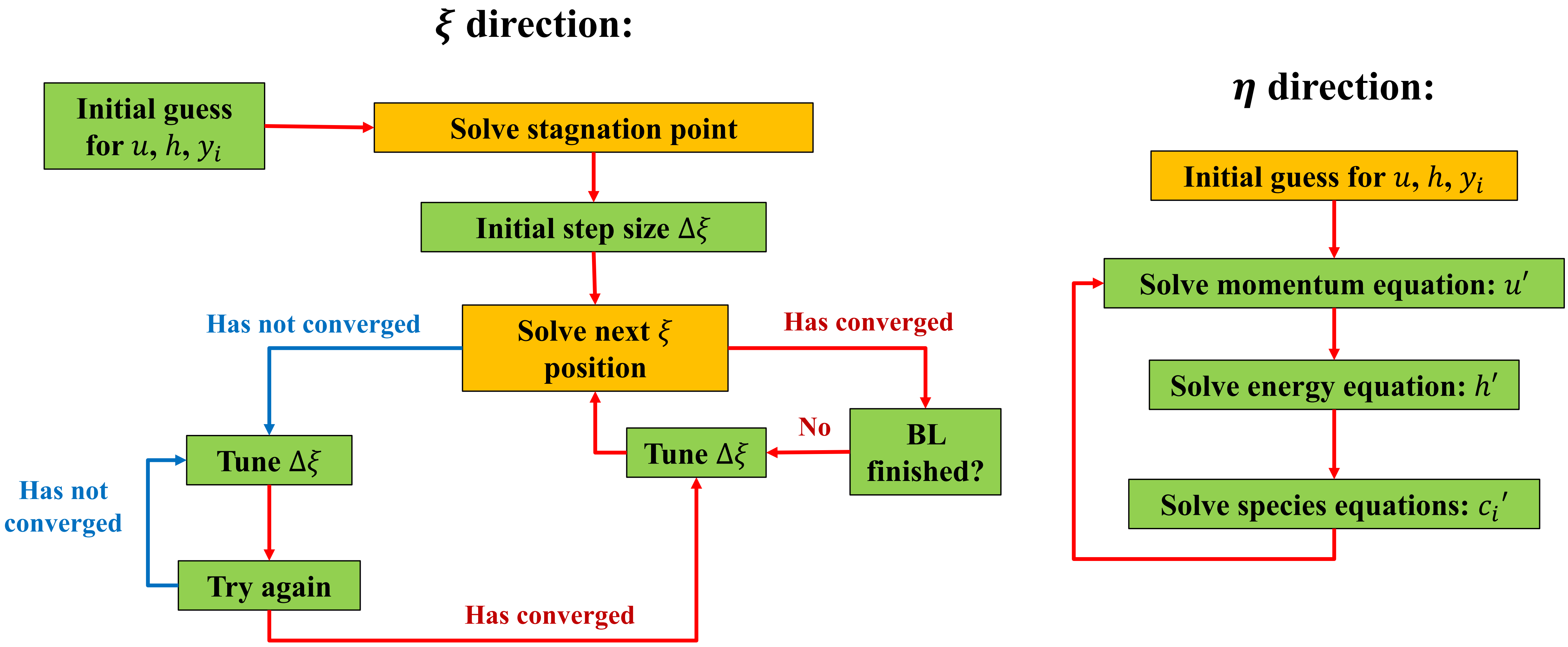
$$w\,(\eta) = \frac{1}{2} w_{n+1}(t^2 + t) + w_n(1 - t^2) + \frac{1}{2} w_{n-1}(t^2 - t) + \Gamma\, t(1 - t^2) + \Lambda t^2 (1 - t^2)$$

- Continuity equation can be simply integrated
- Linearize the coefficients
- Collocate the equations in the nodes and solve

$$\begin{bmatrix} B_1 & A_1 & & & & \\ C_2 & B_2 & A_2 & & & \\ & C_3 & B_3 & A_3 & & \\ & & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot & \\ & & & C_{N-2} & B_{N-2} & A_{N-2} \\ & & & & C_{N-1} & B_{N-1} \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ \cdot \\ \cdot \\ w_{N-2} \\ w_{N-1} \end{pmatrix} = \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ \cdot \\ \cdot \\ D_{N-2} \\ D_{N-1} - A_{N-1} w_N \end{pmatrix}$$

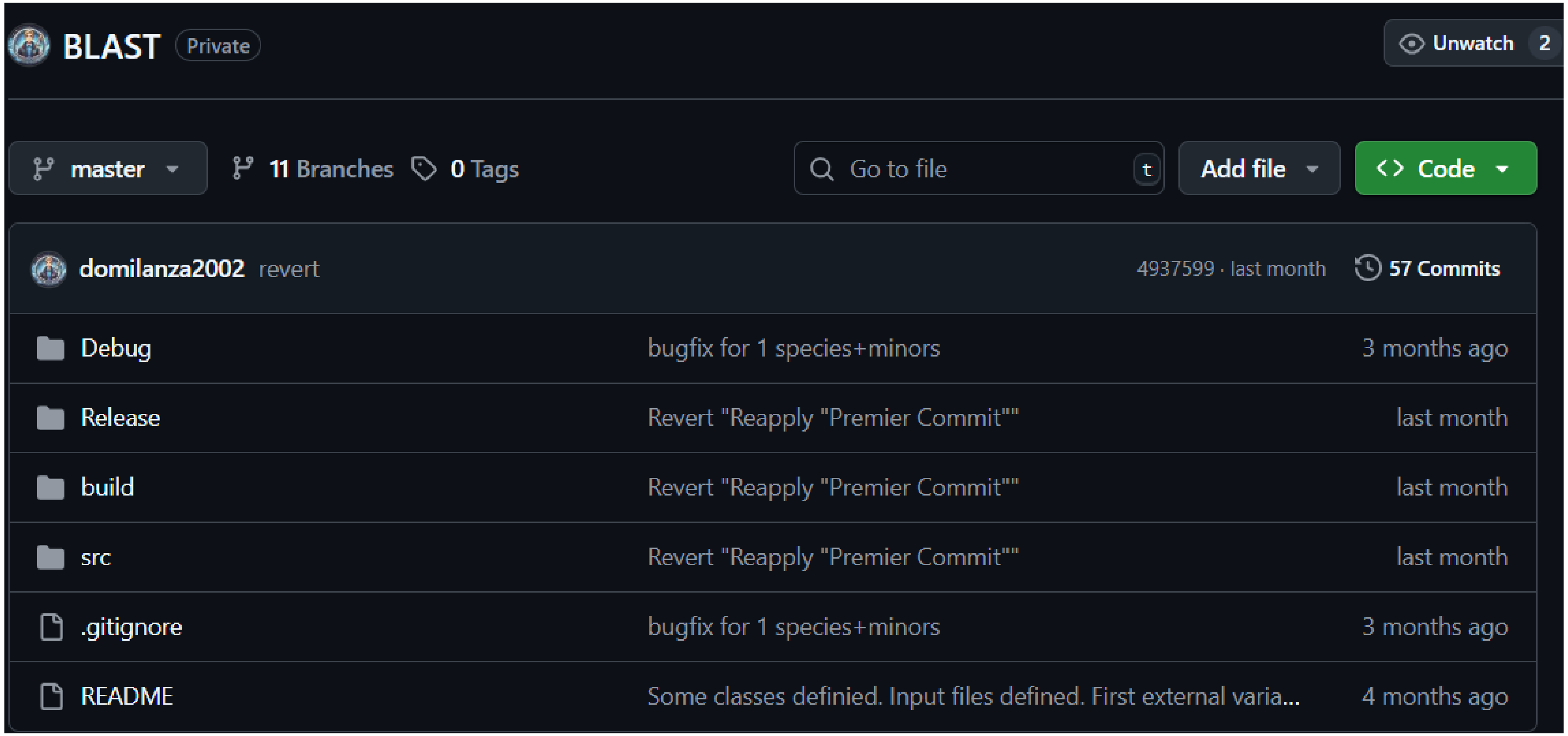


Numerical Method: Resolution Strategy



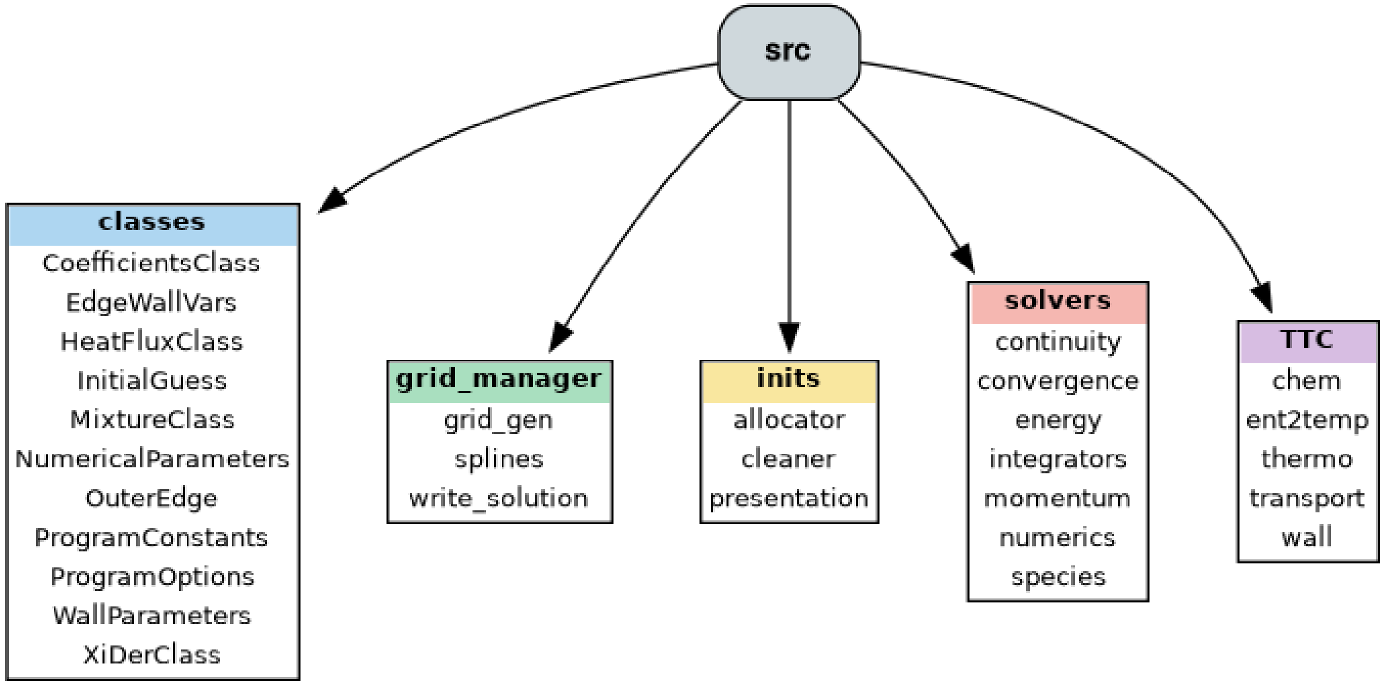
Code Overview

Code written in C++. It will be released as open-source when completed.

