ANALYSIS OF DYNAMICAL SYSTEM BEHAVIORS OF LOOP HEAT PIPES

Triem T. Hoang
TTH Research Inc.
Clifton VA 20124

Thermal & Fluids Analysis Workshop
TFAWS 2023
August 21-25, 2023
NASA Goddard Space Flight Center
Greenbelt, MD

Outline

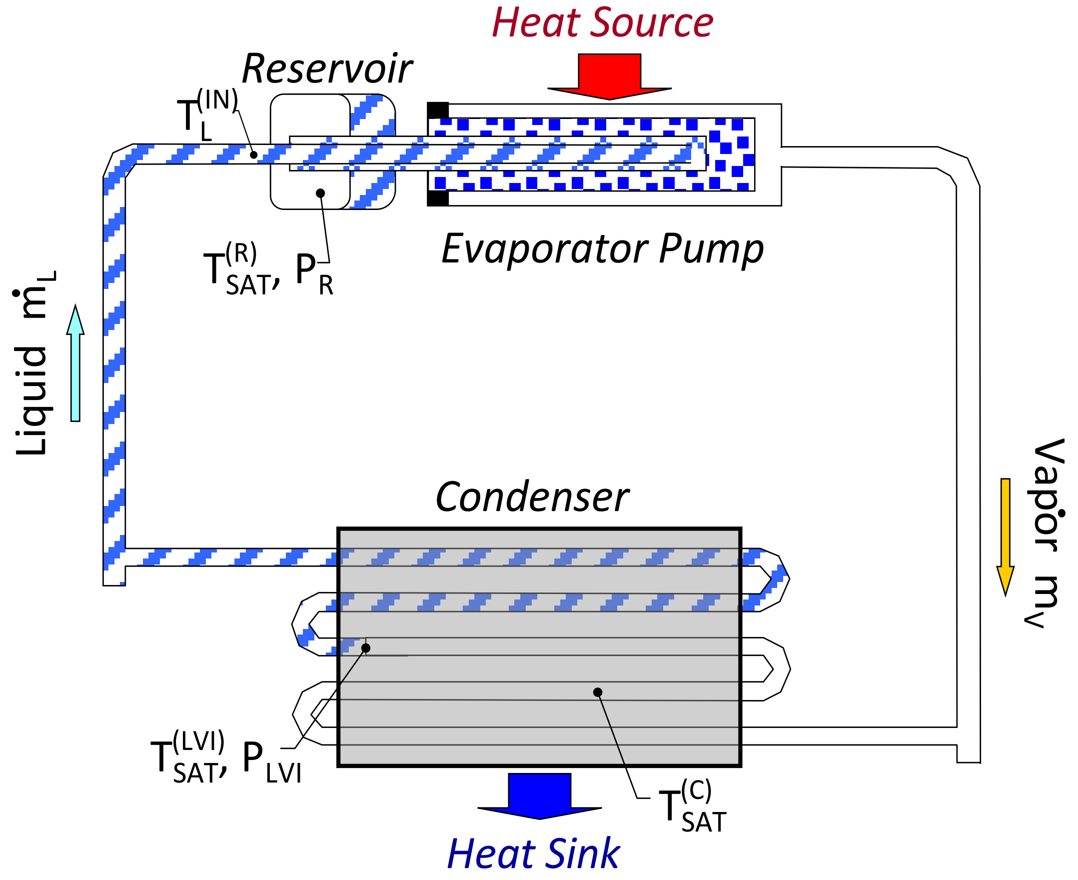
- **Overview of LHP Self-Excited Temperature Oscillations**
- **Development of LHP Linear Stability Theory and Test Data Verification**
- **Analysis of “Dynamical System” Behaviors in LHP Operations**
- **Stable amd Runtime Efficient Solution Method**
- **Model Simulations in Search of Hopf Bifurcation Points**
- **Discussion / Path Forward**
- **Summary**

Acknowledgements

The research endeavor presented herein was initiated and continually funded by the U.S. Naval Research Laboratory (NRL) from 2011 to 2017.

Technical Background

$$\dot{Q}_{IN} = \dot{Q}_1 + \dot{Q}_2$$



$$\dot{Q}_C^{(2\phi)} = \dot{Q}_1 + \dot{Q}_{ENV.}^{(V)}$$

$$\dot{Q}_C^{(L)} = \dot{Q}_2 + \dot{Q}_{ENV.}^{(L)}$$

Loop Heat Pipe (LHP)

- invented in the former Soviet Union in 1970s
- two-phase capillary-pumped heat transport – *no moving part*

Thermal-Fluid Interaction

- heat exchange with environment via components' casing
- fluid "movement" initiated by phase change
- thermal/fluid dynamics of fluid driven by thermal environment

Nominal Operation

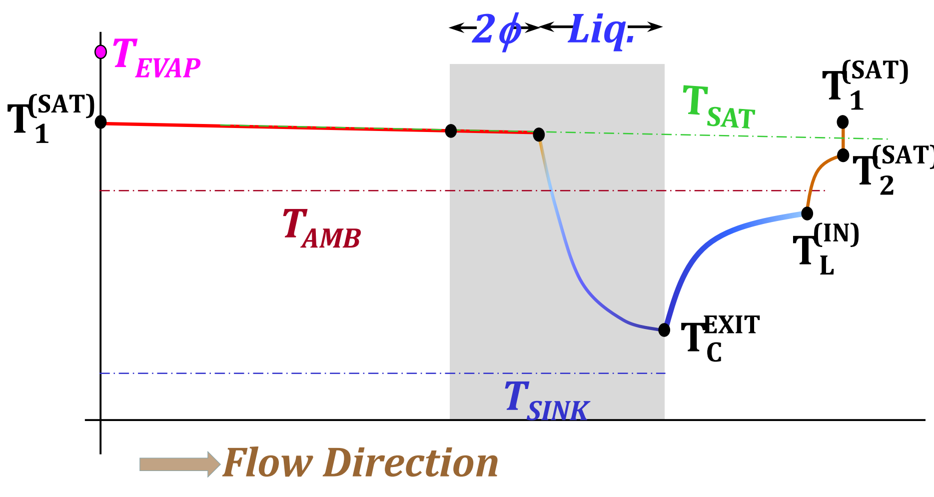
Energy Balance

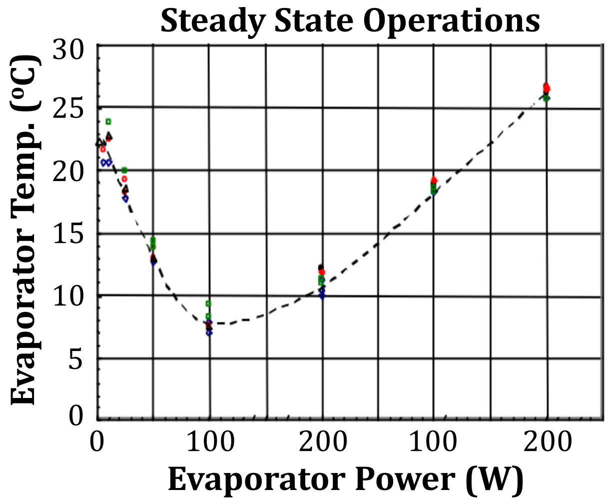
$$\dot{Q}_C^{(2\phi)} = \dot{Q}_1 + \dot{Q}_{ENV.}^{(V)}$$

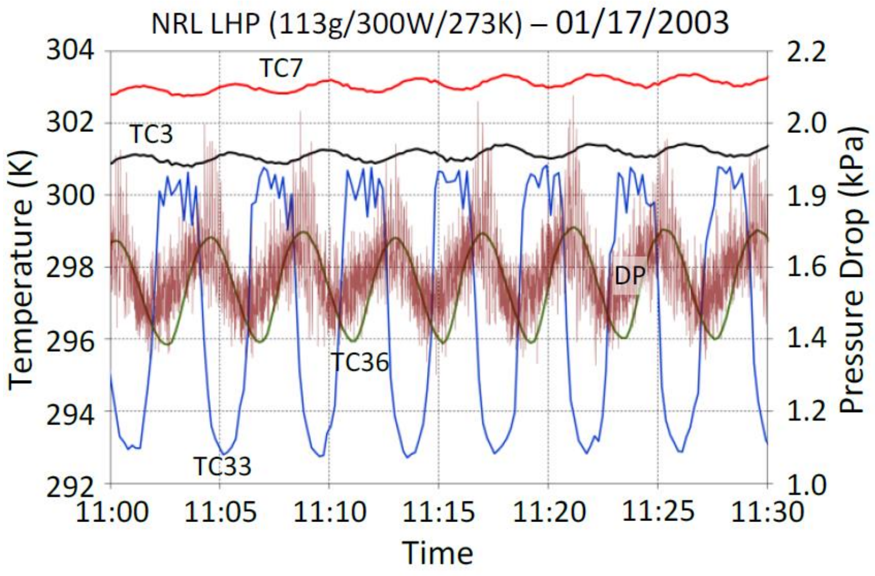
$$\dot{Q}_C^{(L)} = \dot{Q}_2 + \dot{Q}_{ENV.}^{(L)}$$

