Aerothermodynamics of a General One-Dimensional Compressible Flow in a Duct with Area Change, Friction, Heat Transfer, Rotation, Internal Choking, and Normal Shocks: Modeling and Solution

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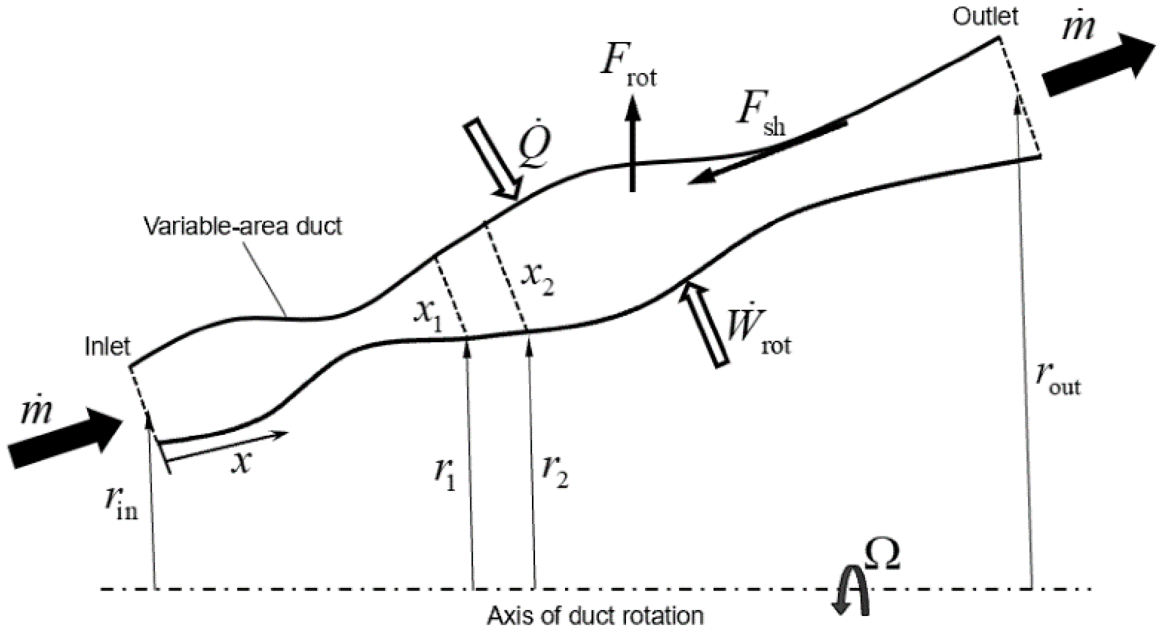
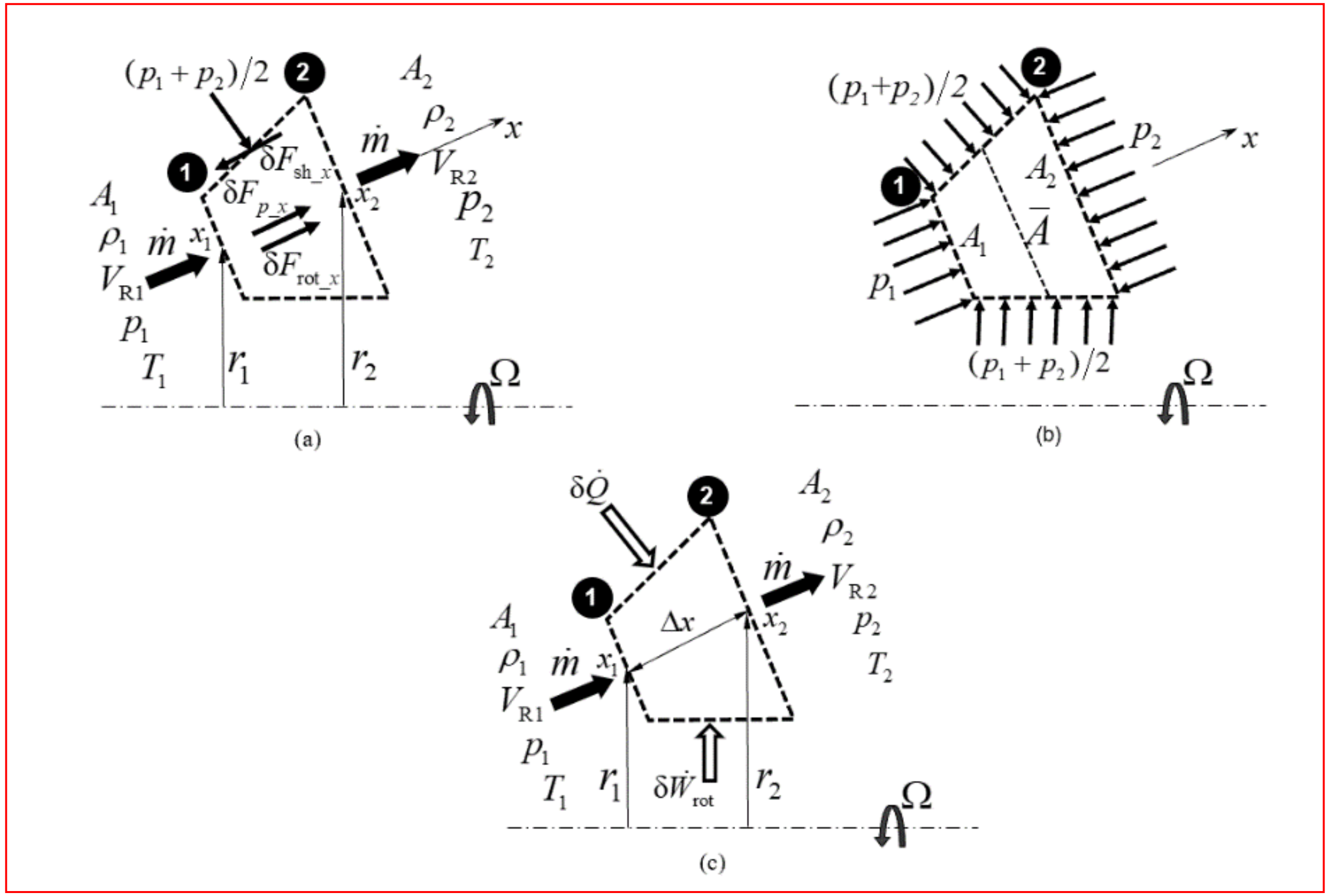
1-D Compressible Flow in a Duct with Combined Effects (1)