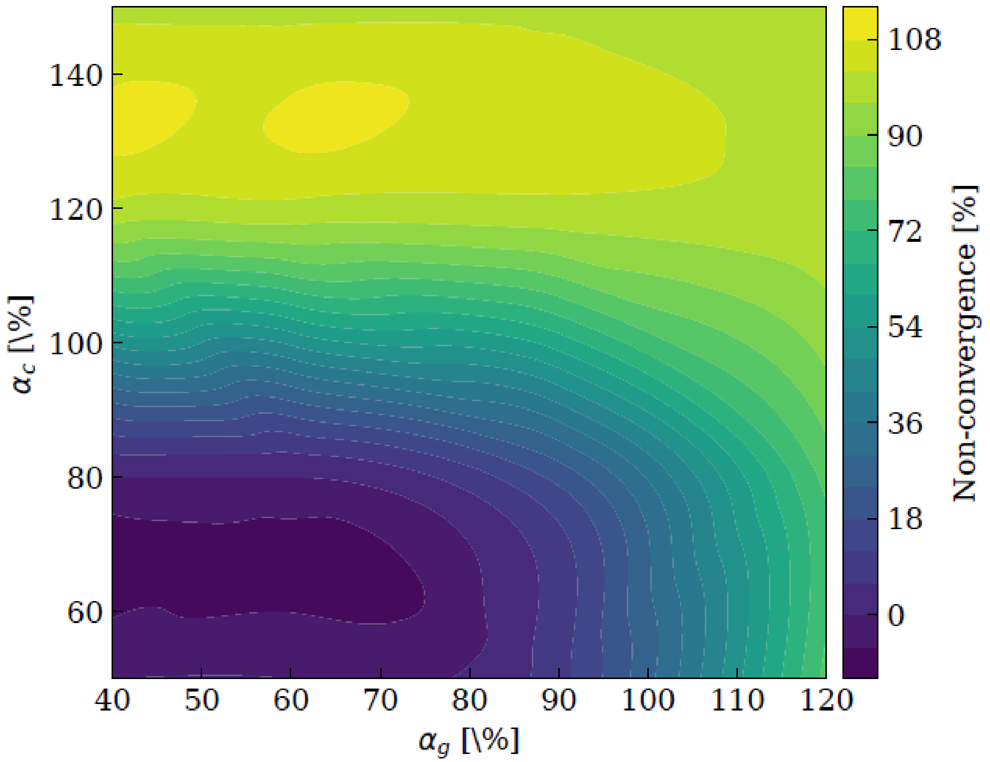


Contact: lanza3@illinois.edu

Code Overview



Code structure

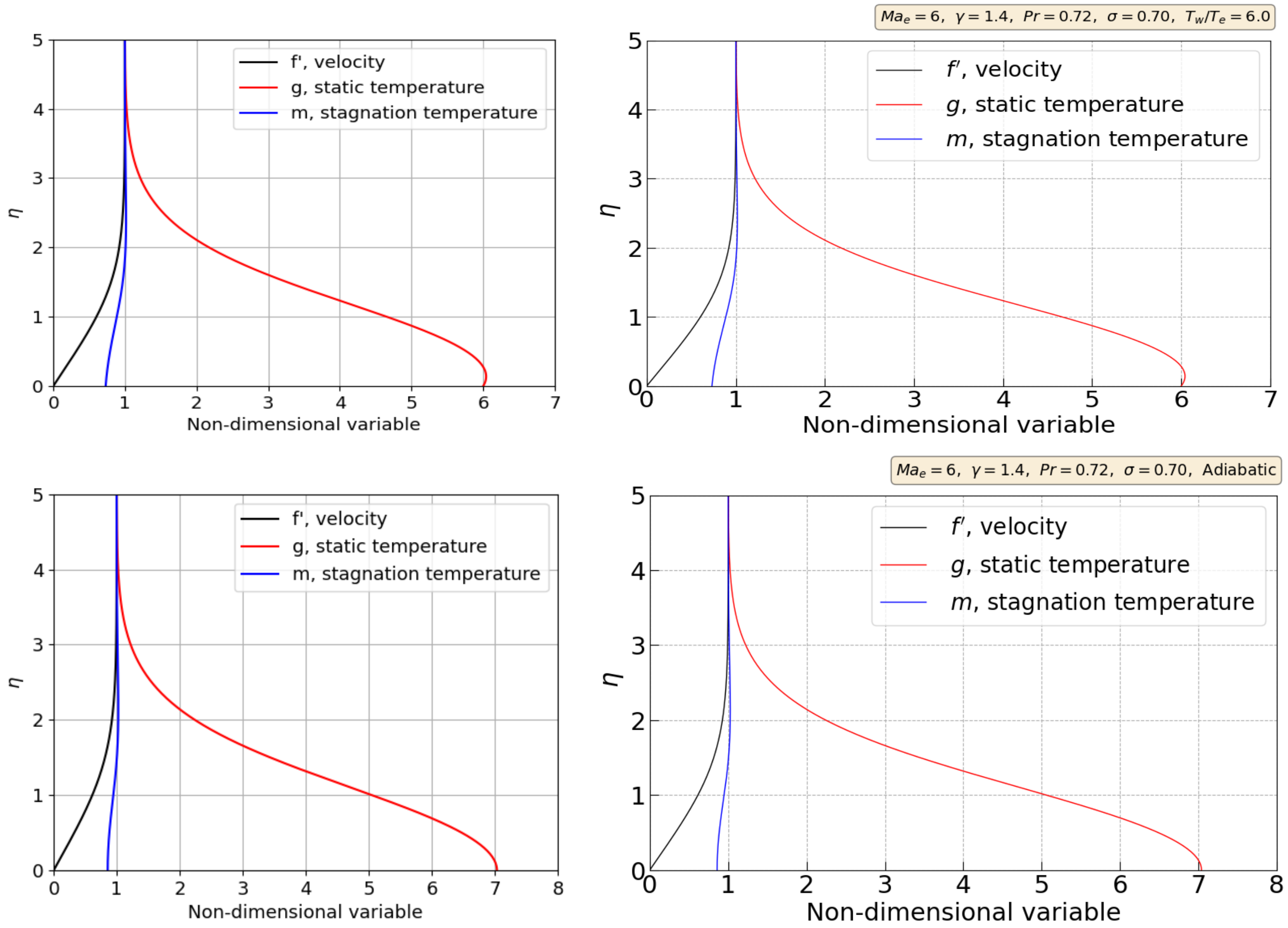


Convergence study

Ideal perfect gas (N_2), flat plate with no pressure gradient

$$f' = \frac{u}{u_e} \qquad g = \frac{h}{h_e}$$

$$m = \frac{T_0}{T_{0e}}$$

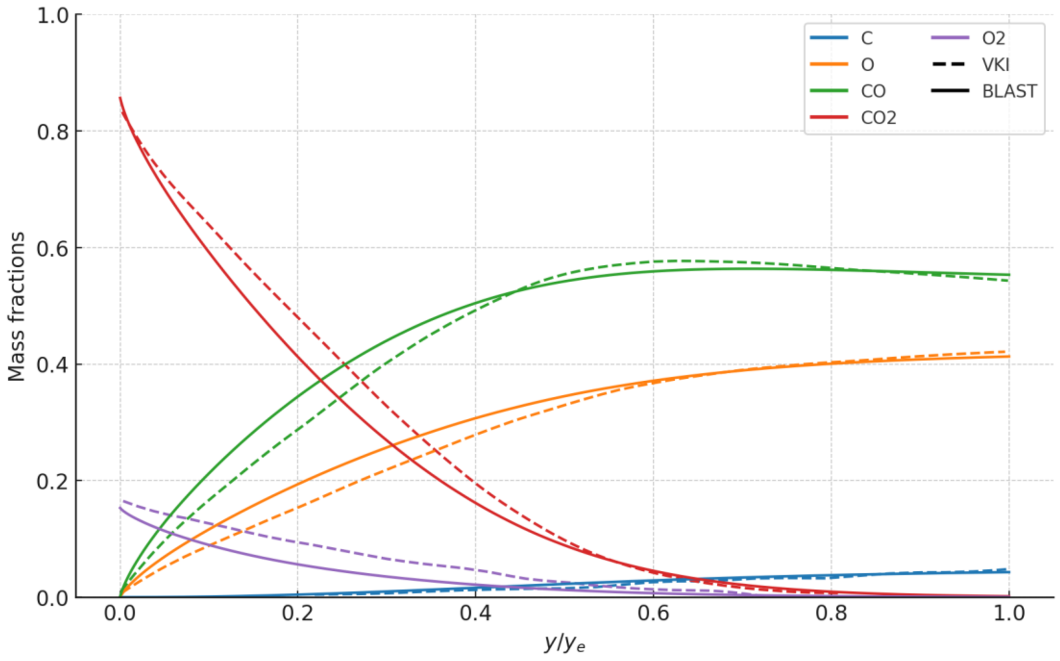


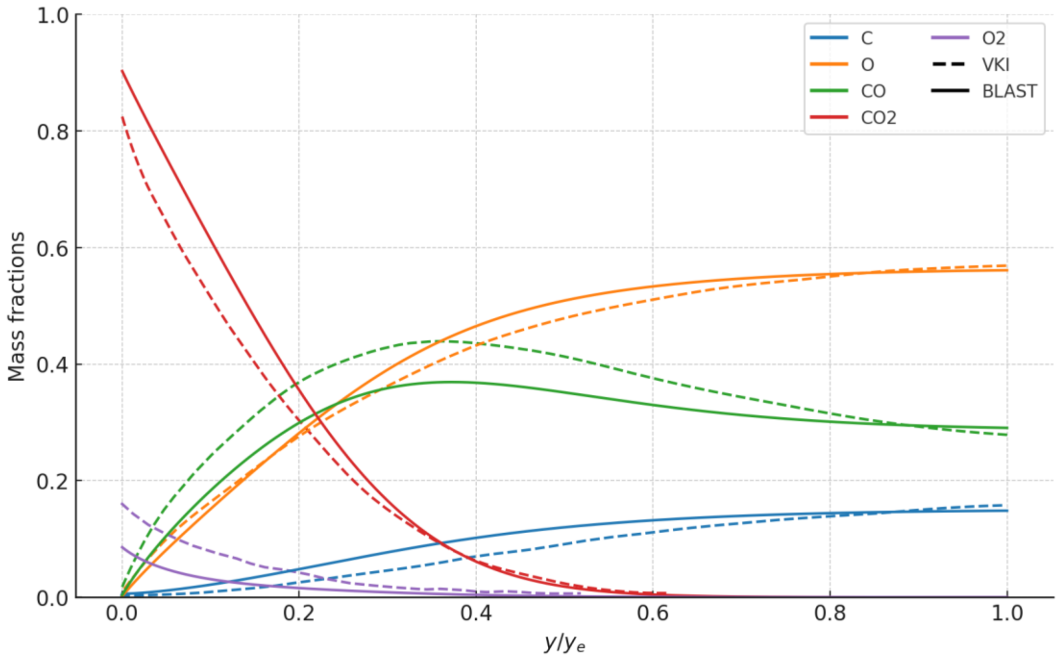
Error on profiles: $10^{-3} – 10^{-1}$ %

Verification

Reactive Cases Comparison

Comparison with **"CO2 Stagnation Line Flow Simulation for Mars Entry Applications"** by P. Rini, A. F. Kolesnikov, S. A. Vasil'evskii, O. Chazot, and G. Degrez (VKI, IPM Moscow, and ULB).



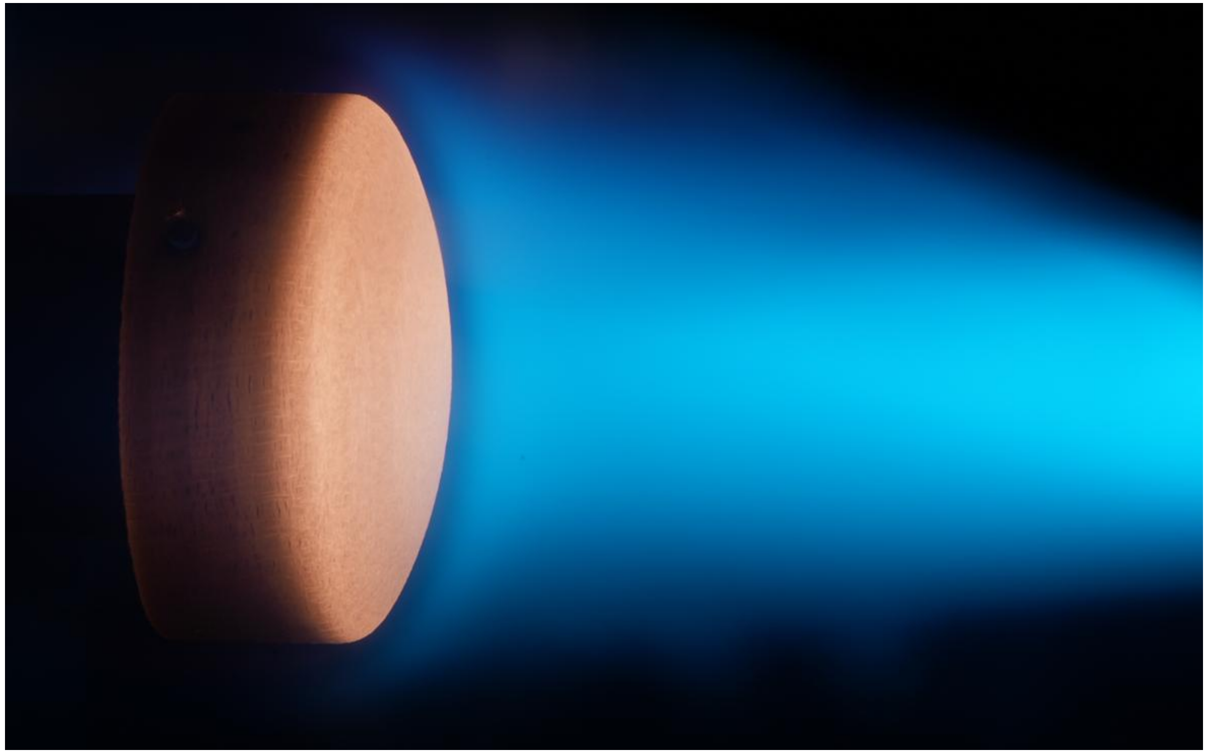


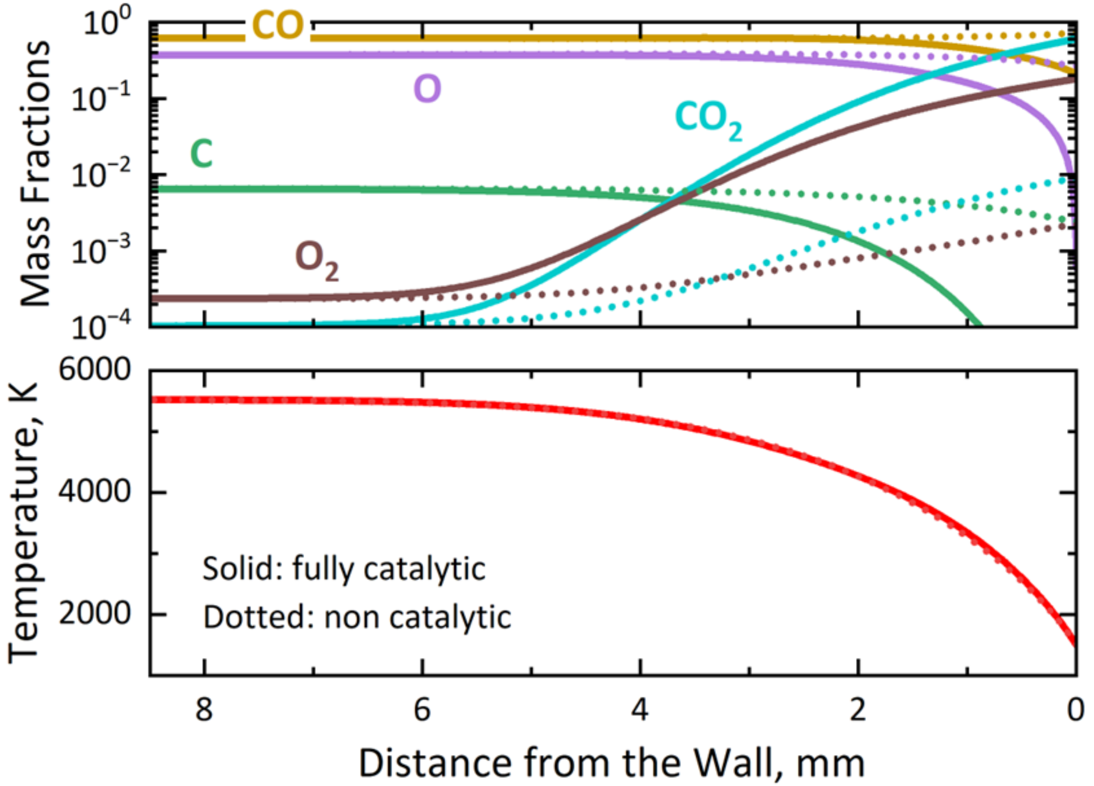
Mass fraction comparison. Fully catalytic cases: on the left, OC1 with $T_w = 300K$*; on the right, OC2 with* $T_w = 2100K$

$\boxed{O + O \rightarrow O_2}$ $\boxed{CO + O \rightarrow CO_2}$

Results

CO2 case inside the Plasmatron with SpaceX starship tile, refractory insulator + RCG coating





Outer edge: $P_0 = 10400\ Pa, T_0 = 5530\ K, T_w = 1500\ K$

$\boldsymbol{O + O \rightarrow O_2}$

$\boldsymbol{CO + O \rightarrow CO_2}$

Conclusions

Key Takeaway

- **BLAST is a boundary layer code for chemically reacting gases in thermal equilibrium and chemical non-equilibrium. It already includes surface catalysis, and an ablation module is under development.**
- **The code will be released as an open-source project on GitHub once development and validation are completed.**
- **New features and analyses are being added in parallel, with the goal of providing stronger support for experimental campaigns.**

THANKS!