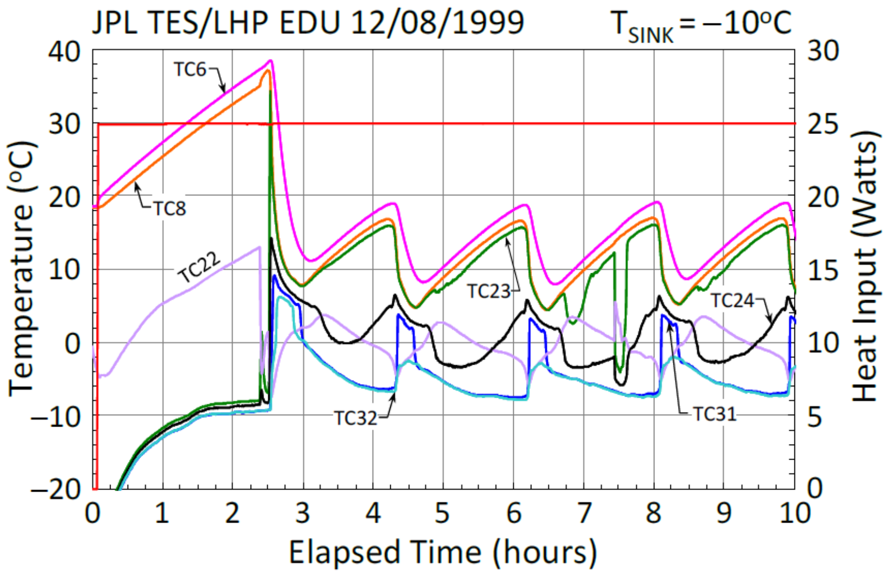
$$\dot{Q}_2 = G_W\left(T_1^{(SAT)} - T_2^{(SAT)}\right)$$

$$\dot{Q}_{ENV.}^{(L)} = \dot{m}_L c_P^{(L)}\left(T_L^{(IN)} - T_C^{EXIT}\right) + G_{A-R}\left(T_{AMB} - T_2^{(SAT)}\right)$$









Development of LHP Linear Stability Theory

High Frequency Low Amplitude (HFLA) Oscillations

$$\frac{\partial \dot{Q}_{IN}}{\partial \dot{T}_{SAT,SS}} > \frac{\dot{Q}_{SC,SS}}{\dot{Q}_{IN}} G_E + \frac{(Mc_P)_R}{\rho_V \lambda \left(V_{RES} - V_{VL} - V_{C,SS}^{(2\phi)}\right)} G_{C,SS}^{(2\phi)} + \frac{\tau_E}{\tau_R} \frac{\partial \dot{Q}_{IN}}{\partial T_{W,SS}^{(E)}} = \Psi_{SS}^{(1)}$$

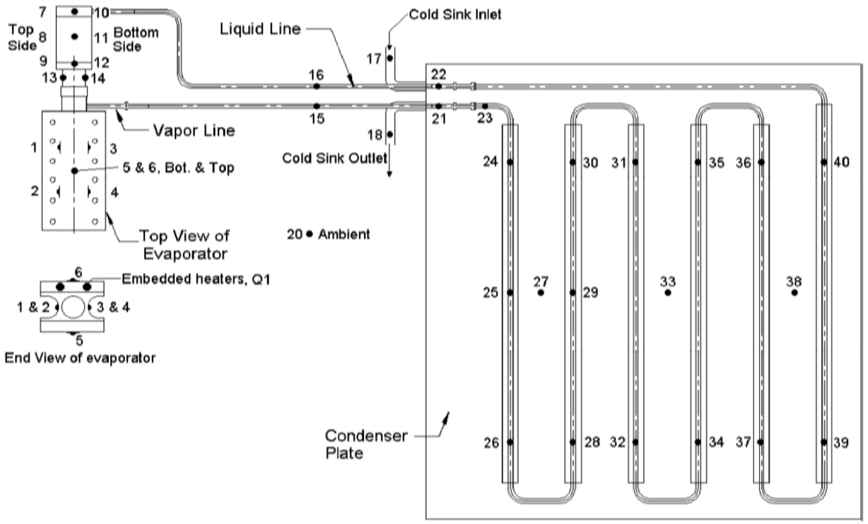
Low Frequency High Amplitude (LFHA) Oscillations

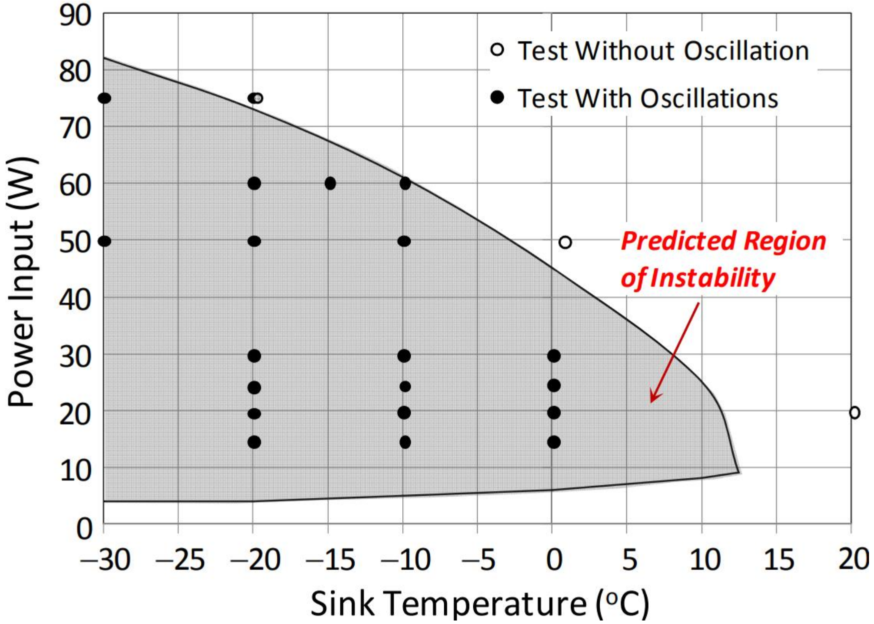
$$\frac{\partial T_{W,SS}^{(E)}}{\partial \dot{Q}_{IN}} < -\frac{1}{\frac{\rho_L}{\rho_V} \frac{\tau_E}{\tau_R} G_E \frac{\dot{Q}_{SC,SS}}{\dot{Q}_{IN}} \left(\frac{V_{RES}}{V_{RES} - V_{VL} - V_{C,SS}^{(2\phi)}}\right)} = -\Psi_{SS}^{(2)}$$

where

$$\tau_E = \frac{(Mc_P)_E}{G_E + G_{C,SS}^{(2\phi)}} \qquad \text{and} \qquad \tau_R = \frac{(Mc_P)_R}{(\dot{Q}_{SC,SS}/\dot{Q}_{1,SS})G_E}$$

Linear Stability Analysis of NASA/JPL LHP





Physical Dimensions and Properties of NASA/JPL LHP

Evaporator

Primary Wick

Material:	Sintered Powder Nickel
Outer Diameter:	24.21mm (0.950")
Inner Diameter:	9.525mm (0.375")
Active Length:	0.1524m (6")
Max. Pore Radius:	1.2µm
Permeability:	4.0×10^{-14}m^2
Effective Conductivity:	7.8W/m-K

Casing/Saddle, 1st Wick, and Attached Thermal Mass

Attached Thermal Mass:	9,080J/K
Thermal Mass-to-Vapor Conductance G_E:	8.16 W/K
Saddle: 7.62cm x 15.24cm x 1.91cm Al 6061	

Vapor Grooves

Number of Channels:	4
Hydraulic Diameter:	0.05"

Transport Lines

Vapor Line

Outer Diameter:	5.54mm
Wall Thickness:	0.508mm
Length:	1.0m

Liquid Line

Outer Diameter:	5.54mm
Wall Thickness:	0.508mm
Length:	1.2264m (incl. bayonet)

Condenser

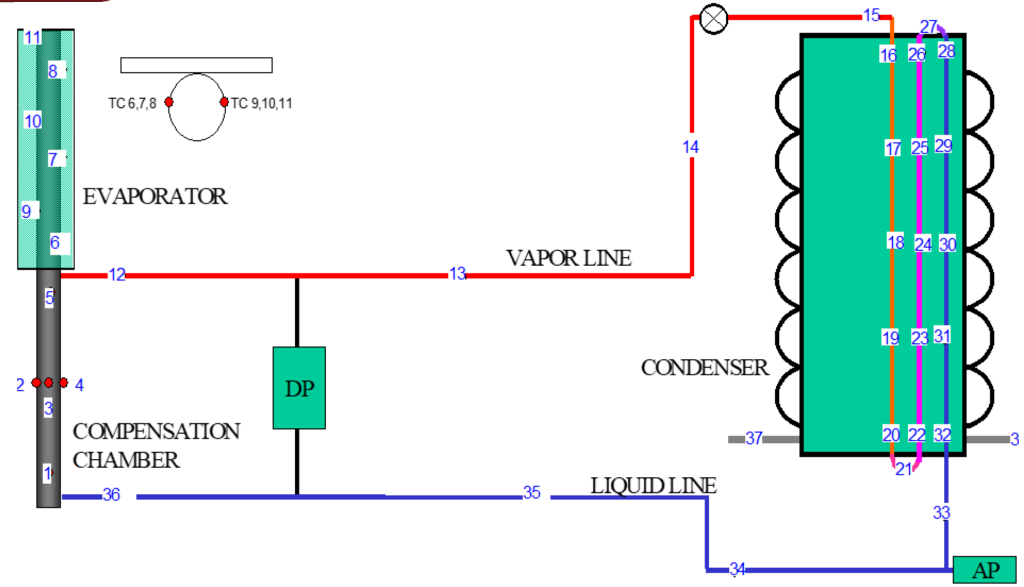
Number of Parallel Passes	1
Heat Exchanger Tubing	
Inner Diameter:	3.99mm
Length:	3.81m (200")
Conductance $G_C^{(MAX)}$:	25W/K