- **Variable-area duct with friction, heat transfer, rotation, choking, and normal shocks.**
- **Combined effects are <u>nonlinearly</u> coupled—the results are not obtainable by linear superposition of individual effects.**
- **Fanno (only friction) and Rayleigh (only heat transfer) flows are not helpful!**



1-D Compressible Flow in a Duct with Combined Effects (2)

- **Control Volumes between Sections at $x = x_1$ and $x = x_2$:**
  - **(a) Force-momentum balance**
  - **(b) Pressure distribution**
  - **(c) Energy conservation**



1-D Compressible Flow in a Duct with Combined Effects (3)

Mass Conservation (Continuity Equation)

$$\dot{m} = \rho_1 V_{R1} A_1 = \bar{\rho} \bar{V}_R \bar{A} = \rho_2 V_{R2} A_2$$

where

$$\bar{\rho} = \frac{\rho_1 + \rho_2}{2}$$

$$\bar{V}_R = \frac{V_{R1} + V_{R2}}{2}$$

$$\bar{A} = \frac{A_1 + A_2}{2}$$

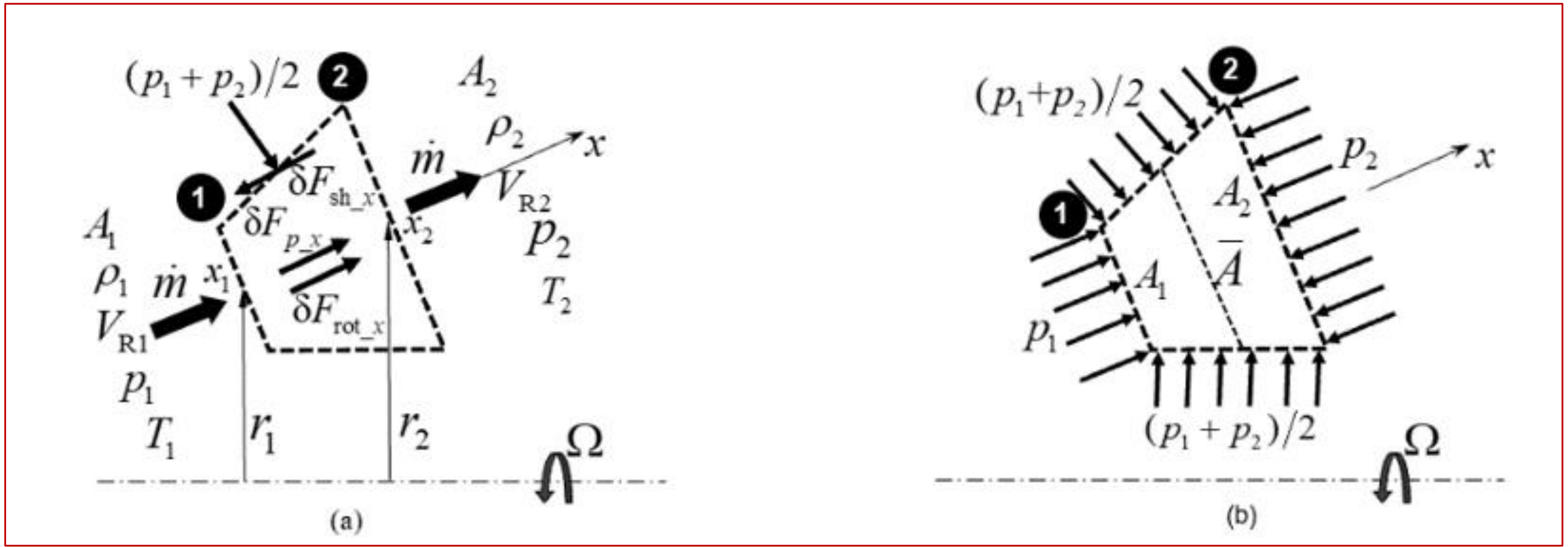
1-D Compressible Flow in a Duct with Combined Effects (4)

Mass Conservation (Continuity Equation)

$$\dot{m} = \frac{\hat{F}_{f0R1} A_1 p_{0R1}}{\sqrt{RT_{0R1}}} = \frac{\hat{F}_{f0R2} A_2 p_{0R2}}{\sqrt{RT_{0R2}}}$$

$$\dot{m} = \frac{\hat{F}_{f1} A_1 p_1}{\sqrt{RT_{0R1}}} = \frac{\hat{F}_{f2} A_2 p_2}{\sqrt{RT_{0R2}}}$$

Linear Momentum Equation



$$\delta F_{p\_x} - \delta F_{\text{sh}\_x} + \delta F_{\text{rot}\_x} = \dot{m} V_{\text{R}2} - \dot{m} V_{\text{R}1} = \dot{m}(V_{\text{R}2} - V_{\text{R}1})$$

$\delta F_{p\_x} \equiv$ **Pressure force;** $\delta F_{\text{sh}\_x} \equiv$ **Shear force;** $\delta F_{\text{rot}\_x} \equiv$ **Rotational body force**