This work is supported by the Office of Naval Research award no. N00014-23-12623

Further details on the Levy-Lees transformation

Levy-Lees Transformation

$$\xi(x) = \int_0^x (\rho\mu)_r u_e r^{2\epsilon}\,\mathrm{d}x$$

$$\eta(x,y) = \frac{u_e r^\epsilon}{(2\xi)^{1/2}} \int_0^y \rho\,\mathrm{d}y$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial x} = (\rho\mu)_r u_e r^{2\epsilon}\frac{\partial}{\partial \xi} + \left(\frac{\partial \eta}{\partial x}\right)\frac{\partial}{\partial \eta}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi}\cancelto{0}{\frac{\partial \xi}{\partial y}} + \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\rho u_e r^\epsilon}{(2\xi)^{1/2}}\frac{\partial}{\partial \eta}$$

$$\frac{\mathrm{d}\,\xi}{\mathrm{d}\,x} = (\rho\mu)_r u_e r^{2\epsilon}$$

$$\frac{\partial f}{\partial \eta} = \frac{u}{u_e}$$

$$\frac{\partial \psi}{\partial y} = \rho u r^\epsilon$$

$$\frac{\partial \psi}{\partial x} = -\rho v r^\epsilon$$

$$f(\xi,\eta) = \frac{\psi}{\sqrt{2\xi}}$$

$$V = -\frac{\partial \psi}{\partial \xi}\sqrt{2\xi}$$

$$V = \frac{2\xi}{\frac{\mathrm{d}\,\xi}{\mathrm{d}\,x}}\left(f'\frac{\partial \eta}{\partial x} + \frac{\rho v r^\epsilon}{\sqrt{2\xi}}\right)$$

Transformed equations

$$f' = \frac{u}{u_e} \qquad g = \frac{h}{h_e}$$

$$\mathcal{J}_i^\eta = \frac{r^\epsilon (2\xi)^{1/2}}{\mathrm{d}\xi/\mathrm{d}x} J_i^y$$

$$f' = \frac{u}{u_e} \qquad g = \frac{h}{h_e}$$

$$\beta = \frac{2\xi}{u_e}\frac{\mathrm{d}u_e}{\mathrm{d}\xi}$$

$$\mathcal{C} = \frac{\rho\mu}{(\rho\mu)_r}$$

$$\mathrm{Pr} = \frac{\mu c_{p,\mathrm{fr}}}{\lambda_{\mathrm{fr}}}$$

Continuity

$$2\xi\frac{\partial f'}{\partial \xi} + \frac{\partial V}{\partial \eta} + f' = 0 \tag{74}$$

Species Continuity

$$2\xi f'\frac{\partial c_i}{\partial \xi} + V\frac{\partial c_i}{\partial \eta} = -\frac{\partial \mathcal{J}_i^\eta}{\partial \eta} + \dot{\mathcal{W}}_i \tag{75}$$

x Momentum

$$2\xi f'\frac{\partial f'}{\partial \xi} + V\frac{\partial f'}{\partial \eta} = \beta\left(\frac{\rho_e}{\rho} - (f')^2\right) + \frac{\partial}{\partial \eta}\left(\mathcal{C}\frac{\partial f'}{\partial \eta}\right) \tag{76}$$

Energy

$$\begin{aligned} 2\xi f'\frac{\partial g}{\partial \xi} + V\frac{\partial g}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{\mathcal{C}}{\mathrm{Pr}}\frac{\partial g}{\partial \eta}\right) + \frac{u_e^2}{h_e}\left[\mathcal{C}\left(\frac{\partial f'}{\partial \eta}\right)^2 - \beta\frac{\rho_e}{\rho}f'\right] - \frac{\partial}{\partial \eta}\left(\frac{\mathcal{C}}{\mathrm{Pr}}\sum_{i=1}^{N_s}\frac{\partial c_i}{\partial \eta}\frac{h_i}{h_e}\right) \\ - \frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\mathcal{J}_i^\eta\frac{h_i}{h_e}\right) - \frac{p}{h_e}\frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\frac{\chi_i \mathcal{J}_i^\eta}{\rho_i}\right) - 2\xi f'\frac{g}{h_e}\frac{\partial h_e}{\partial \xi} \end{aligned} \tag{77}$$

Transformed equations: boundary conditions

The transformed equations satisfy the following boundary conditions at the wall ($\eta = 0$):

$$f'(\xi, 0) = 0 \tag{78}$$

$$V(\xi, 0) = 0 \tag{79}$$

$$g(\xi, 0) = g_w(\xi) = h_w/h_e \quad \text{or} \quad \text{adiabatic} \tag{80}$$

$$J^y_{i,w} = \dot{w}_{i,w} \tag{81}$$

Or equilibrium wall

while at edge ($\eta \to \infty$):

$$f' \to 1 \tag{82}$$

$$g \to 1 \tag{83}$$

$$c_i \to c_{i,e} \tag{84}$$

Adiabatic:

$$\frac{\partial g}{\partial \eta} = \sum_{i=1}^{N_s} \frac{\partial c_i}{\partial \eta} \frac{h_i}{h_e} + \frac{Pr}{\mathcal{C}} \sum_{i=1}^{N_s} \frac{h_i}{h_e} \mathcal{J}_i^\eta + \frac{p}{h_e} \frac{Pr}{\mathcal{C}} \sum_{i=1}^{N_s} \chi_i \frac{\mathcal{J}_i^\eta}{\rho_i}$$

Transformed equations: Stagnation Point

Continuity

$$\frac{\partial V}{\partial \eta} + f' = 0 \tag{90}$$

Species Continuity

$$V\frac{\partial c_i}{\partial \eta} = -\frac{\partial \mathcal{J}_i^\eta}{\partial \eta} + \dot{\mathcal{W}}_i \tag{91}$$

x Momentum

$$V\frac{\partial f'}{\partial \eta} = \beta\left(\frac{\rho_e}{\rho} - (f')^2\right) + \frac{\partial}{\partial \eta}\left(\mathcal{C}\frac{\partial f'}{\partial \eta}\right) \tag{92}$$

Energy

$$V\frac{\partial g}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{\mathcal{C}}{\mathrm{Pr}}\frac{\partial g}{\partial \eta}\right) + \frac{u_e^2}{h_e}\left[\mathcal{C}\left(\frac{\partial f'}{\partial \eta}\right)^2 - \beta\frac{\rho_e}{\rho}f'\right] - \frac{\partial}{\partial \eta}\left(\frac{\mathcal{C}}{\mathrm{Pr}}\sum_{i=1}^{N_s}\frac{\partial c_i}{\partial \eta}\frac{h_i}{h_e}\right)$$
$$-\frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\mathcal{J}_i^\eta\frac{h_i}{h_e}\right) - \frac{p}{h_e}\frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\frac{\chi_i \mathcal{J}_i^\eta}{\rho_i}\right) \tag{93}$$

If a Hiemenz (or Homann)'s flow is considered, then $u_e \to 0$ for $\xi \to 0$, and the energy equation simplify further:

$$V\frac{\partial g}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{\mathcal{C}}{\mathrm{Pr}}\frac{\partial g}{\partial \eta}\right) - \frac{\partial}{\partial \eta}\left(\frac{\mathcal{C}}{\mathrm{Pr}}\sum_{i=1}^{N_s}\frac{\partial c_i}{\partial \eta}\frac{h_i}{h_e}\right) - \frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\mathcal{J}_i^\eta\frac{h_i}{h_e}\right) - \frac{p}{h_e}\frac{\partial}{\partial \eta}\left(\sum_{i=1}^{N_s}\frac{\chi_i \mathcal{J}_i^\eta}{\rho_i}\right) \tag{94}$$

Resolution strategy: finite thickness

New inputs: $v_e,\ \delta,\ \frac{\partial}{\partial y}\left(\frac{\partial u_e}{\partial x}\right)$

$$\hat{\eta}(x,y) = \mathcal{K}\frac{u_e r^{\epsilon}}{(2\xi)^{1/2}}\int_0^y \rho\,\mathrm{d}y = \mathcal{K}\eta$$

$$\mathcal{K} = \frac{1}{\delta}\frac{(2\xi)^{1/2}}{u_e r^{\epsilon}}\int_0^{\hat{\eta}_{\max}} 1/\rho\,\mathrm{d}\hat{\eta}$$

$$\frac{\partial}{\partial \eta} = K\frac{\partial}{\partial \hat{\eta}}$$

$$\tilde{V} = \mathcal{K}V$$

$$\frac{\partial p}{\partial x} = -\rho_e u_e\frac{\partial u_e}{\partial x} - \rho_e v_e\frac{\partial u_e}{\partial y}$$

Continuity

$$\frac{\partial \tilde{V}}{\partial \hat{\eta}} + f' = 0 \qquad (13$$

Species Continuity

$$\tilde{V}\frac{\partial c_i}{\partial \hat{\eta}} = -\mathcal{K}\frac{\partial \tilde{\mathcal{J}}_i^{\hat{\eta}}}{\partial \hat{\eta}} + \dot{\tilde{\mathcal{W}}}_i \qquad (13$$

x Momentum

$$\tilde{V}\frac{\partial f'}{\partial \hat{\eta}} = \frac{1}{2}\left(\frac{\rho_e}{\rho} - (f')^2\right) + \frac{1}{2}\frac{\rho_e v_e}{\rho\left(\left(\frac{\partial u_e}{\partial x}\right)_{x=0}\right)^2}\frac{\partial}{\partial y}\left(\frac{\partial u_e}{\partial x}\right)_{x=0} + \frac{\partial}{\partial \hat{\eta}}\left(\mathcal{K}^2\mathcal{C}f'\right) \qquad (13$$

Energy

$$\tilde{V}\frac{\partial g}{\partial \hat{\eta}} = \frac{\partial}{\partial \hat{\eta}}\left(\mathcal{K}^2\frac{\mathcal{C}}{\mathrm{Pr}}\frac{\partial g}{\partial \hat{\eta}}\right) - \frac{\partial}{\partial \hat{\eta}}\left(\mathcal{K}^2\frac{\mathcal{C}}{\mathrm{Pr}}\sum_{i=1}^{N_s}\frac{\partial c_i}{\partial \hat{\eta}}\frac{h_i}{h_e}\right) - \frac{\partial}{\partial \hat{\eta}}\left(\mathcal{K}\sum_{i=1}^{N_s}\tilde{\mathcal{J}}_i^{\eta}\frac{h_i}{h_e}\right) - \frac{p}{h_e}\frac{\partial}{\partial \hat{\eta}}\left(\mathcal{K}\sum_{i=1}^{N_s}\frac{\chi_i\mathcal{J}_i^{\eta}}{\rho_i}\right) \qquad (13$$

Further details on the numerical method

Numerical Solution

Non-linear 2nd order parabolic differential equations

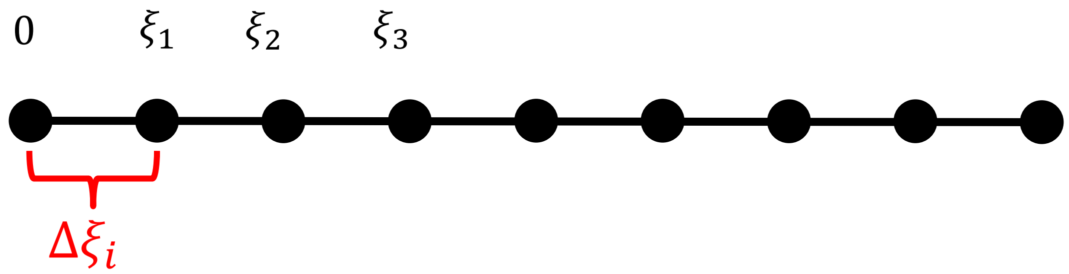
$$a(w)\,\frac{\partial^2 w}{\partial \eta^2} + b(w)\,\frac{\partial w}{\partial \eta} + c(w)\,w + e\,\frac{\partial w}{\partial \xi} = d(w)$$

$$\begin{aligned} \eta &= 0: \quad f\frac{\partial w}{\partial \eta} + gw = h \\ \eta &= \eta_\delta: \quad w = w_\delta \end{aligned}$$

Except continuity.
Species must be modified.

Numerical Solution: Xi Discretization

2nd order Lagrangian polynomial



$$w(\xi_m,\eta) = L_m(\xi_m)w_m(\eta) + L_{m-1}(\xi_m)w_{m-1}(\eta) + L_{m-2}(\xi_m)w_{m-2}(\eta)$$

$$\boxed{L_{m-i}(\xi) = \frac{\prod_{j=0,j\neq i}^{2}(\xi - \xi_{m-j})}{\prod_{j=0,j\neq i}^{2}(\xi_{m-i} - \xi_{m-j})}}$$

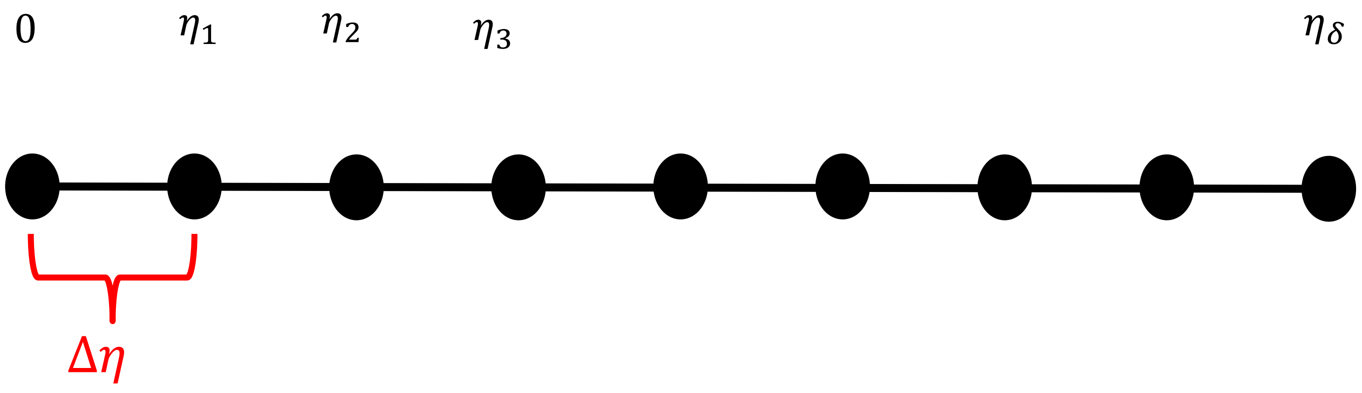
$$\frac{\partial w(\xi_m,\eta)}{\partial \xi} = \sum_{i=0}^{2} \Lambda_{2,m-i} w_{m-i}$$

$$\begin{aligned}
\Lambda_{2,m} &= \frac{1}{\xi_m - \xi_{m-1}} + \frac{1}{\xi_m - \xi_{m-1}} \\
\Lambda_{2,m-1} &= \frac{\xi_m - \xi_{m-2}}{(\xi_{m-1} - \xi_m)(\xi_{m-1} - \xi_{m-2})} \\
\Lambda_{2,m-2} &= \frac{\xi_m - \xi_{m-1}}{(\xi_{m-2} - \xi_m)(\xi_{m-2} - \xi_{m-1})}
\end{aligned}$$

Numerical Solution: Eta Discretization

For the m-th station in the ξ direction:

$$a(w_m)\frac{\partial^2 w_m}{\partial \eta^2} + b(w_m)\frac{\partial w_m}{\partial \eta} + \left(c(w_m) + e(w_m)\Lambda_{2,m}\right)w_m = d(w_m) - e\sum_{i=1}^{2}\Lambda_{2,m-i}w_{m-i}$$