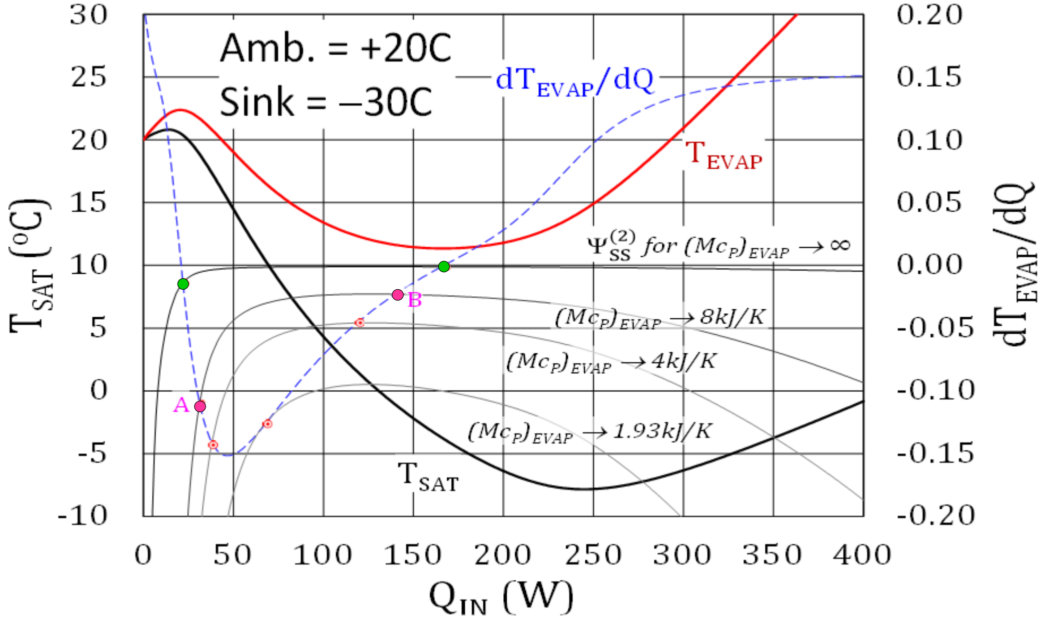
Reservoir

Outer Diameter:	43.94mm
Wall Thickness:	2.20mm
Active Length:	0.08023m
Thermal Mass $(Mc_p)_R$:	190J/K
Conductance G_R:	22W/K

Linear Stability Theory has been verified against test data from various LHPs

Previous Research at U.S. Naval Research Laboratory





Physical Dimensions and Properties of NRL LHP

Evaporator

Primary Wick

Material:	Sintered Powder Nickel
Outer Diameter:	24.21mm (0.950”)
Inner Diameter:	9.525mm (0.375”)
Active Length:	0.3048m (12”)
Max. Pore Radius:	1.3µm
Permeability:	$1.3x10^{-14} m^2$
Effective Conductivity:	7.80W/m-K

Casing/Saddle, 1st Wick, and Attached Thermal Mass

Thermal Mass of Heater Plate + Saddle + Casing:	~~1,575J/K~~ → **8kJ/K**
Thermal Mass-to-Vapor Conductance G_E:	35.80 W/K

Vapor Grooves

Number of Channels:	4
Hydraulic Diameter:	0.05”

Transport Lines

Vapor Line

Outer Diameter:	4.76mm
Wall Thickness:	0.508mm
Length:	1.524m

Liquid Line

Outer Diameter:	4.76mm
Wall Thickness:	0.508mm
Length:	1.96m (incl. bayonet)

Condenser

Number of Parallel Passes	1
Heat Exchanger Tubing	
Inner Diameter:	3.744mm
Length:	2.032m (80”)
Conductance $G_C^{(MAX)}$:	12.00W/K

Reservoir

Outer Diameter:	25.4mm
Wall Thickness:	1.27mm
Active Length:	0.127m
Thermal Mass $(Mc_P)_R$:	135.80J/K
Conductance G_R:	16.50W/K

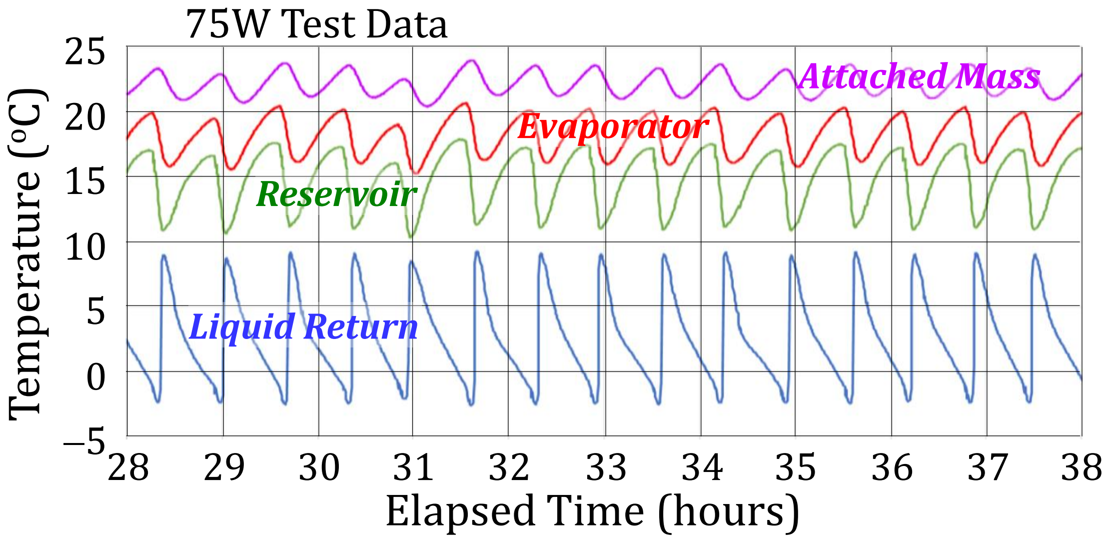
No LFHA Oscillation with Thermal Mass < 1.93kJ/K

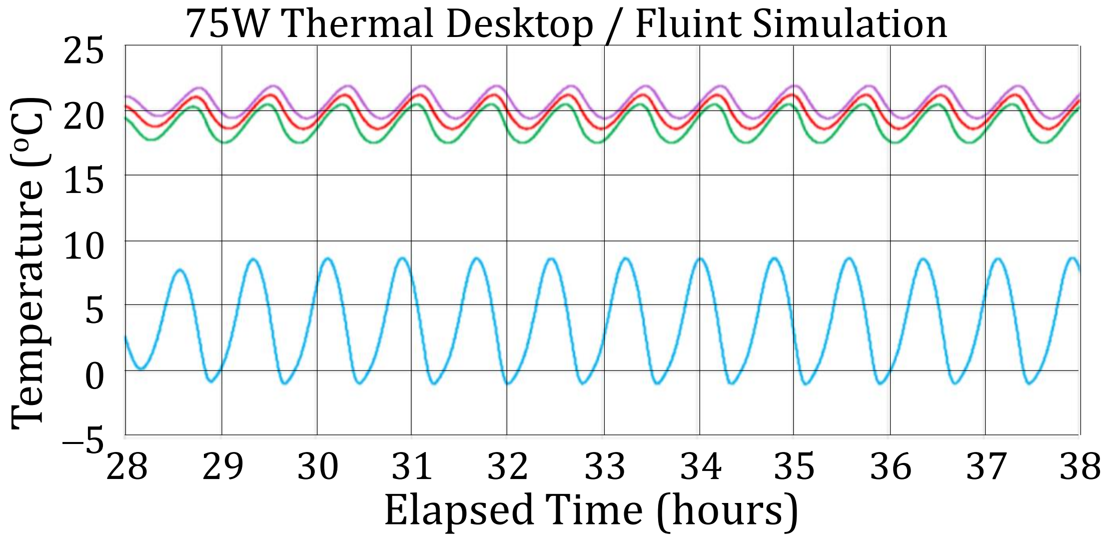
LFHA Self-Excited Oscillation Regimes

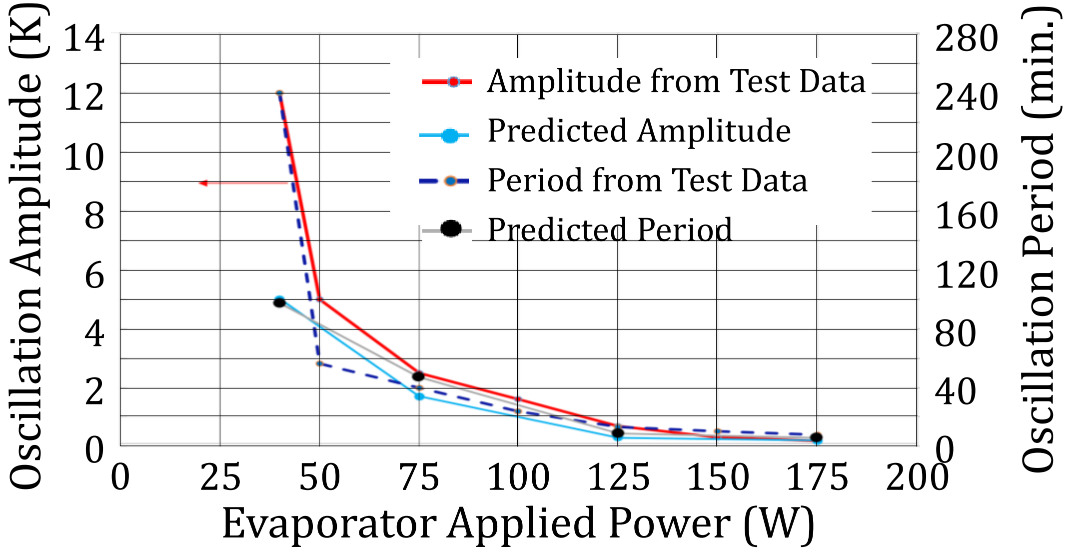
20W – 165W with Attached Thermal Mass → ∞