- **Pressure force (δF_{p_x})**

Assuming average pressure on the lateral surface $=(p_1 + p_2)/2$**, we obtain**

$$\delta F_{p\_x} = p_1 A_1 + \frac{1}{2}(p_1 + p_2)(A_2 - A_1) - p_2 A_2$$

$$\delta F_{p\_x} = p_1 \left(\frac{A_1 + A_2}{2} \right) - p_2 \left(\frac{A_1 + A_2}{2} \right)$$

$$\boxed{\delta F_{p\_x} = (p_1 - p_2)\left(\frac{A_1 + A_2}{2} \right) = (p_1 - p_2)\bar{A}}$$

1-D Compressible Flow in a Duct with Combined Effects (7)

- **Shear force ($\delta F_{\mathrm{sh\_}x}$)**

$$\delta F_{\mathrm{sh\_}x} = \bar{A}\,\bar{f}\,\frac{\Delta x}{D_{\mathrm{h}}}\,\frac{1}{2}\,\bar{\rho}\bar{V}_{\mathrm{R}}^{2}$$

where $\bar{f}$ is the average value of the Darcy friction factor over the lateral control surface.

- **Rotational body force ($\delta F_{\mathrm{rot\_}x}$)**

$$\delta F_{\mathrm{rot\_}x} = \bar{A}\bar{\rho}\,\Omega^{2}\left(\frac{r_2^2 - r_1^2}{2}\right)$$

1-D Compressible Flow in a Duct with Combined Effects (8)

- **The linear momentum equation**

$$\delta F_{p\_x} - \delta F_{\text{sh}\_x} + \delta F_{\text{rot}\_x} = \dot{m} V_{\text{R}2} - \dot{m} V_{\text{R}1} = \dot{m}(V_{\text{R}2} - V_{\text{R}1})$$

becomes

$$(p_1 - p_2)\bar{A} - \bar{A}\,\bar{f}\,\frac{\Delta x}{D_{\text{h}}}\frac{1}{2}\bar{\rho}\bar{V}_{\text{R}}^2 + \bar{A}\bar{\rho}\,\Omega^2\left(\frac{r_2^2 - r_1^2}{2}\right) = \dot{m}(V_{\text{R}2} - V_{\text{R}1})$$

$$(p_1 - p_2) - \bar{f}\,\frac{\Delta x}{D_{\text{h}}}\frac{1}{2}\bar{\rho}\bar{V}_{\text{R}}^2 + \bar{\rho}\,\Omega^2\left(\frac{r_2^2 - r_1^2}{2}\right) = \frac{\dot{m}(V_{\text{R}2} - V_{\text{R}1})}{\bar{A}}$$

giving

$$p_2 = p_1 - \bar{f}\,\frac{\Delta x}{D_{\text{h}}}\frac{1}{2}\bar{\rho}\bar{V}_{\text{R}}^2 + \bar{\rho}\,\Omega^2\left(\frac{r_2^2 - r_1^2}{2}\right) - \frac{\dot{m}(V_{\text{R}2} - V_{\text{R}1})}{\bar{A}}$$

1-D Compressible Flow in a Duct with Combined Effects (9)

Thus, we can write

$$p_2 = p_1 - \Delta p_{\mathrm{f}} + \Delta p_{\mathrm{rot}} - \Delta p_{\mathrm{mom}}$$

where

$$\Delta p_{\mathrm{f}} = \bar{f}\,\frac{\Delta x}{D_{\mathrm{h}}}\,\frac{1}{2}\,\bar{\rho}\bar{V}_{\mathrm{R}}^{2}$$

$$\Delta p_{\mathrm{rot}} = \bar{\rho}\,\Omega^{2}\left(\frac{r_2^2 - r_1^2}{2}\right)$$

$$\Delta p_{\mathrm{mom}} = \frac{\dot{m}(V_{\mathrm{R2}} - V_{\mathrm{R1}})}{\bar{A}}$$

Note

We can also obtain Δp_{rot} by integrating the radial equilibrium equation $\mathrm{d}p/\mathrm{d}r = \bar{\rho} r\,\Omega^2$ over the CV from section 1 to section 2.