Numerical Solution: Continuity Equation

$$2\xi \frac{\partial F}{\partial \xi} + \frac{\partial V}{\partial \eta} + F = 0$$

$$2\xi \sum_{i=0}^{2} \Lambda_{2,m-i} F_{m-i} + \frac{\partial V}{\partial \eta} + F = 0$$

$$V_m(\eta) = \int_0^{\eta} \left(-(2\xi \Lambda_{2,m} + 1) F_m - 2\xi(\Lambda_{2,m-1} F_{m-1} + \Lambda_{2,m-2} F_{m-2})\right) \mathrm{d}\eta$$

Numerical Solution: Eta Discretization

For the m-th station in the ξ direction:

$$a(w_m)\frac{\partial^2 w_m}{\partial \eta^2} + b(w_m)\frac{\partial w_m}{\partial \eta} + (c(w_m) + e(w_m)\Lambda_{2,m})w_m = d(w_m) - e\sum_{i=1}^{2}\Lambda_{2,m-i}w_{m-i}$$

4th order Hermitian polynomial:

$$w(\eta) = \frac{1}{2}w_{n+1}(t^2+t) + w_n(1-t^2) + \frac{1}{2}w_{n-1}(t^2-t) + \Gamma t(1-t^2) + \Lambda t^2(1-t^2)$$

$$t = \frac{\eta - \eta_n}{\Delta \eta}$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta} = \frac{1}{\Delta\eta}\frac{\mathrm{d}}{\mathrm{d}t}$$

Numerical solution: Non linear Coefficients

$$a(w_m)\frac{\partial^2 w_m}{\partial \eta^2} + b(w_m)\frac{\partial w_m}{\partial \eta} + (c(w_m) + e(w_m)\Lambda_{2,m})w_m = d(w_m) - e\sum_{i=1}^{2} \Lambda_{2,m-i} w_{m-i}$$

Remove non-linearity of a, b, c, e by using the (m-1)-th station (or previous iteration)

$$a\frac{\partial^2 w_m}{\partial \eta^2} + b\frac{\partial w_m}{\partial \eta} + \left(c + e\Lambda_{2,m}\right) w_m = d(w_m) - e\sum_{i=1}^{2} \Lambda_{2,m-i} w_{m-i}$$

Divide d in 2 parts:

Constant **Linearized**

$$d = \bar{d}(w_{m-1}) + F(w_m)$$

Numerical solution: Tridiagonal System

$$
\begin{bmatrix}
B_1 & A_1 & & & & \\
C_2 & B_2 & A_2 & & & \\
 & C_3 & B_3 & A_3 & & \\
 & & \cdot & \cdot & \cdot & \\
 & & \cdot & \cdot & \cdot & \\
 & & & C_{N-2} & B_{N-2} & A_{N-2} \\
 & & & & C_{N-1} & B_{N-1}
\end{bmatrix}
\begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ \cdot \\ \cdot \\ w_{N-2} \\ w_{N-1} \end{pmatrix}
=
\begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ \cdot \\ \cdot \\ D_{N-2} \\ D_{N-1} - A_{N-1} w_N \end{pmatrix}
$$

Solved using Thomas' algorithm

Numerical Solution: Species Equation

$$2\xi F \frac{\partial y_i}{\partial \xi} + V \frac{\partial y_i}{\partial \eta} + \frac{\partial \mathcal{J}_i^\eta}{\partial \eta} = \dot{W}_i$$

Issue: first-order differential equation → **Add fictitious fluxes**

Add and subtract: $J_{f,i} = \rho D \frac{\partial y_i}{\partial y}$

$-\rho D \left.\frac{\partial y_i}{\partial y}\right|_m$ **2nd order differential equation**

$+\rho D \left.\frac{\partial y_i}{\partial y}\right|_{m-1}$ **Fictitious flux**

Numerical Solution: Species Equation

Regarding the boundary condition, use the catalytic fluxes:

$$\boxed{J^y_{i,w} = \dot{w}_{i,\text{cat}}}$$

$$\boxed{J^y_{i,w} - \rho D \frac{\partial y_i}{\partial y} = \dot{w}_{i,\text{cat}} - J_{f,i}}$$

$$\boxed{-\rho D \frac{\partial y_i}{\partial y} = \dot{w}_{i,\text{cat}} - J_{f,i} - J^y_{i,w}}$$

$$\boxed{f \frac{\partial w}{\partial \eta} + g w = h}$$

$$\dot{W}_{i,m} = \dot{W}_{i,m-1} + \sum_{j=1}^{N_s} \left(\frac{\partial \dot{W}_i}{\partial y_j} \right)_{m-1} (y_{j,m} - y_{j,m-1})$$

Coupling between the species equations -> Block-tridiagonal matrix!

Further details on the verification

Verification: Reactive case

Table 1 Operating conditions for the VKI-Plasmatron.

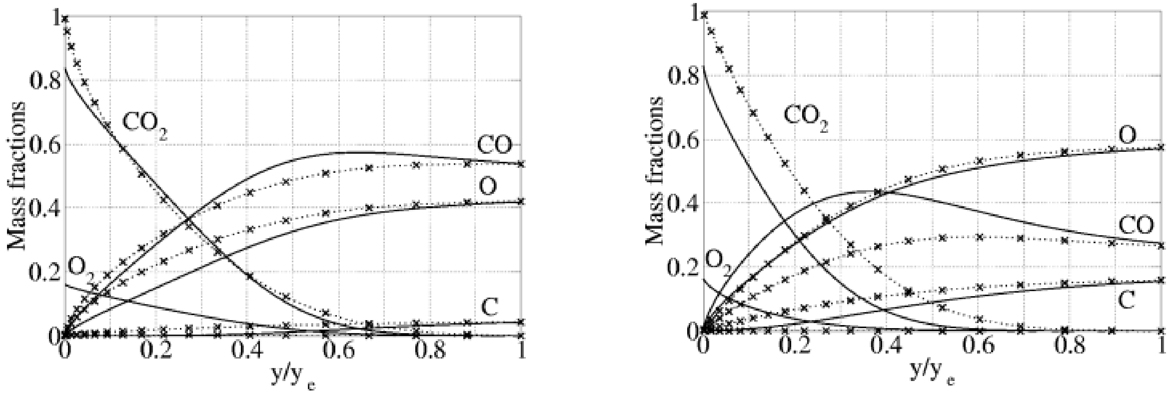
OC	P_{pl} [kW]	P_{static}	P_{Pitot} [Pa]	q_w^{exp} [MW/m^2]
1	140	7000	13.80	0.665
2	180	7000	31.92	1.232

Table 3 Rebuild conditions [T_w= 300 K] for two operating conditions.

	OC	T_e [K]	h_e [MJ/kg]	V_s [m/s]
VKI	1	5909	14.53	92.82
IPM	1	5932	14.71	93.10
VKI	2	6514	25.96	161.88
IPM	2	6556	26.67	162.80

Table 4 Bulk chemistry reactions (M∈S).

Dissociation reactions
$CO_2+M \leftrightarrow CO+O+M$
$O_2+M \leftrightarrow O+O+M$
$CO+M \leftrightarrow C+O+M$
Neutral exchange reactions
$CO+O \leftrightarrow C+O_2$
$CO_2+O \leftrightarrow CO+O_2$



a) OC1, T_w=300 K.

d) OC2, T_w=2100 K.

Pietro Rini, Anatoly Kolesnikov, Sergei Vasil'evskii, Olivier Chazot and Gérard Degrez. "CO2 Stagnation Line Flow Simulation for Mars Entry Applications," AIAA 2005-5206. 38th AIAA Thermophysics Conference. June 2005.

Verification: Ideal Perfect Gas

Verification cases: Calorically perfect and ideal gas

Continuity

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{138}$$

x Momentum

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\mathrm{d}p_e}{\mathrm{d}x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{139}$$

Energy

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + u \frac{\mathrm{d}p_e}{\mathrm{d}x} + \mu \left(\frac{\partial u}{\partial y}\right)^2 \tag{140}$$

1. **Flat plate case with no pressure gradient, Pr=1, given Tw**
2. **Flat plate case with no pressure gradient, Pr=1, adiabatic**
3. **Flat Plate case with no pressure gradient, variable Pr, given Tw**
4. **Flat Plate case with no pressure gradient, variable Pr, adiabatic**

Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=1, given Tw

- Calorically perfect and ideal gas: air, $C_p = 1005\, J/KgK$, $\gamma = 1.4$, $R = 287\, J/KgK$
- Flat plate, no pressure gradient->constant edge velocity->constant edge temperature

$$\frac{dp_e}{dx} = 0 \;\text{->}\; \frac{du_e}{dx} = -\frac{1}{\rho_e u_e}\frac{dp_e}{dx} = 0 \;\text{->} u_e = const.\; h_0 = C_p T_0 = const = C_p T_e + \frac{1}{2}\frac{{u_e}^2}{C_p} \;\text{->}\; T_e = const$$

- T_w is given

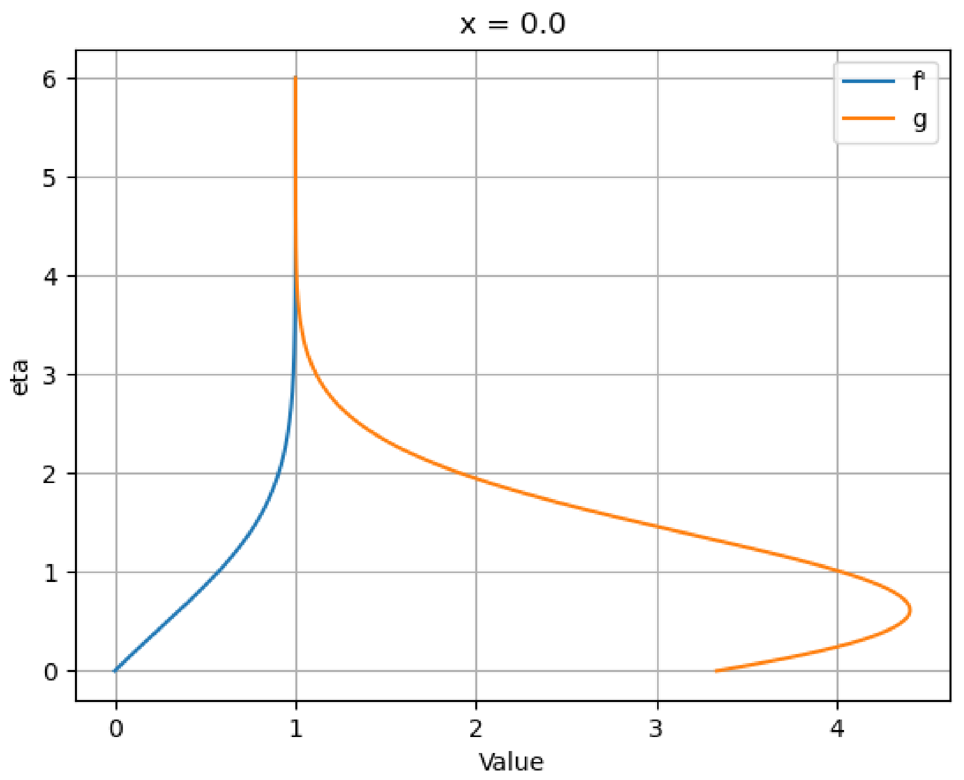
$$\boxed{\begin{aligned} \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \\ \rho u \frac{\partial T_0}{\partial x} + \rho v \frac{\partial T_0}{\partial y} &= \frac{\partial}{\partial y}\left(k \frac{\partial T_0}{\partial y}\right) \end{aligned}}$$

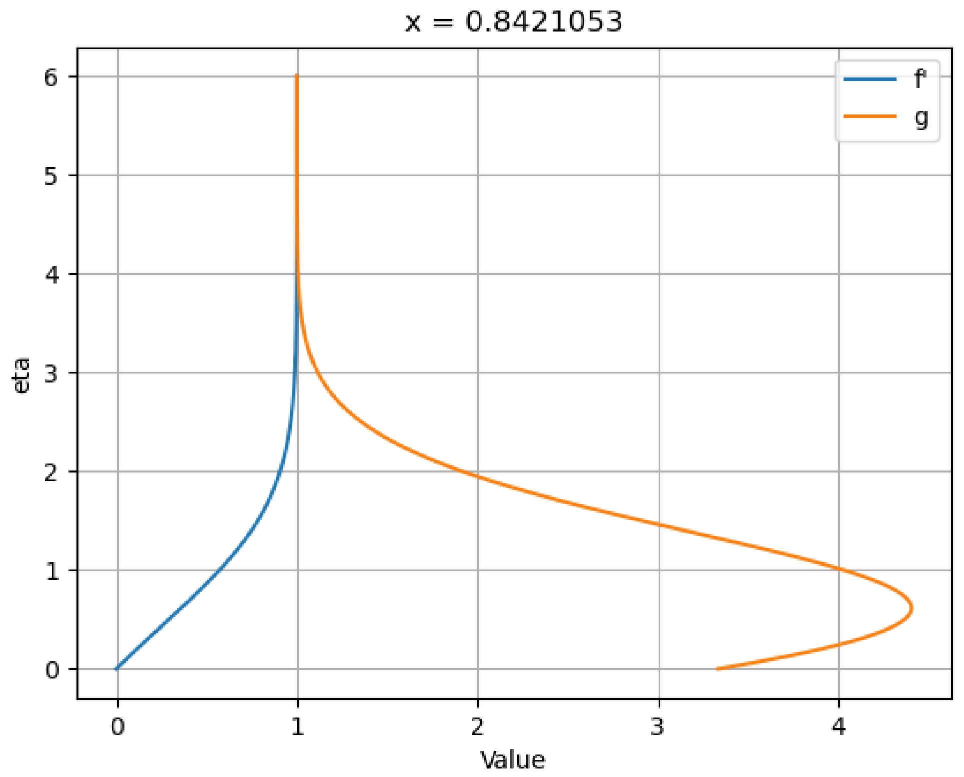
- $Pr = 1$-> viscosity proportional to thermal conductivity: $k = \frac{\mu C_p}{Pr} = \mu C_p$
- Viscosity follows a power law: $\mu = \mu_0 \left(\frac{T}{T_0}\right)^n, \mu_0 = 1.72 \cdot 10^{-5}, T_0 = 0\,°C,\; n = 0.7$
- Boundary layer with 20 points along x, from 0 to 1, and 300 points along y, with eta from 0 to 6

Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=1, given Tw

- $T_e = 1500\ K$, $P = 101325\ Pa$, $u_e = 5000\ m/s$, $T_w = 5000\ K$ -> $Ma_e = 6.44$