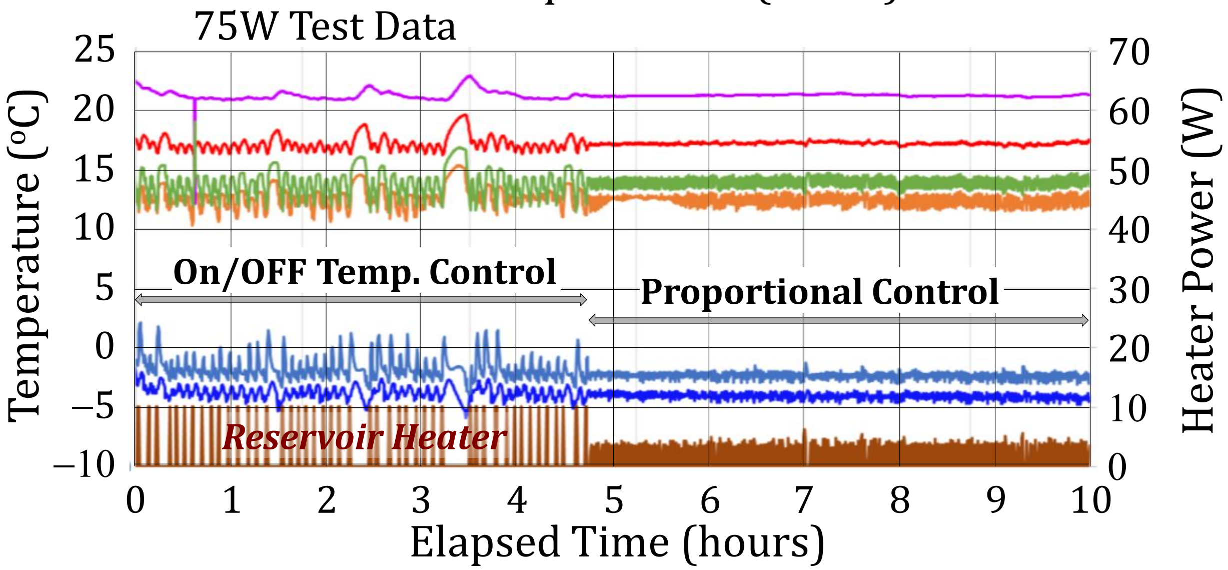
30W – 145W with Attached Thermal Mass = 8kJ/K

Attached Thermal Mass = 13kJ/K and Sink Temps. = –10°C









Scope of Current Research

- Primary Objectives
  - “deep dive” into dynamical system behaviors of LHP thermal-fluid interaction in all plausible operating scenarios / regimes
  - develop accurate / efficient analytical tools to predict LHP system performance for large number of conditions (even in uncharted territories)
  - eliminate / mitigate ill effects of LHP (temperature) oscillations
- Technical Approach
  - simplify governing equations by exploiting unique characteristics of problems at hand
  - select suitable solution scheme(s) that are numerically stable and runtime efficient
  - conduct *far-reaching* analytical investigation of LHP performance in relevant conditions
  - search for sensible methods to mitigate / control LHP oscillations

Fluid "Dynamical" System

$$\frac{d\bar{V}_C^{(2\phi)}}{d\bar{t}} = \frac{1}{\bar{\rho}_L}\left(-\bar{\dot{Q}}_C^{(2\phi)} + \bar{\dot{m}}_L\right)$$

$$\tau_2 \frac{d\bar{\dot{m}}_L}{d\bar{t}} = \frac{\bar{P}_{LVI} - \bar{P}_{SAT}\left(\bar{T}_{SAT}^{(R)}\right) - \left[\bar{R}_B^{(L)} + \bar{R}_{LL}^{(L)} + \bar{R}_{CL}^{(L)}\left(1 - \frac{\bar{V}_C^{(2\phi)}}{\bar{V}_C}\right)\right]\bar{\dot{m}}_L}{\left[\frac{\bar{L}_B}{\bar{A}_B} + \frac{\bar{L}_{LL}}{\bar{A}_{LL}} + \frac{\bar{L}_{CL}}{\bar{A}_{CL}}\left(1 - \frac{\bar{V}_C^{(2\phi)}}{\bar{V}_C}\right)\right]}$$

$$\tau_3 \frac{d\bar{T}_{SAT}^{(E)}}{d\bar{t}} = \frac{1}{\left(\frac{\partial\bar{\rho}_V}{\partial\bar{T}}\right)_{SAT}\left(\bar{V}_{VL} + \bar{V}_C^{(2\phi)}\right)}\left(\frac{\bar{\dot{Q}}_1 - \bar{\dot{Q}}_C^{(2\phi)}}{\bar{\lambda}} - \bar{\rho}_E^{(V)}\frac{d\bar{V}_C^{(2\phi)}}{d\bar{t}}\right)$$

$$\tau_4 \frac{dT_{SAT}^{(R)}}{dt} = \frac{1}{\left(\frac{\partial\bar{\rho}_V}{\partial\bar{T}}\right)_{SAT}\left(\bar{V}_{LHP}^{(V)} - \bar{V}_{VL} - \bar{V}_C^{(2\phi)}\right)}\left(\frac{-\eta\bar{\dot{Q}}_{SC}^{(MAX)} + \bar{\dot{Q}}_2 + \bar{\dot{Q}}_R^{(W)} + \bar{\dot{Q}}_R^{(L)}}{\bar{\lambda}} + \bar{\rho}_R^{(V)}\frac{d\bar{V}_C^{(2\phi)}}{d\bar{t}}\right)$$

Heat Exchange with Environment

$$\tau_5 \frac{d\bar{T}_W^{(E)}}{d\bar{t}} = \bar{\dot{Q}}_{IN} - \bar{\dot{Q}}_E$$

$$\tau_6 \frac{d\bar{T}_W^{(R)}}{d\bar{t}} = -\bar{\dot{Q}}_R^{(W)} - \bar{G}_{R-\infty}\left(\bar{T}_W^{(R)} - \bar{T}_\infty\right)$$

$$\tau_7 \frac{d\bar{T}_L^{(R)}}{d\bar{t}} = -\bar{\dot{Q}}_R^{(L)} - \bar{G}_{L-\infty}\left(\bar{T}_L^{(R)} - \bar{T}_\infty\right) - (1-\eta)\bar{\dot{Q}}_{SC}^{(MAX)}$$

$$\frac{\partial\bar{h}_F}{\partial\bar{t}} + \bar{\dot{m}}_L\frac{\partial\bar{h}_F}{\partial\bar{\xi}} + \bar{g}_{F-\infty}\left(\bar{T}_F - \bar{T}_\infty\right) = 0$$

Multi-Scale Perturbation Problem

$$\tau_1 = \frac{M_{LHP}^{(F)}\lambda_{@293K}}{G_E \Delta T_{REF}} \qquad \tau_2 = \frac{\frac{2}{(\pi A_{REF})^{1/2}}}{\Delta P_{REF} M_{LHP}^{(F)}} \qquad \tau_3 = \frac{V_{LHP}\Delta T_{REF}\left(\frac{\partial \rho_V}{\partial T}\right)_{@293K}}{M_{LHP}^{(F)}} \qquad \tau_4 = \tau_3$$

$$\tau_5 = \frac{(Mc_P)_E \Delta T_{REF}}{M_{LHP}^{(F)}\lambda_{@293K}} \qquad \tau_6 = \frac{(Mc_P)_R^{(W)}}{M_{LHP}^{(F)}\lambda_{@293K}} \qquad \tau_7 = \frac{(Mc_P)_R^{(L)}}{M_{LHP}^{(F)}\lambda_{@293K}} \qquad \tau_8 = \tau_1$$

$$\frac{d\bar{T}_k}{d\bar{t}} = G_j(\bar{T}_k, \bar{T}_k', \bar{X}_i, \bar{X}_i', \bar{X}_j, \bar{X}_j', \bar{t})$$ for Thermal Nodes k's in time scale t_{REF}

$$\varepsilon\frac{d\bar{X}_i}{d\bar{t}} = F_i(\bar{T}_k, \bar{T}_k', \bar{X}_i, \bar{X}_i', \bar{X}_j, \bar{X}_j', \bar{t})$$ for LHP liquid nodes i's in time scale εt_{REF}

$$\varepsilon^2\frac{d\bar{X}_j}{d\bar{t}} = F_j(\bar{T}_k, \bar{T}_k', \bar{X}_i, \bar{X}_i', \bar{X}_j, \bar{X}_j', \bar{t})$$ for LHP vapor nodes j's in time scale $\varepsilon^2 t_{REF}$

Asymptotic Expansion Series Solutions

$$\bar{t} = \bar{\tau}_0 + \frac{\bar{\tau}_1}{\varepsilon} + \frac{\bar{\tau}_2}{\varepsilon^2} \quad \Rightarrow \quad \frac{d}{d\bar{t}} = \left(\frac{\partial}{\partial\bar{\tau}_0} + \varepsilon\frac{\partial}{\partial\bar{\tau}_1} + \varepsilon^2\frac{\partial}{\partial\bar{\tau}_2}\right)$$

$$\frac{\partial\bar{T}_k^{(0)}}{\partial\bar{\tau}_0} = G_k^{(0)} \qquad F_i^{(0)} = 0 \qquad F_j^{(0)} = 0$$

$$\frac{\partial\bar{T}_k^{(1)}}{\partial\bar{\tau}_0} = G_k^{(1)} - \frac{\partial\bar{T}_k^{(0)}}{\partial\bar{\tau}_1} \qquad \frac{\partial\bar{X}_i^{(0)}}{\partial\bar{\tau}_0} = F_i^{(1)} \qquad F_j^{(1)} = 0$$

$$\frac{\partial\bar{T}_k^{(2)}}{\partial\bar{\tau}_0} = G_k^{(2)} - \frac{\partial\bar{T}_k^{(1)}}{\partial\bar{\tau}_1} - \frac{\partial\bar{T}_k^{(0)}}{\partial\bar{\tau}_2} \qquad \frac{\partial\bar{X}_i^{(1)}}{\partial\bar{\tau}_0} = F_i^{(2)} - \frac{\partial\bar{X}_i^{(0)}}{\partial\bar{\tau}_1} \qquad \frac{\partial\bar{X}_j^{(0)}}{\partial\bar{\tau}_0} = F_j^{(2)}$$