1-D Compressible Flow in a Duct with Combined Effects (10)

Energy Equation

$$T_{0R2} = T_{0R1} + \left(\Delta T_{0R}\right)_{ht} + \left(\Delta T_{0R}\right)_{rot} + \left(\Delta T_{0R}\right)_{CCT}$$

$\left(\Delta T_{0R}\right)_{ht} \equiv$ **Change in total temperature due to heat transfer**

$\left(\Delta T_{0R}\right)_{rot} \equiv$ **Change in total temperature due to rotational work transfer**

$\left(\Delta T_{0R}\right)_{CCT} \equiv$ **Heat transfer and work transfer coupling correction term**

- **Heat Transfer**

$$\frac{T_{\mathrm{w}} - T_{0\mathrm{R2}}}{T_{\mathrm{w}} - T_{0\mathrm{R1}}} = e^{-\eta}$$

where

$$\eta = (A_{\mathrm{w}} h)/(\dot{m} c_p)$$

giving

$$\left(\Delta T_{0\mathrm{R}}\right)_{\mathrm{ht}} = (T_{0\mathrm{R}2} - T_{0\mathrm{R}1})_{\mathrm{ht}} = (T_{\mathrm{w}} - T_{0\mathrm{R}1})(1 - e^{-\eta})$$

- **Rotational Work Transfer**

Method 1—Using Constancy of Rothalpy:

$$I = T_{0R2} - \frac{\Omega^2 r_2^2}{2c_p} = T_{0R1} - \frac{\Omega^2 r_1^2}{2c_p}$$

giving

$$\left(\Delta T_{0R}\right)_{\text{rot}} = \left(T_{0R2} - T_{0R1}\right)_{\text{rot}} = \frac{\Omega^2 (r_2^2 - r_1^2)}{2c_p}$$

- **Coupling correction term – derived by Sultanian (2015)**

$$\left(\Delta T_{0R}\right)_{\text{CCT}} = -\frac{\Omega^2 (r_2 - r_1) r_1 \eta}{2c_p}$$

1-D Compressible Flow in a Duct with Combined Effects (13)

Energy Equation

$$T_{0R2} = T_{0R1} + \left(\Delta T_{0R}\right)_{\text{ht}} + \left(\Delta T_{0R}\right)_{\text{rot}} + \left(\Delta T_{0R}\right)_{\text{CCT}}$$

Finally, we obtain

$$T_{0R2} = \left\{T_w - (T_w - T_{0R1})\, e^{-\eta}\right\} + \frac{\Omega^2 (r_2^2 - r_1^2)}{2c_p} - \frac{\Omega^2 (r_2 - r_1) r_1 \eta}{2c_p}$$

Solution Method with Internal Choking and Normal Shocks

- **Treat the duct as a two-point (inlet-outlet) boundary value problem.**
- **Divide the duct into multiple, solution-adaptive, 1-D control volumes.**
- **Knowing the conditions at inlet, numerically march the solution to the outlet and iterate until the solution satisfies the outlet boundary conditions.**
- **Place a choked section ($M = 1$) to coincide with the boundary of two adjacent control volumes.**
- **Place an internal normal shock within a narrow control volume and use the normal shock relations to establish conditions at the outlet of this control volume.**
- **This solution method is valid for all Mach numbers!**

Concluding Remarks

- **The present method of modeling a general 1-D compressible flow in a variable-area duct with wall friction, heat transfer, rotation, internal choking, and normal shocks is fully physics-based.**
- **The iterative numerical solution method is robust and free from any convergence problem.**
- **The modeling and solution methods are valid for all Mach numbers!**
- **The present method can be used in a compressible duct flow network—gas turbine airfoil internal cooling and internal air systems design applications.**
- **The present method can be used for the primary validation of any related design tool—both commercial and inhouse!**

THANK YOU.