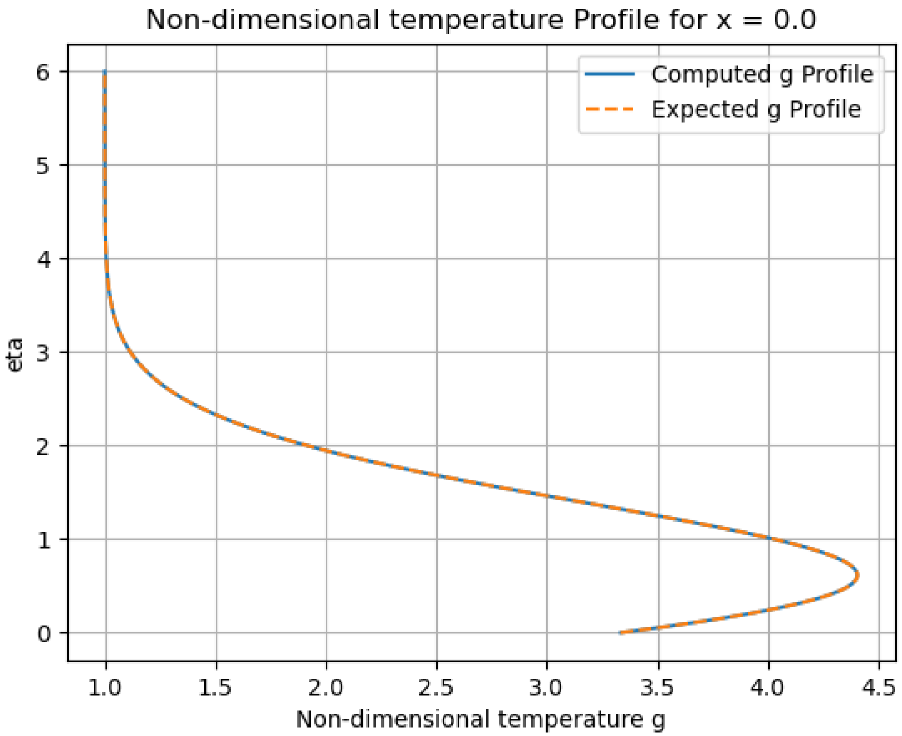


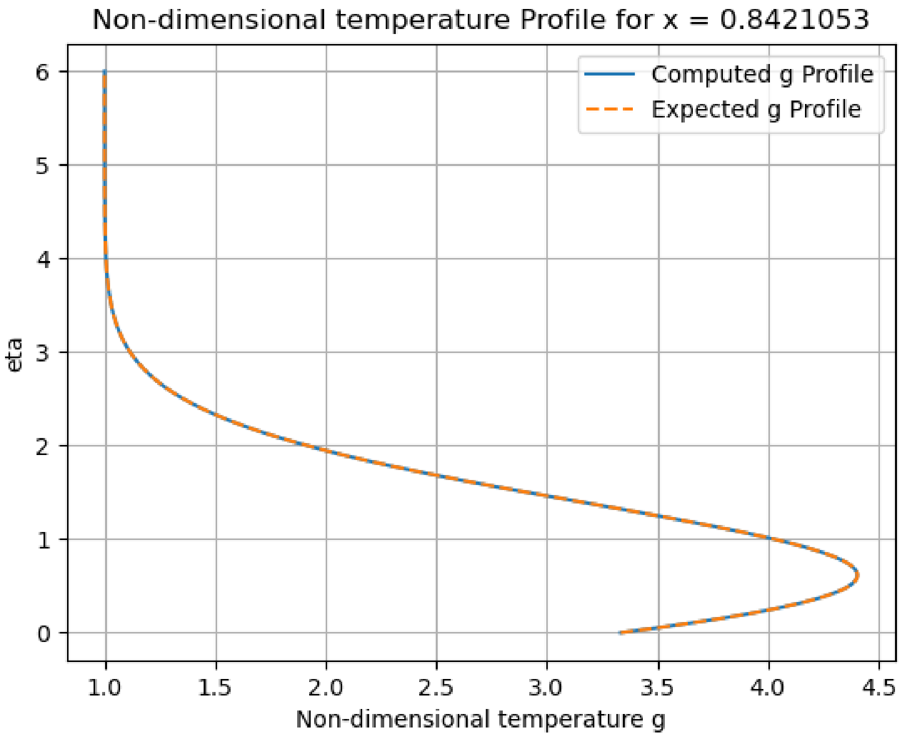
Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=1, given Tw

- $T_e = 1500\ K,\ P = 101325\ Pa,\ u_e = 5000\ m/s,\ T_w = 5000\ K$

$$T - T_w = (T_{0e} - T_w)\left(\frac{u}{u_e}\right) - \frac{\gamma - 1}{2}\mathrm{Ma}_e^2 T_e \left(\frac{u}{u_e}\right)^2$$



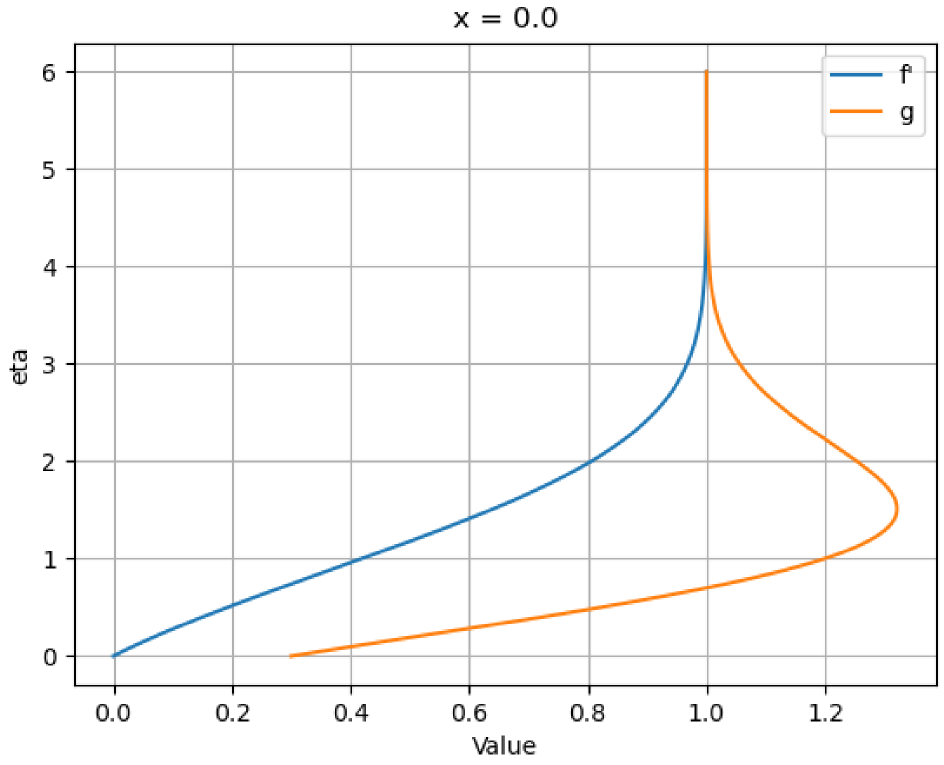


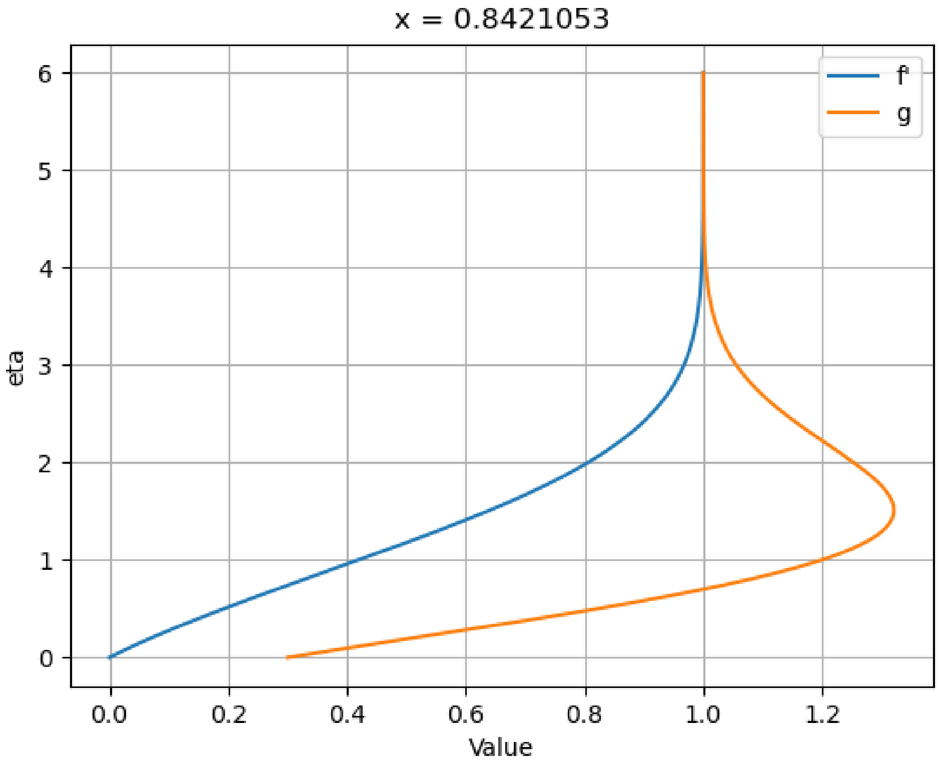
Max g error = 1.03185e-3
Max T error = 1.55 K
%Te error = 0.1%

Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=1, given Tw

- $T_e = 5000\ K, P = 101325\ Pa, u_e = 5000\ m/s, T_w = 1500\ K$ -> $Ma_e = 3.53$



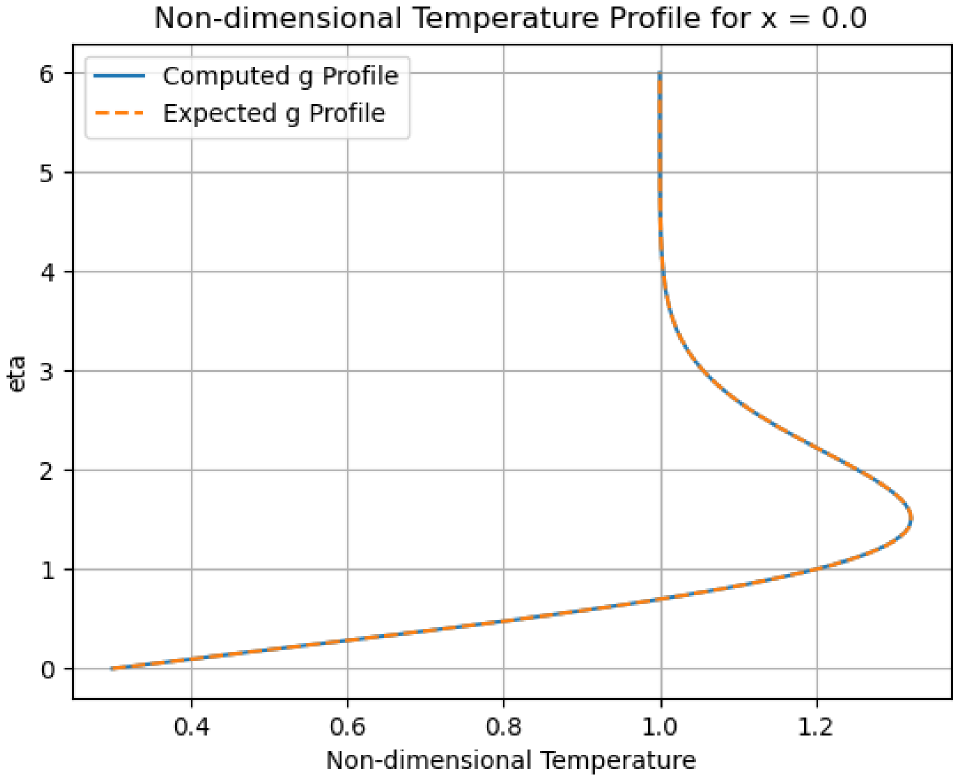


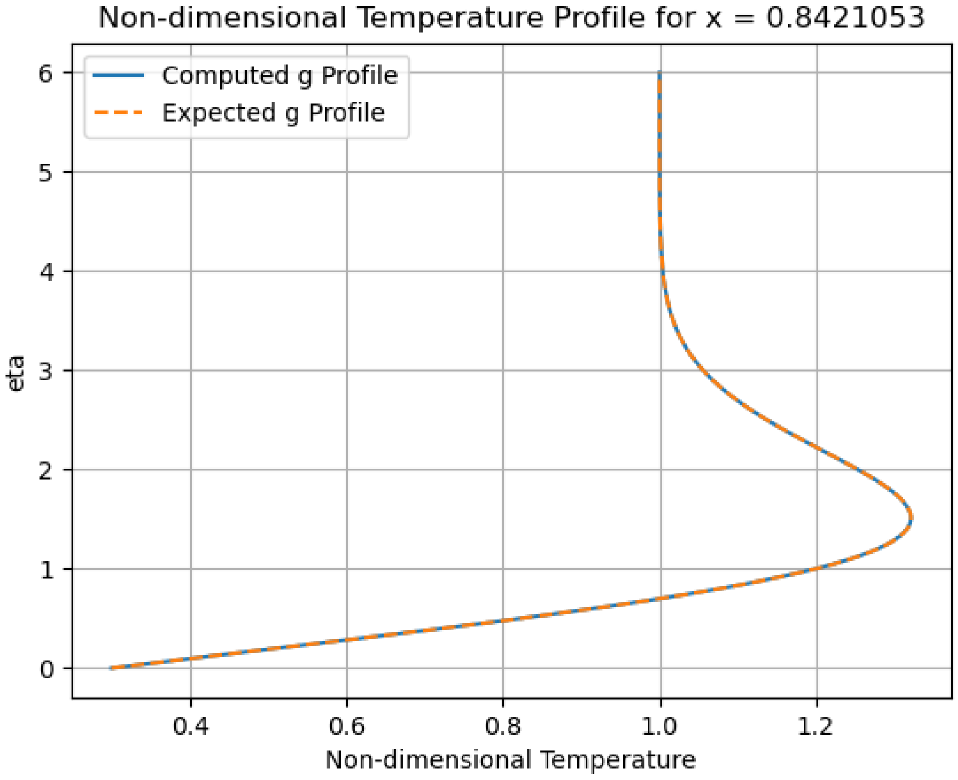
Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=1, given Tw

- $T_e = 5000\ K,\ P = 101325\ Pa,\ u_e = 5000\ m/s,\ T_w = 1500\ K$

$$T - T_w = (T_{0e} - T_w)\left(\frac{u}{u_e}\right) - \frac{\gamma - 1}{2}\mathrm{Ma}_e^2 T_e \left(\frac{u}{u_e}\right)^2$$





Max g error = 3.095e-4
Max T error = 1.54 K
%Tw error = 0.1%

Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=1, adiabatic

- **Calorically perfect and ideal gas: air,** $C_p = 1005\,J/KgK,\ \gamma = 1.4,\ R = 287\,J/KgK$

- **Flat plate, no pressure gradient->constant edge velocity->constant edge temperature**

$$\frac{dp_e}{dx} = 0 \text{ -> } \frac{du_e}{dx} = -\frac{1}{\rho_e u_e}\frac{dp_e}{dx} = 0 \text{ ->} u_e = const.\ h_0 = C_p T_0 = const = C_p T_e + \frac{1}{2}\frac{{u_e}^2}{C_p} \text{ -> } T_e = const$$