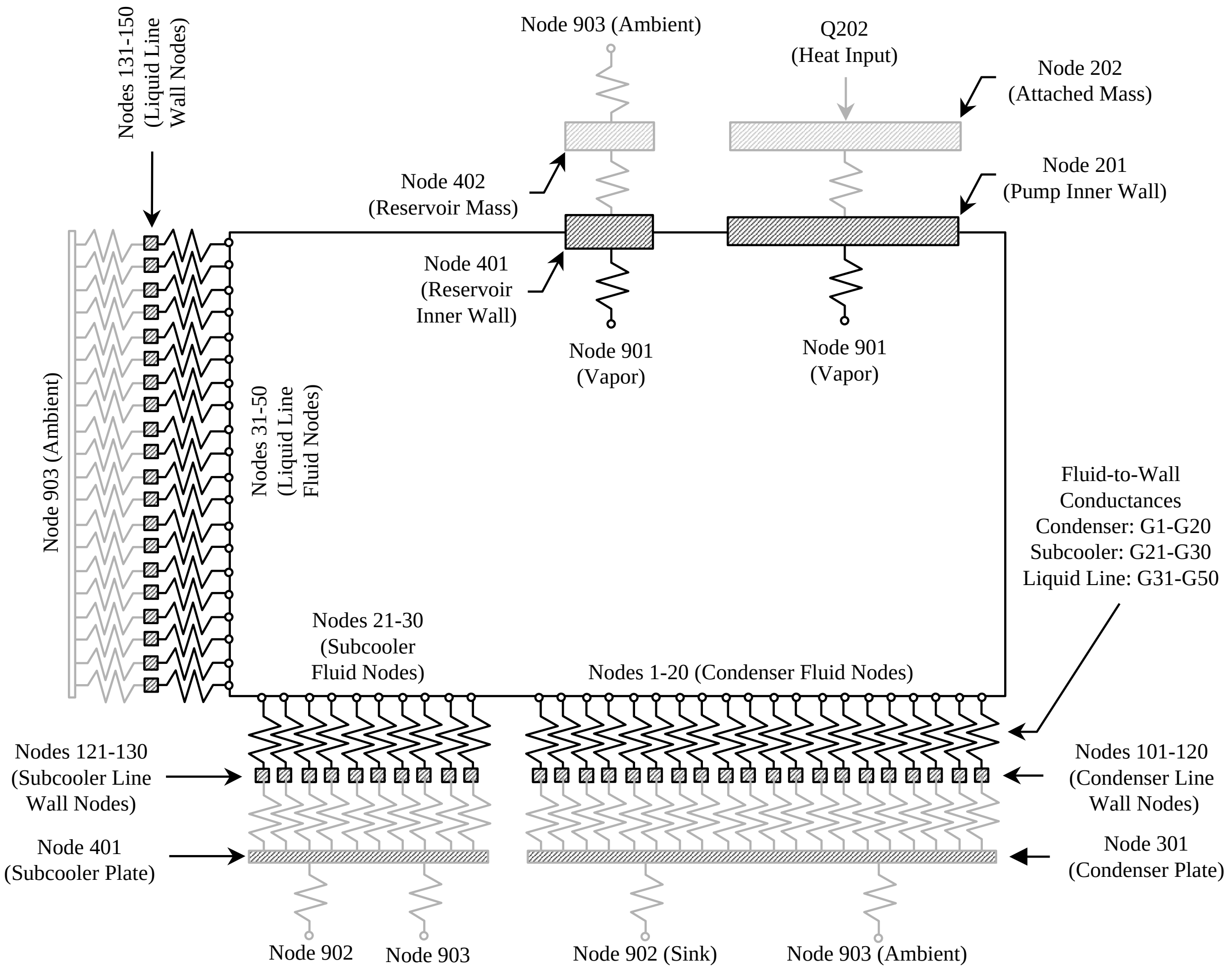
$$\bar{T}_k = \bar{T}_k^{(0)} + \varepsilon\bar{T}_k^{(1)} + \varepsilon^2\bar{T}_k^{(2)} \qquad \bar{T}_k^{(0)}, \bar{T}_k^{(1)}, \bar{T}_k^{(2)} \text{ are all of Order}(1)$$

$$\bar{X}_i = \bar{X}_i^{(0)} + \varepsilon\bar{X}_i^{(1)} + \varepsilon^2\bar{X}_i^{(2)} \qquad \bar{X}_i^{(0)}, \bar{X}_i^{(1)}, \bar{X}_i^{(2)} \text{ are all of Order}(1)$$

$$\bar{X}_j = \bar{X}_j^{(0)} + \varepsilon\bar{X}_j^{(1)} + \varepsilon^2\bar{X}_j^{(2)} \qquad \bar{X}_j^{(0)}, \bar{X}_j^{(1)}, \bar{X}_j^{(2)} \text{ are all of Order}(1)$$

Numerical Method for NRL LHP Transient Modeling

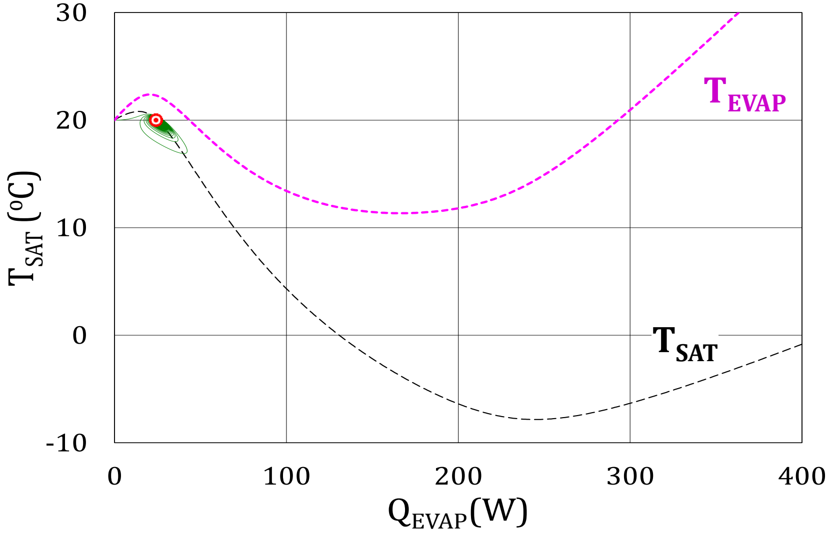
- Adaptive Runge-Kutta-Fehlberg
- Model Assumptions for Simulations
  - incompressible flow (Mach No. < 0.2)
  - no mechanical moving part
  - a successful LHP start-up precedes loop operations to be simulated
  - wicks in working condition
  - single-pass condenser / subcooler
  - no gravity-assist mode of operation
- Computer Code
  - written in BASIC as part of Excel Macro
  - spreadsheets utilized as I/O medium
  - thermophysical properties of working fluid from NIST database



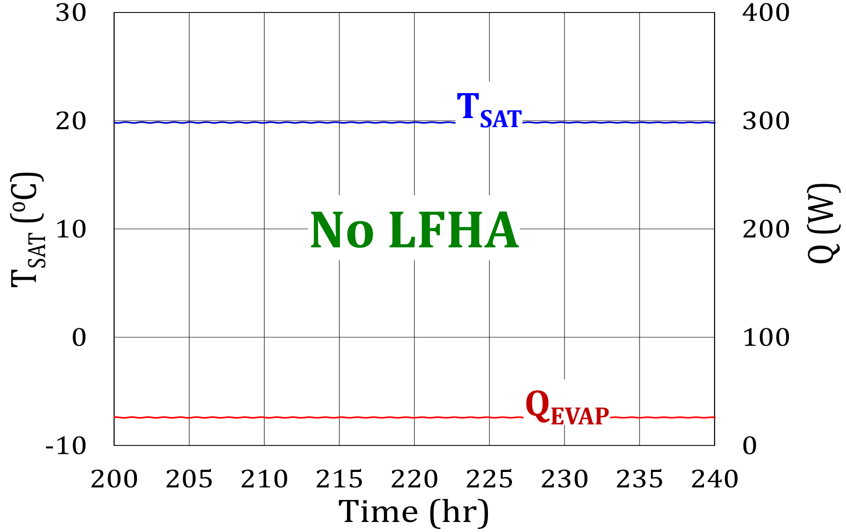
Analysis of Hopf Bifurcations in NRL LHP

Attached Thermal Mass = 8.0kJ/K

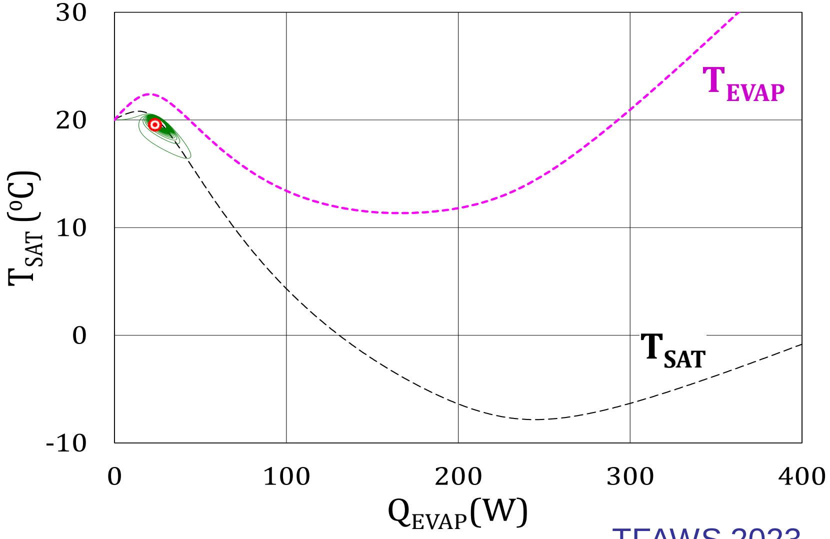
Q_{IN} = 29.9W, T_{AMB} = +20°C & T_{SINK} =–30°C