- **Adiabatic wall:** $q_w = -k\frac{\partial T}{\partial y} = 0$ **->** $\frac{\partial g}{\partial \eta} = 0$

$$\boxed{\begin{aligned} \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} &= \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \\ \rho u\frac{\partial T_0}{\partial x} + \rho v\frac{\partial T_0}{\partial y} &= \frac{\partial}{\partial y}\left(k\frac{\partial T_0}{\partial y}\right) \end{aligned}}$$

- $Pr = 1$**-> viscosity proportional to thermal conductivity:** $k = \frac{\mu C_p}{Pr} = \mu C_p$

- **Viscosity follows a power law:** $\mu = \mu_0\left(\frac{T}{T_0}\right)^n, \mu_0 = 1.72\cdot 10^{-5}, T_0 = 0\,°C,\ n = 0.7$

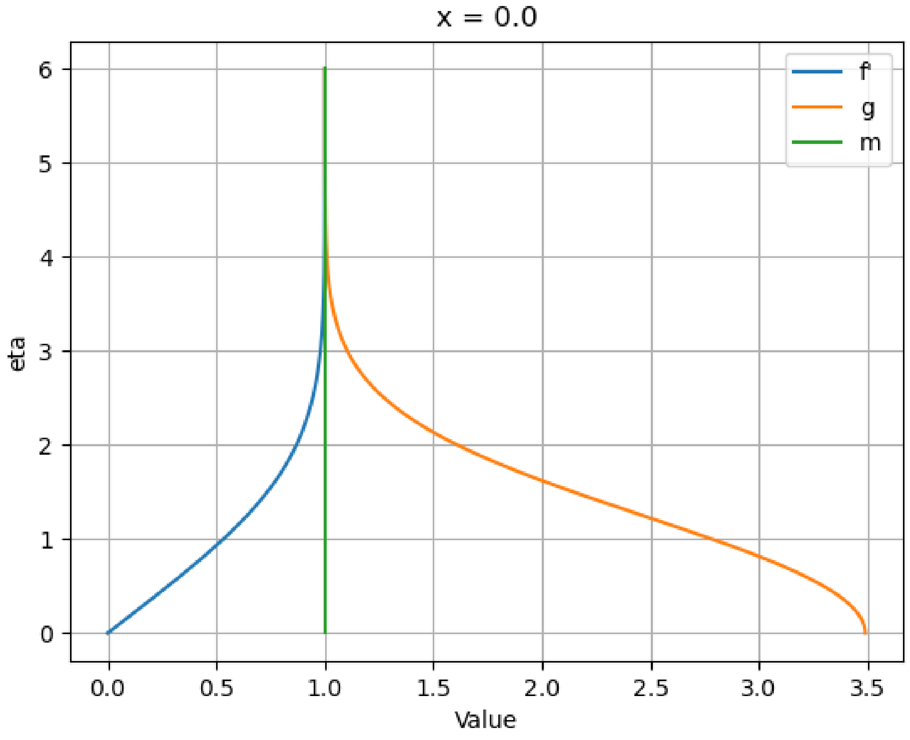
- **Boundary layer with 20 points along x, from 0 to 1, and 300 points along y, with eta from 0 to 6**

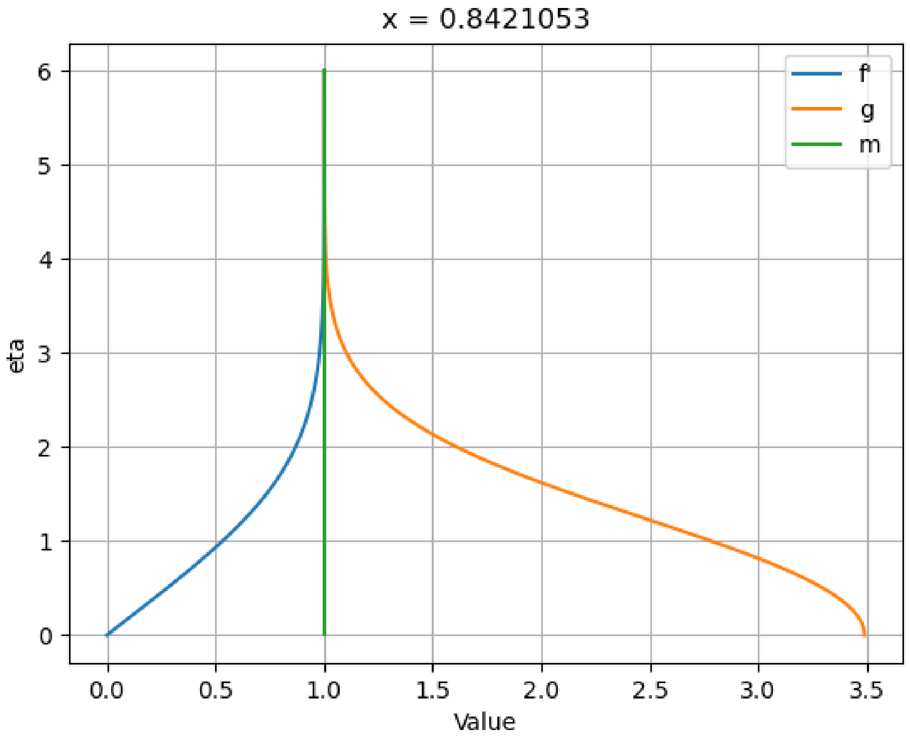
Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=1, adiabatic

$T_e = 5000\ K$, $P = 101325\ Pa$, $u_e = 5000\ m/s$ -> $Ma_e = 3.52$

$$\boxed{m = \frac{T_0}{T_{0e}} = \frac{T_e}{T_{0e}} g + \frac{{u_e}^2}{2C_p T_{0e}} f'^2}$$



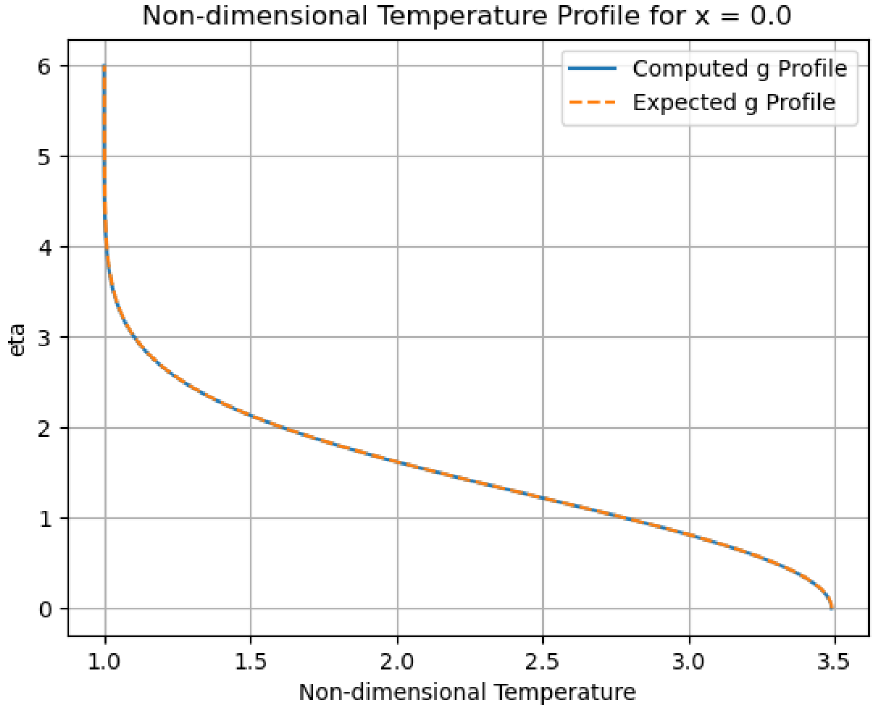


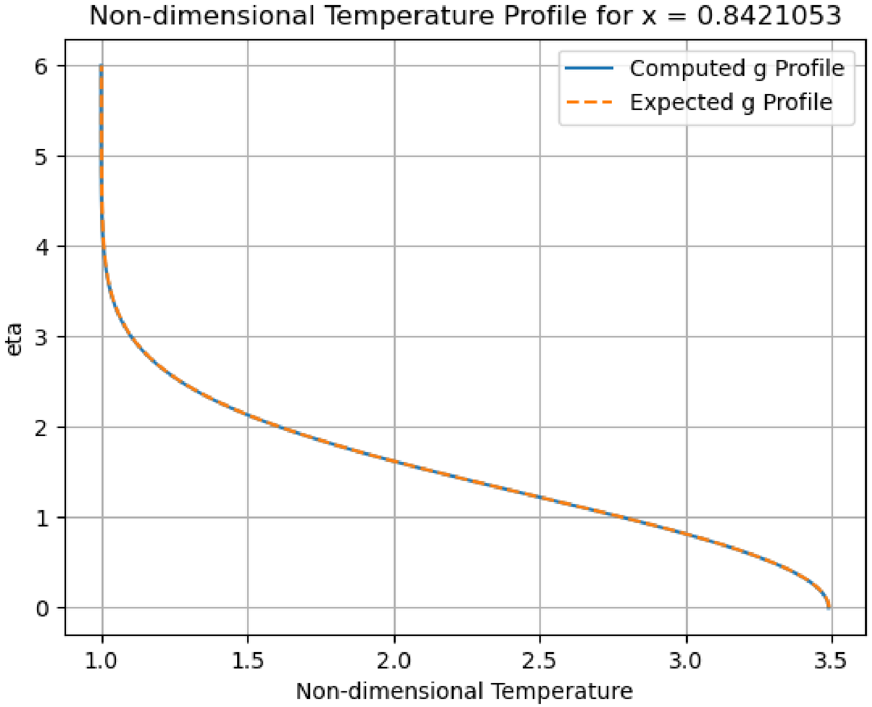
Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=1, adiabatic

$T_e = 5000\ K, P = 101325\ Pa, u_e = 5000\ m/s$ -> $Ma_e = 3.52$

$$T = T_0 - \frac{u^2}{2C_p}$$





Max g error = 1.238e-3
Max T error = 6.19 K
%Te error = 0.13%

Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, variable Pr, given Tw

- Calorically perfect and ideal gas: air, $C_p = 1005\, J/KgK,\, \gamma = 1.4,\, R = 287\, J/KgK$

- Flat plate, no pressure gradient->constant edge velocity->constant edge temperature

$$\frac{dp_e}{dx} = 0 \;\text{->}\; \frac{du_e}{dx} = -\frac{1}{\rho_e u_e}\frac{dp_e}{dx} = 0 \;\text{->} u_e = const.\; h_0 = C_p T_0 = const = C_p T_e + \frac{1}{2}\frac{{u_e}^2}{C_p} \;\text{->}\; T_e = const$$

$$(\mathcal{C}f'')' + ff'' = \beta\left[(f')^2 - \frac{\rho_e}{\rho}\right]$$

$$\left(\frac{\mathcal{C}}{\mathrm{Pr}}g'\right)' + fg' = \frac{2\xi}{h_e}\left[-f'g\frac{\mathrm{d}h_e}{\mathrm{d}\xi} + \frac{\rho_e\mu_e}{\rho}f'\frac{\mathrm{d}u_e}{\mathrm{d}\xi}\right] - \mathcal{C}\mathrm{Ec}(f'')^2$$

- T_w is given

- $Pr = const$-> viscosity proportional to thermal conductivity: $k = \frac{\mu C_p}{Pr}$

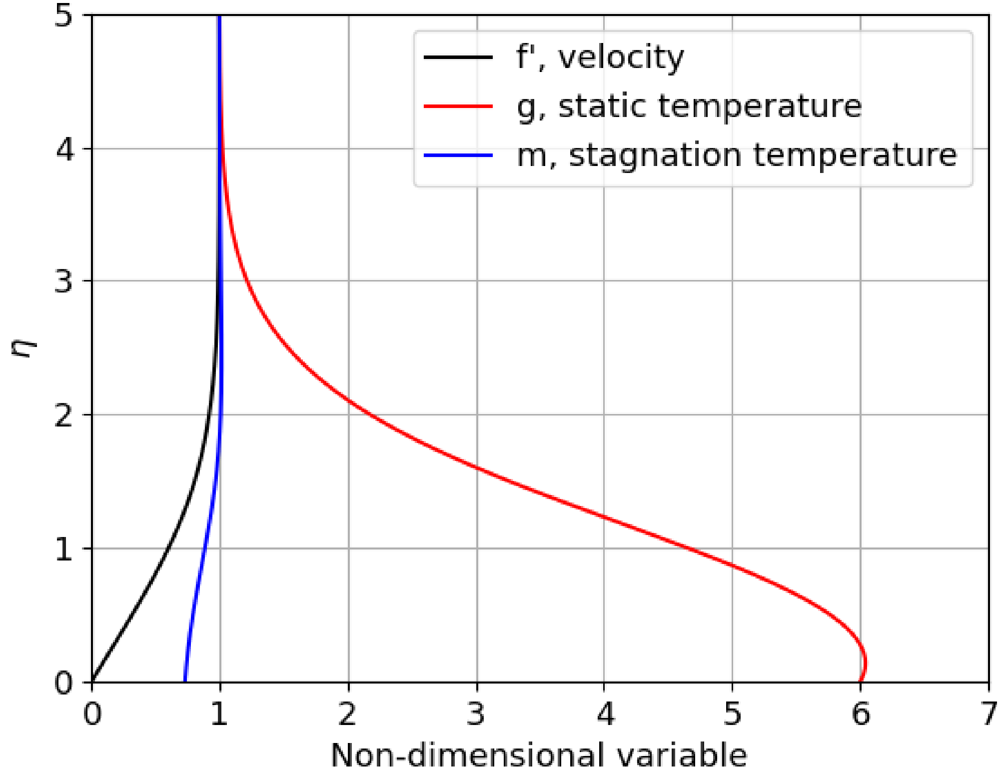
- Viscosity follows a power law: $\mu = \mu_0\left(\frac{T}{T_0}\right)^n, \mu_0 = 1.72\cdot 10^{-5}, T_0 = 0\,°C,\; n = 0.7$