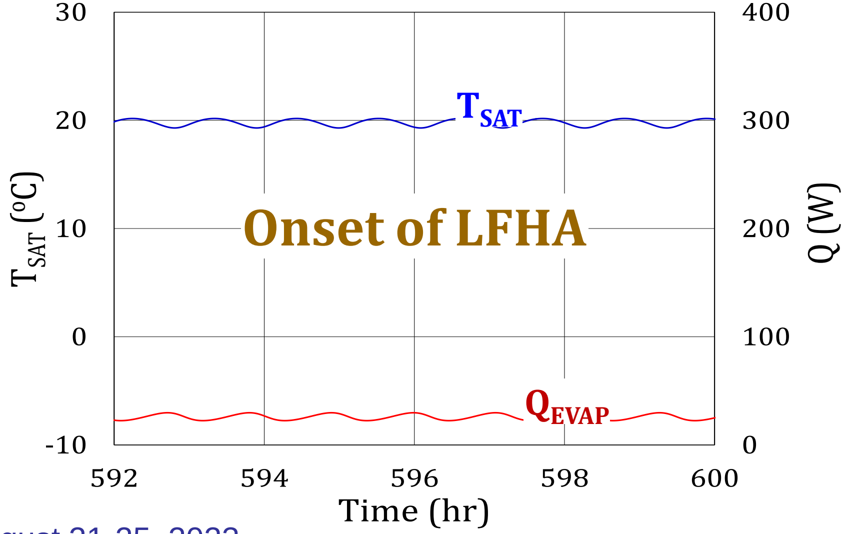
Q_{IN} = 29.9W, T_{AMB} = +20°C & T_{SINK} =–30°C



Q_{IN} = 29.95W, T_{AMB} = +20°C & T_{SINK} =–30°C

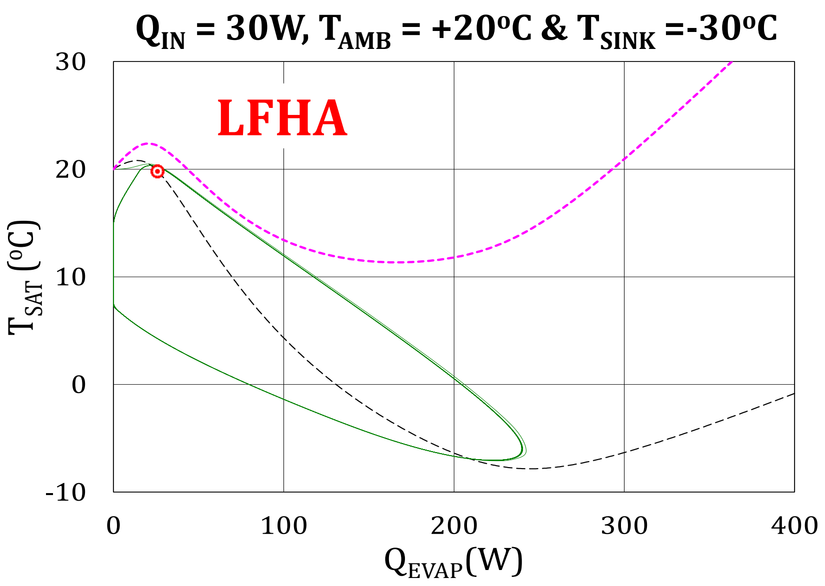


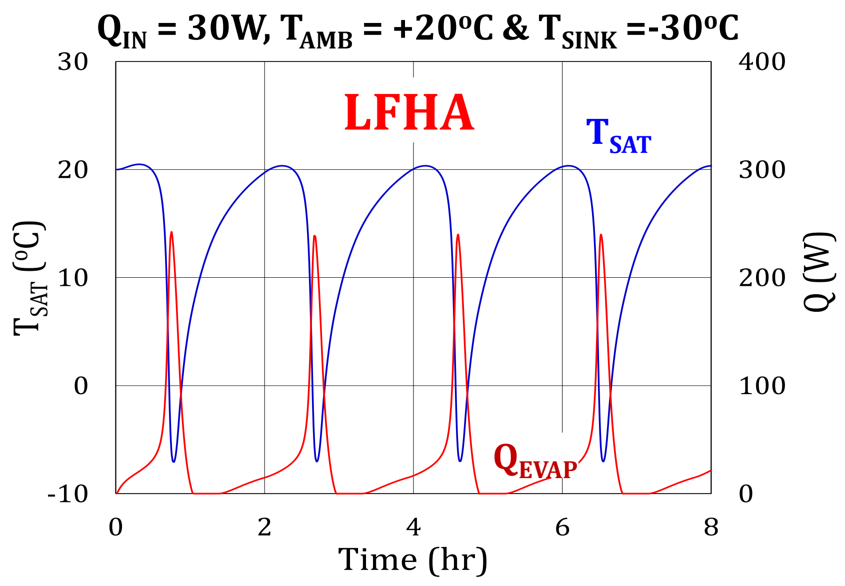
Q_{IN} = 25.95W, T_{AMB} = +20°C & T_{SINK} =–30°C

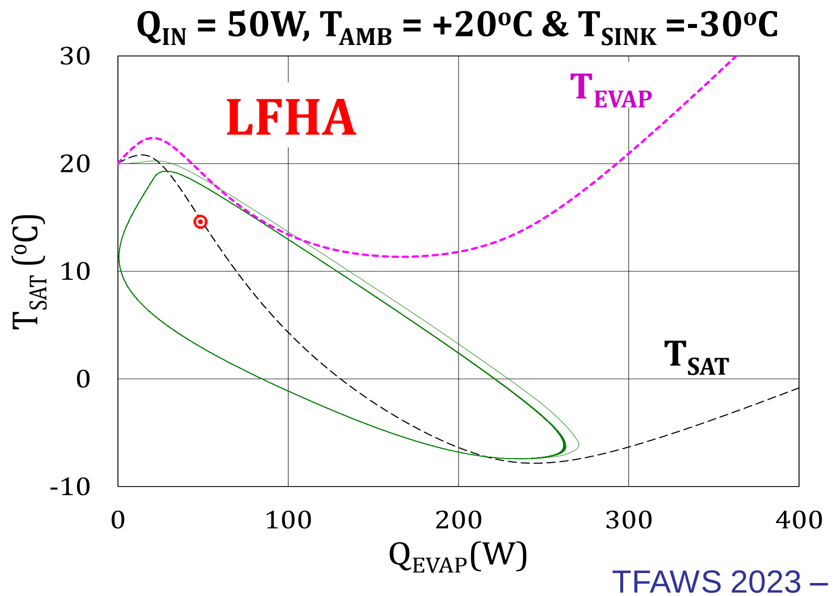


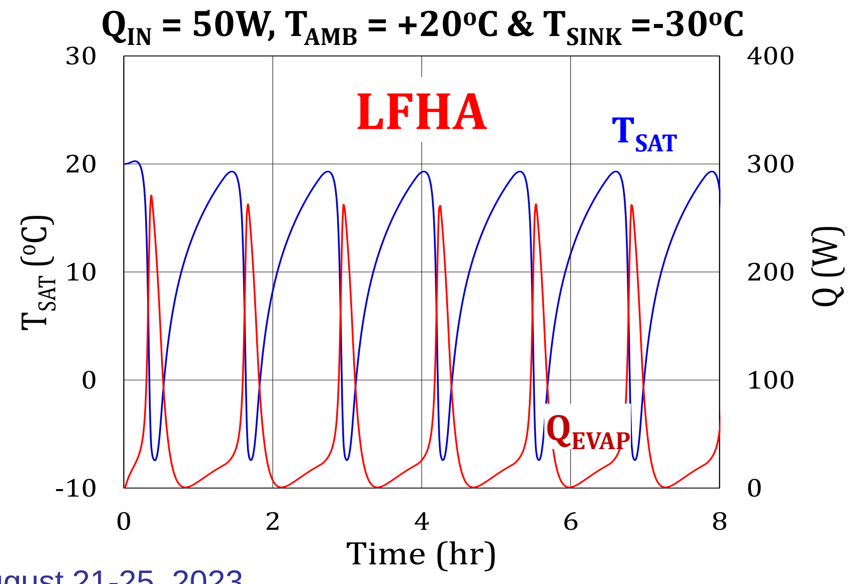
Analysis of Hopf Bifurcations in NRL LHP