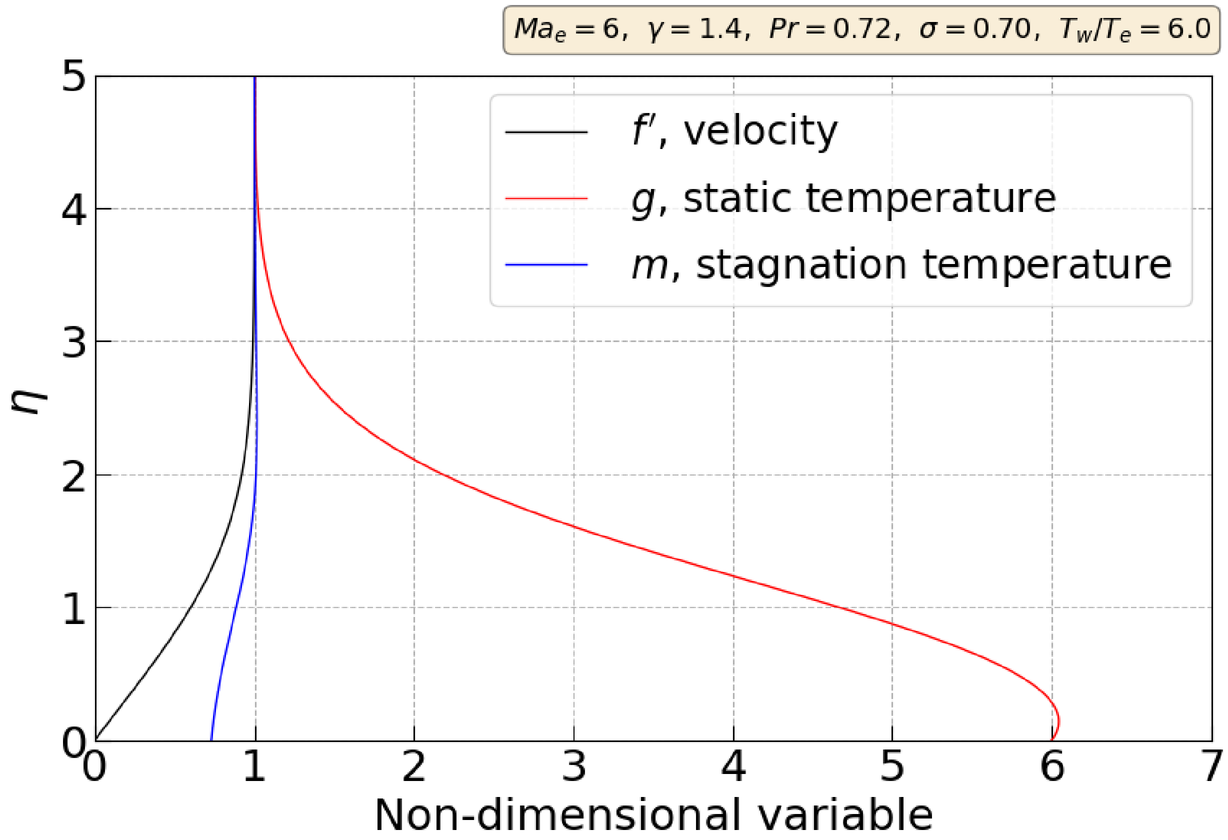
- Boundary layer with 20 points along x, from 0 to 1, and 300 points along y, with eta from 0 to 6

Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=0.72, given Tw

$$Ma_e = 6,\ T_e = 1500\ K,\ (P = 101325\ Pa)\ \text{->}\ u_e = 4658\ m/s, \qquad \frac{T_w}{T_e} = 6\ \rightarrow T_w = 9000\ K,\ Pr = 0.72$$

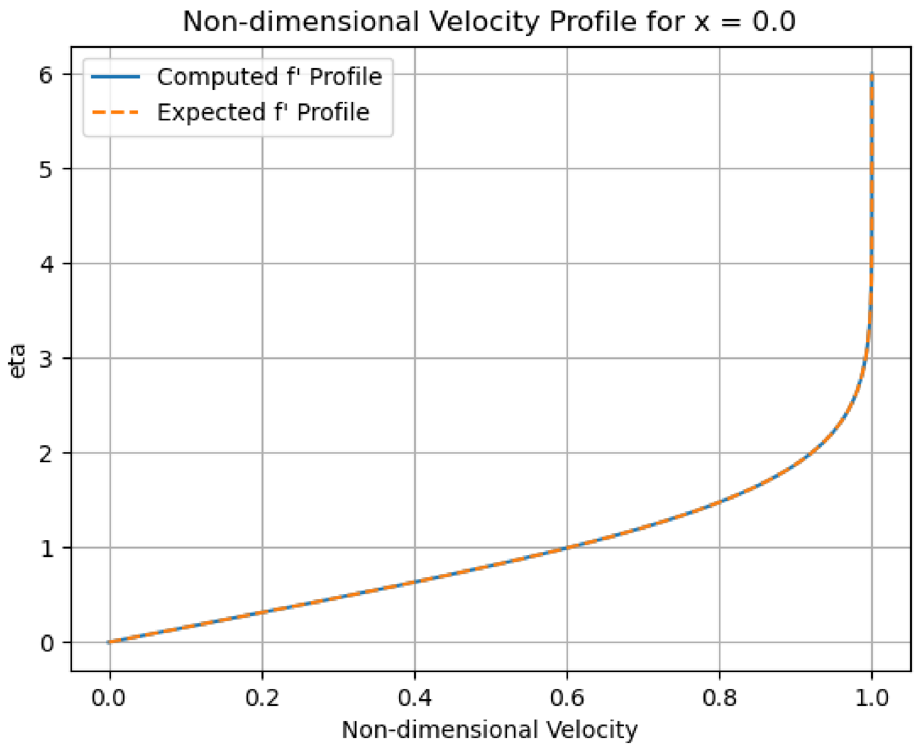


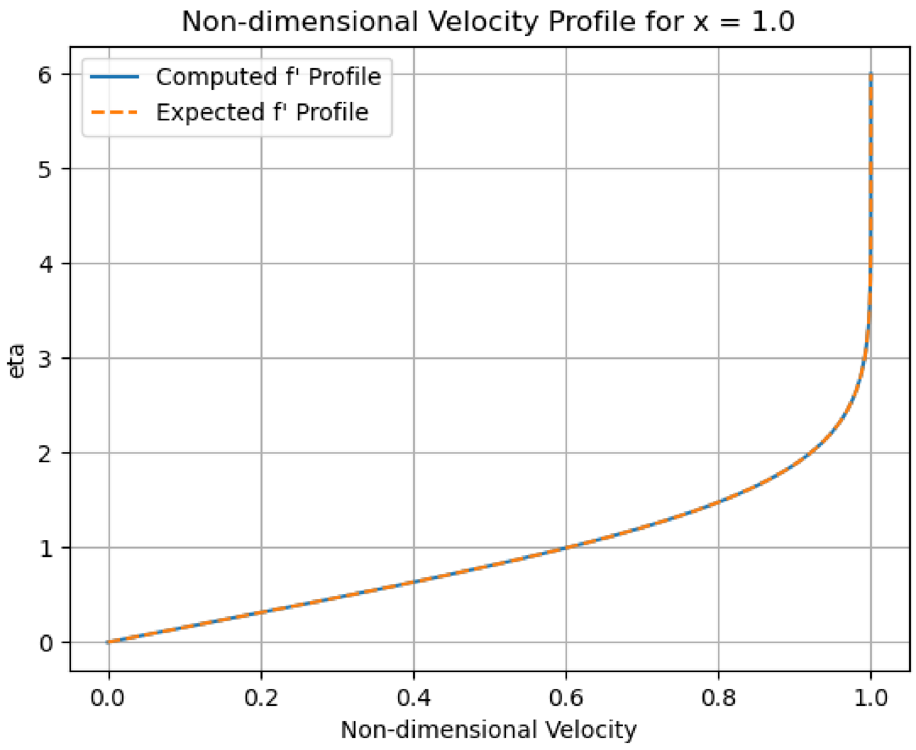


Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=0.72, given Tw

$$Ma_e = 6,\ T_e = 1500\ K,\ (P = 101325\ Pa) \rightarrow u_e = 4658\ m/s, \quad \frac{T_w}{T_e} = 6\ \rightarrow T_w = 9000\ K,\ Pr = 0.72$$

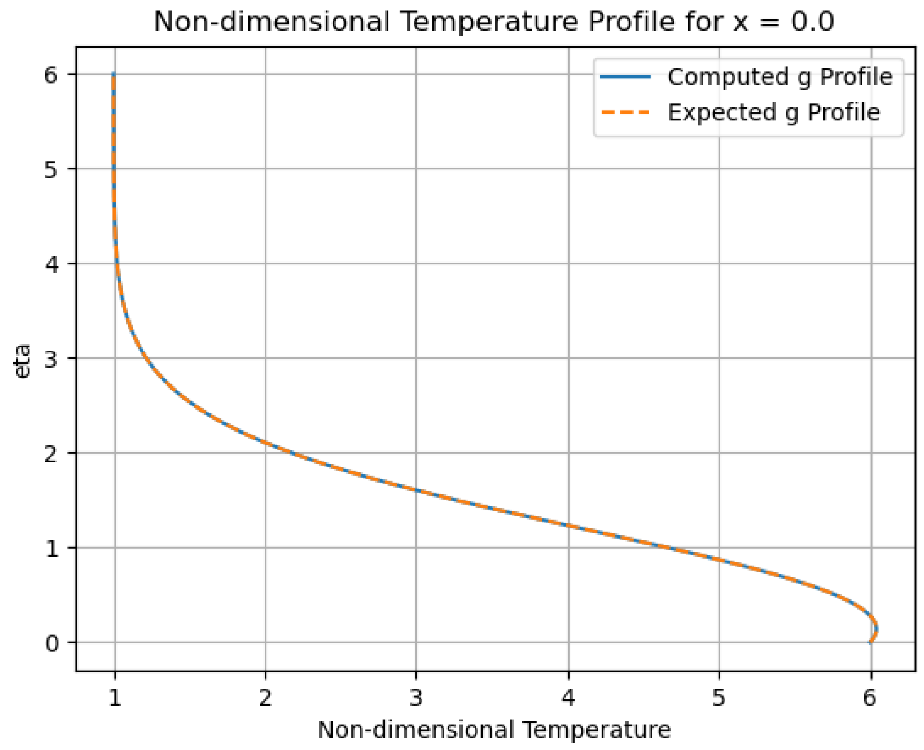


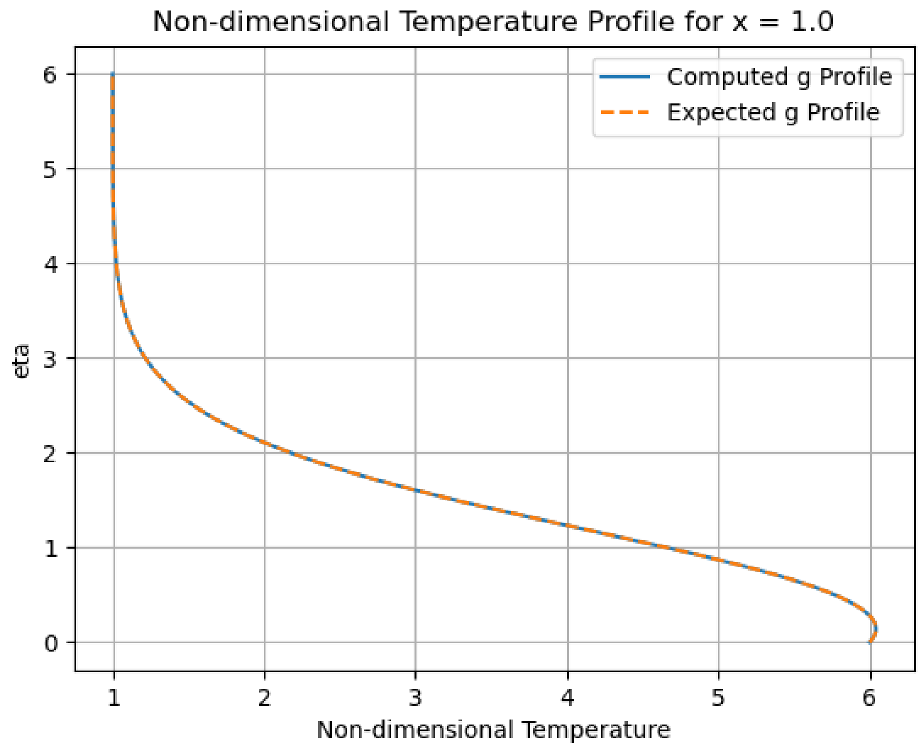


Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=0.72, given Tw

$$Ma_e = 6,\ T_e = 1500\ K, (P = 101325\ Pa) \text{ -> } u_e = 4658\ m/s, \quad \frac{T_w}{T_e} = 6\ \rightarrow T_w = 9000\ K, Pr = 0.72$$





Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=0.72, given Tw

$$Ma_e = 6,\ T_e = 1500\ K, (P = 101325\ Pa) \rightarrow u_e = 4658\ m/s, \quad \frac{T_w}{T_e} = 6\ \rightarrow T_w = 9000\ K, Pr = 0.72$$

Error on non-dimensional velocity profile f': 6.95e-6
Error on velocity profile u: 0.033 m/s
%Error on velocity profile:0.000695%

Error on non-dimensional temperature profile g: 6.98e-4
Error on temperature profile T: 1.04 K
%Error on temperature profile:0.0698%

Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, variable Pr, adiabatic wall

- **Calorically perfect and ideal gas: air, $C_p = 1005\, J/KgK,\ \gamma = 1.4,\ R = 287\, J/KgK$**
- **Flat plate, no pressure gradient->constant edge velocity->constant edge temperature**

$$\frac{dp_e}{dx} = 0 \ \text{->}\ \frac{du_e}{dx} = -\frac{1}{\rho_e u_e}\frac{dp_e}{dx} = 0 \ \text{->} u_e = const.\ h_0 = C_p T_0 = const = C_p T_e + \frac{1}{2}\frac{{u_e}^2}{C_p} \ \text{->}\ T_e = const$$

- **Adiabatic wall:** $q_w = -k\frac{\partial T}{\partial y} = 0$ **->** $\frac{\partial g}{\partial \eta} = 0$

$$(\mathcal{C} f'')' + f f'' = \beta\left[(f')^2 - \frac{\rho_e}{\rho}\right]$$

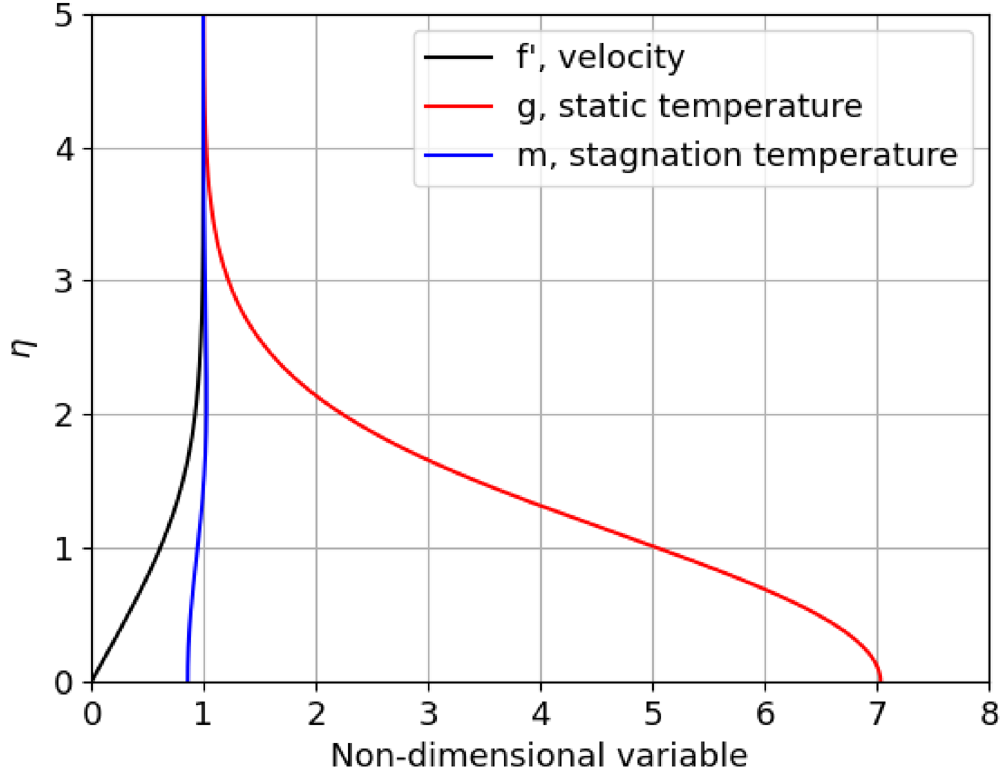
$$\left(\frac{\mathcal{C}}{\mathrm{Pr}} g'\right)' + f g' = \frac{2\xi}{h_e}\left[-f' g \frac{\mathrm{d}h_e}{\mathrm{d}\xi} + \frac{\rho_e \mu_e}{\rho} f' \frac{\mathrm{d}u_e}{\mathrm{d}\xi}\right] - \mathcal{C}\mathrm{Ec}(f'')^2$$

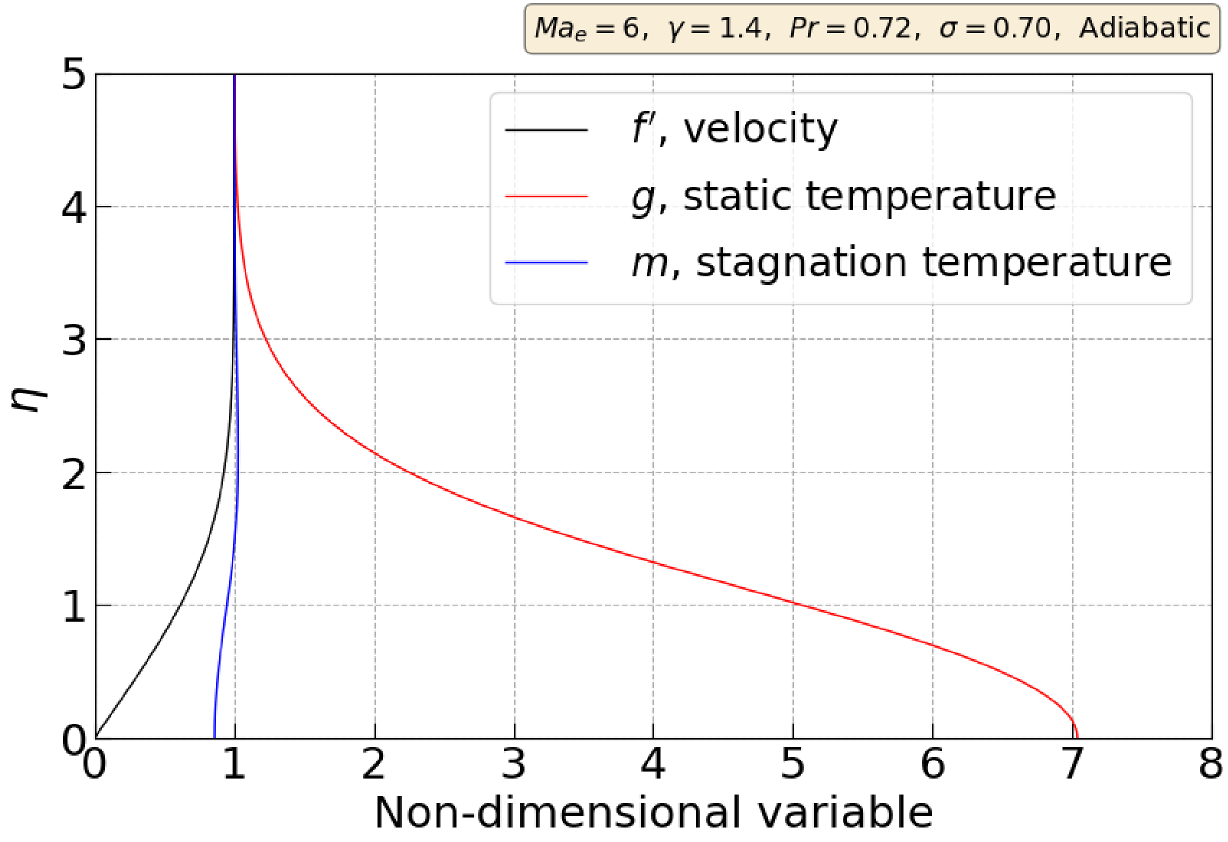
- $Pr = const$**-> viscosity proportional to thermal conductivity:** $k = \frac{\mu C_p}{Pr}$
- **Viscosity follows a power law:** $\mu = \mu_0 \left(\frac{T}{T_0}\right)^n, \mu_0 = 1.72 \cdot 10^{-5}, T_0 = 0\,°C,\ n = 0.7$
- **Boundary layer with 20 points along x, from 0 to 1, and 300 points along y, with eta from 0 to 6**

Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=0.72, adiabatic wall

$Ma_e = 6,\ T_e = 1500\ K, (P = 101325\ Pa)$ -> $u_e = 4658\ m/s$, adiabatic, $Pr = 0.72$

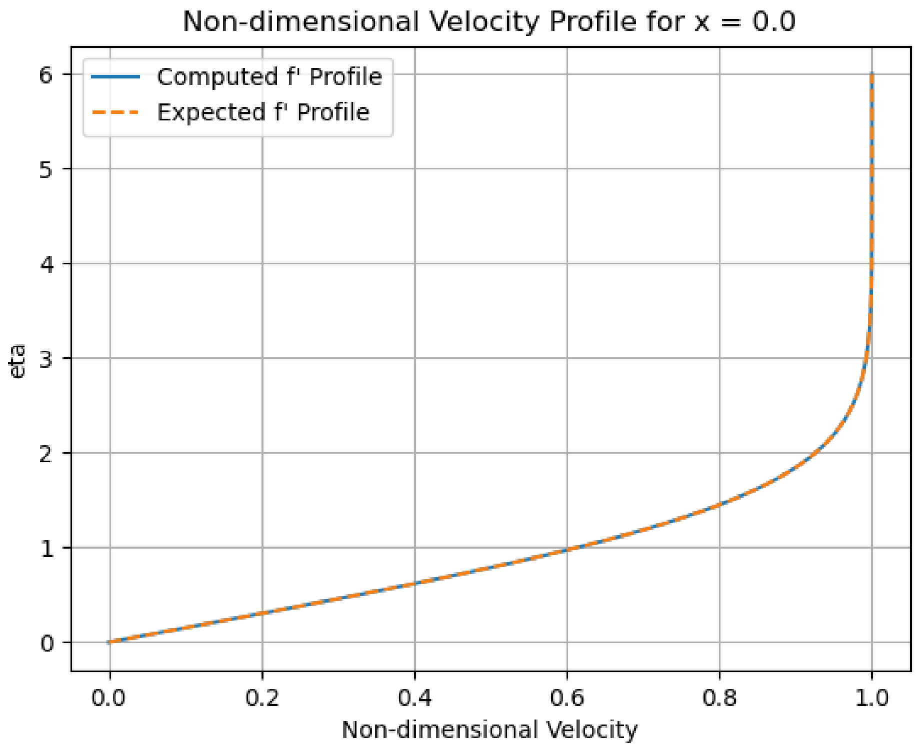


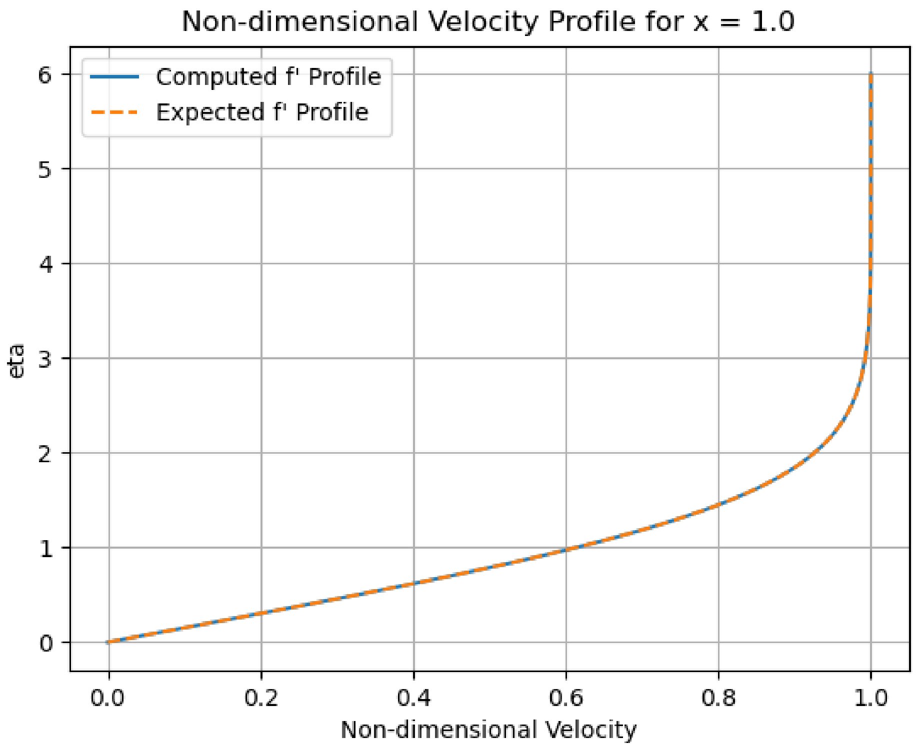


Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=0.72, , adiabatic wall

$Ma_e = 6,\ T_e = 1500\ K$, ($P = 101325\ Pa$) -> $u_e = 4658\ m/s$, adiabatic, $Pr = 0.72$

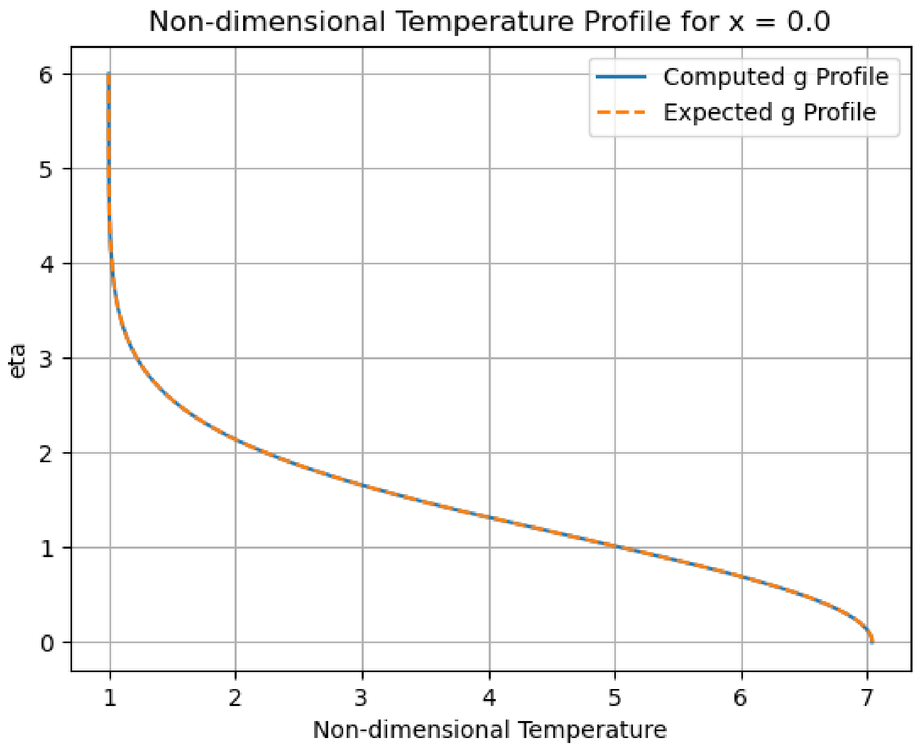


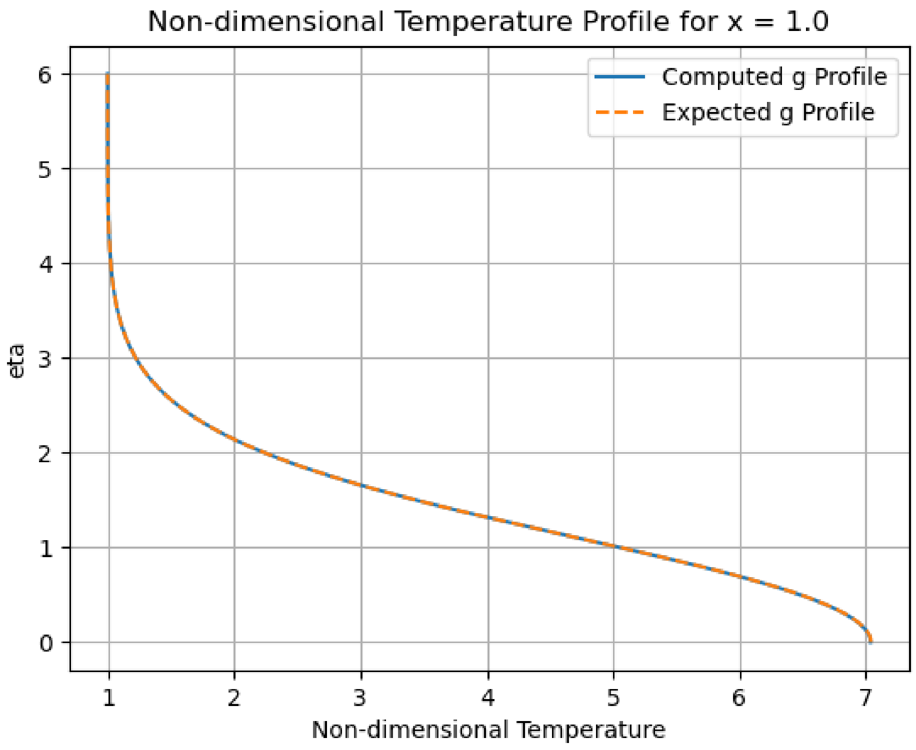


Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=0.72, , adiabatic wall

$Ma_e = 6,\ T_e = 1500\ K, (P = 101325\ Pa)$ -> $u_e = 4658\ m/s$, adiabatic, $Pr = 0.72$





Verification: Ideal Perfect Gas

Plate flow, no pressure gradient, Pr=0.72, adiabatic

$Ma_e = 6,\ T_e = 1500\ K$, $(P = 101325\ Pa)$ -> $u_e = 4658\ m/s$, adiabatic, $Pr = 0.72$

Error on non-dimensional velocity profile f': 3.15e-5
Error on velocity profile u: 0.14 m/s
%Error on velocity profile:0.00315%

Error on non-dimensional temperature profile g: 3.0e-3
Error on temperature profile T: 4.5 K
%Error on temperature profile:0.3%