Attached Thermal Mass = 8.0kJ/K





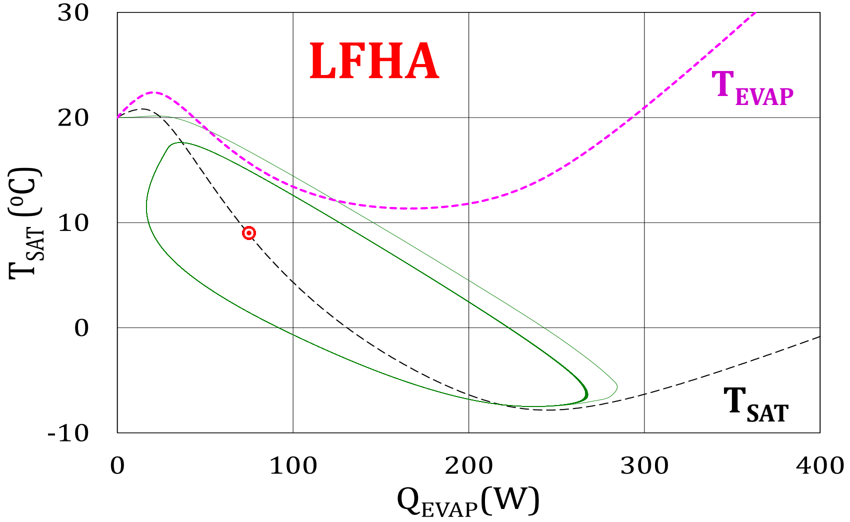




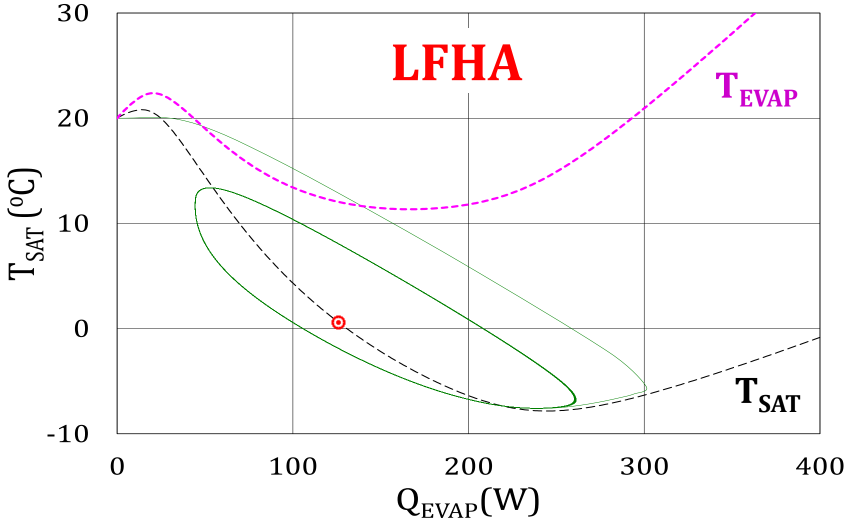
Analysis of Hopf Bifurcations in NRL LHP

Attached Thermal Mass = 8.0kJ/K

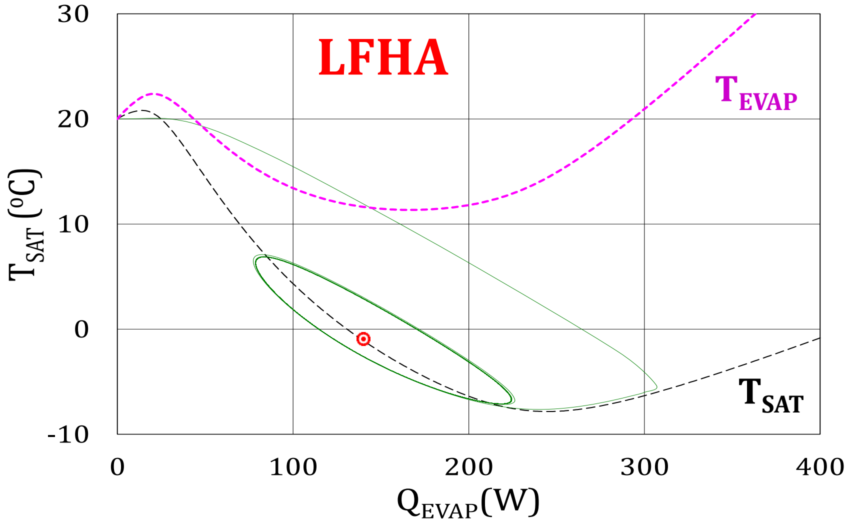
Q_{IN} = 75W, T_{AMB} = +20°C & T_{SINK} =–30°C



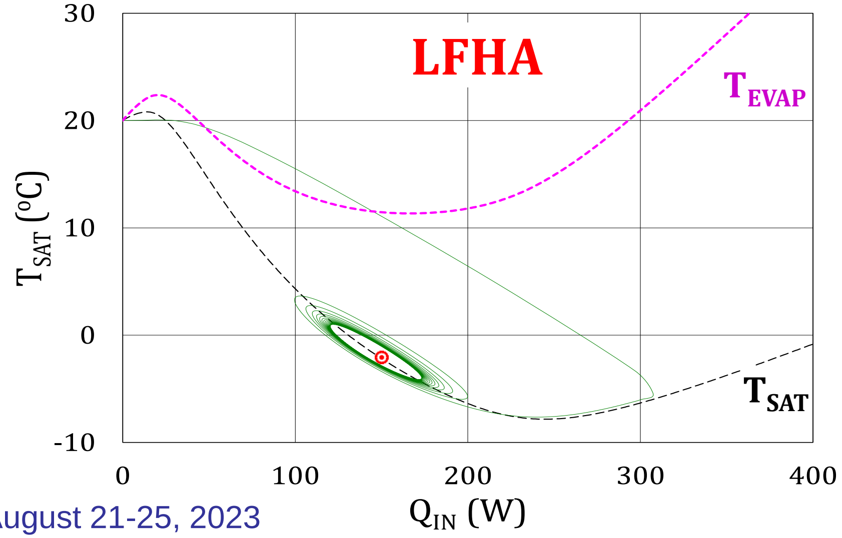
Q_{IN} = 120W, T_{AMB} = +20°C & T_{SINK} =–30°C



Q_{IN} = 140W, T_{AMB} = +20°C & T_{SINK} =–30°C

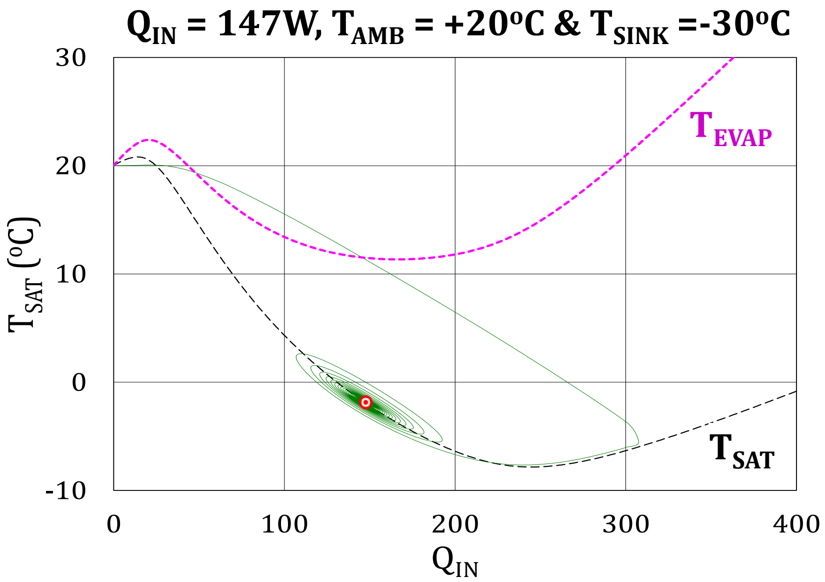


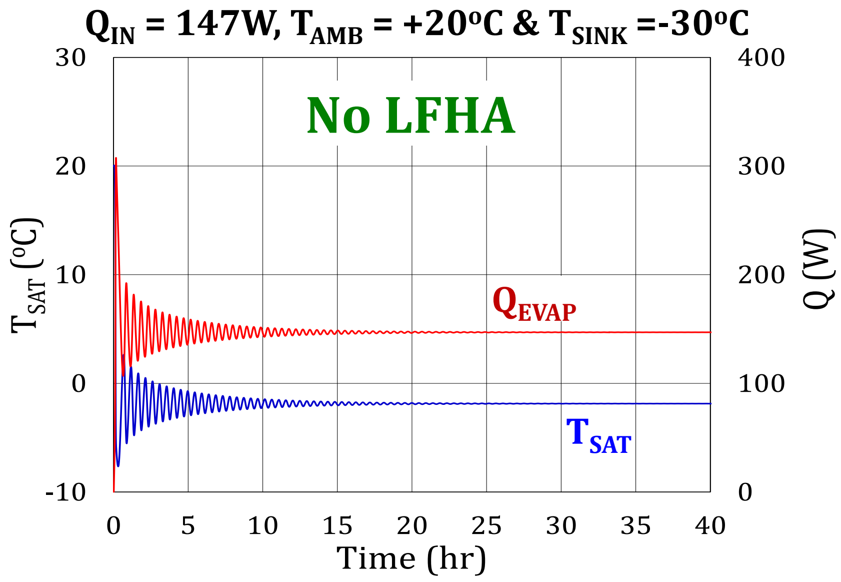
Q_{IN} = 145W, T_{AMB} = +20°C & T_{SINK} =–30°C

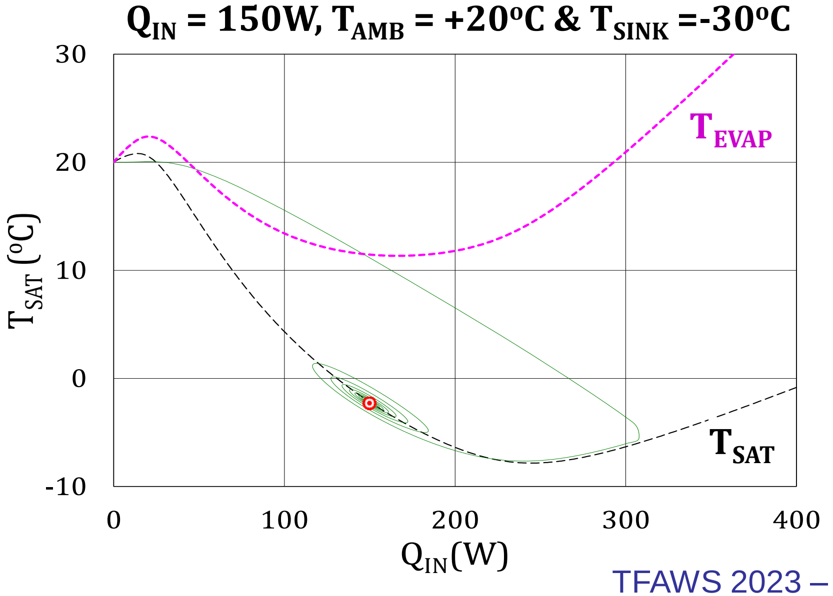


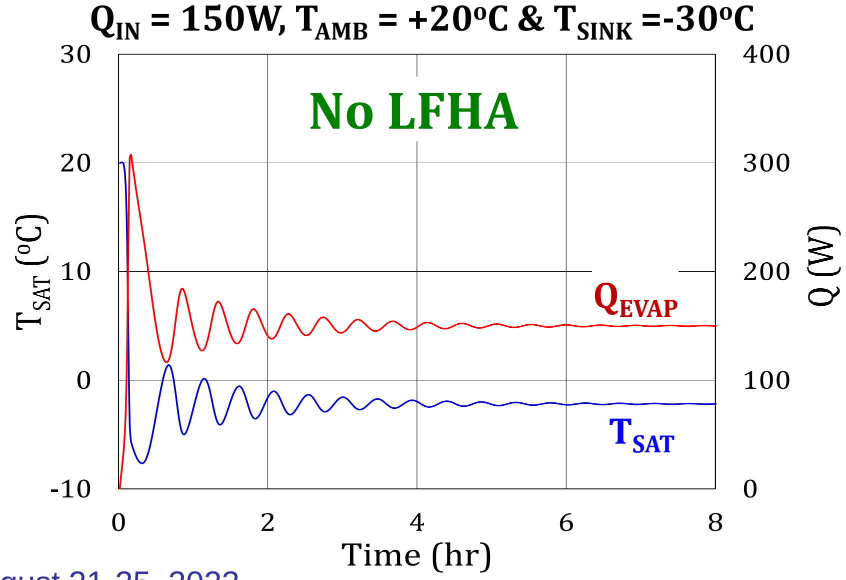
Analysis of Hopf Bifurcations in NRL LHP

Attached Thermal Mass = 8.0kJ/K



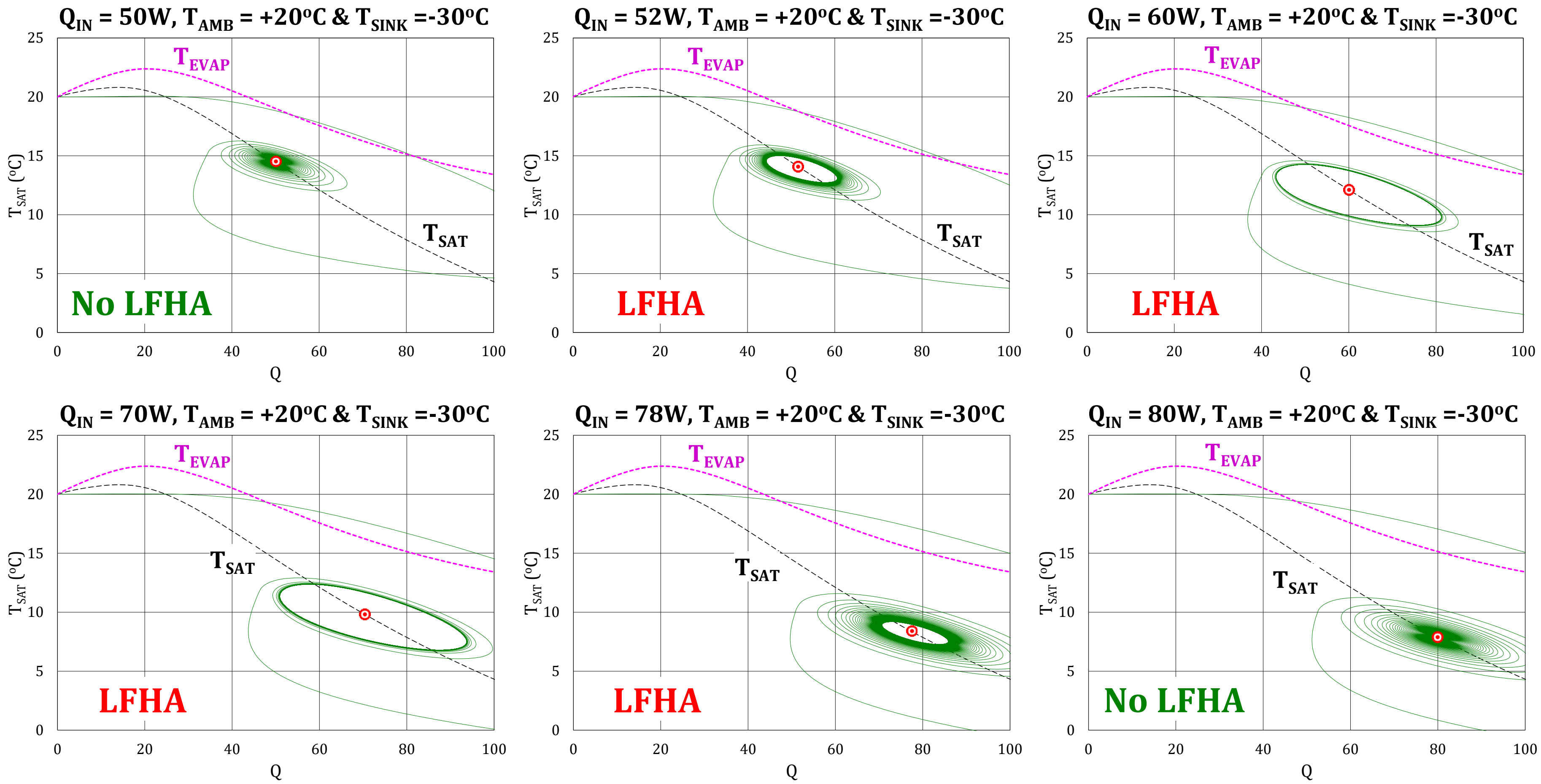






Analysis of Hopf Bifurcations in NRL LHP

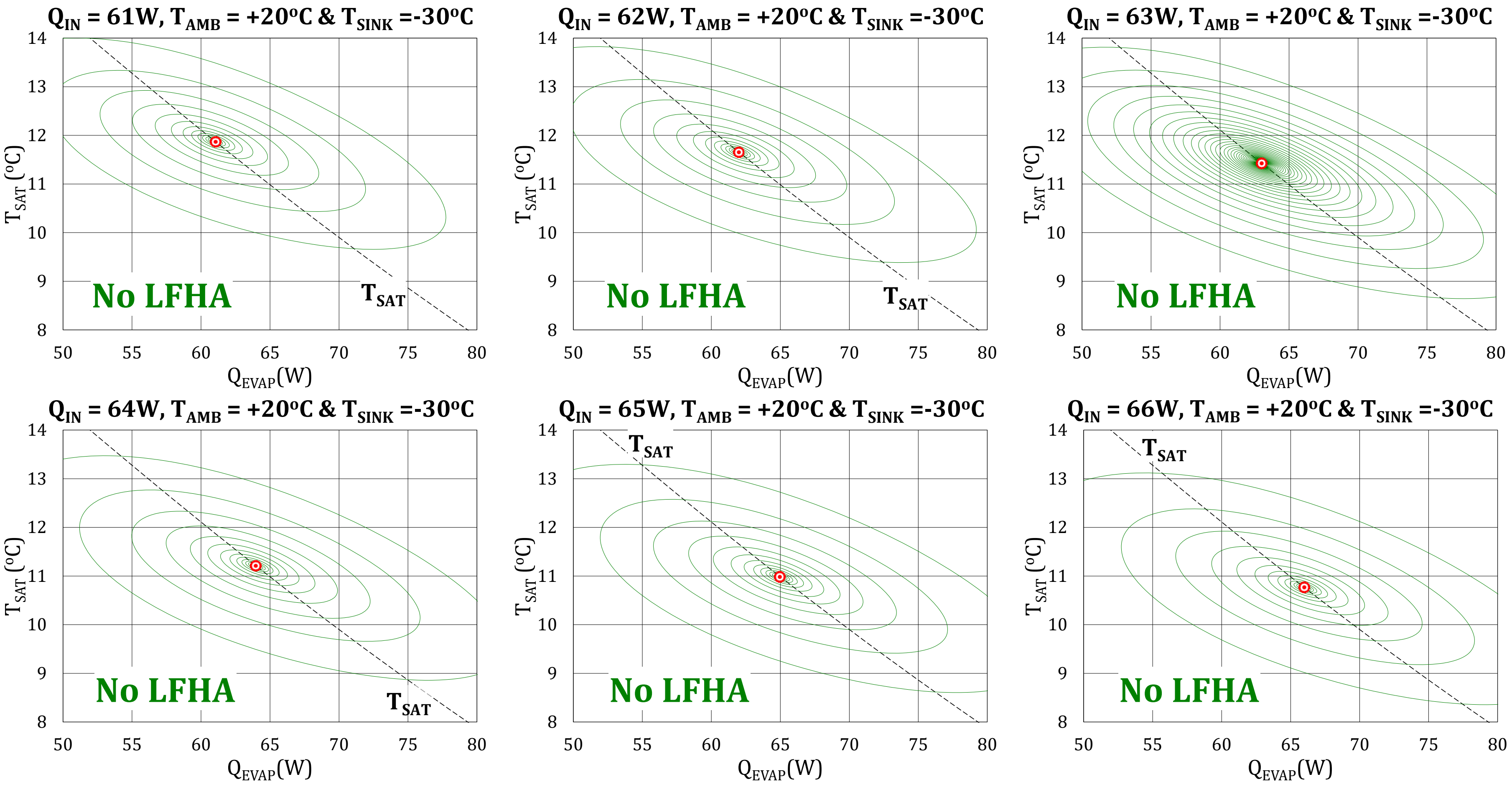
Attached Thermal Mass = 2.0kJ/K



Range of input power for LFHA Oscillations is reduced with decreasing attached thermal mass

Look Ma, No LFHAs!

Attached Thermal Mass = 1.95kJ/K



No LFHA with thermal mass of 1,950 J/K or less

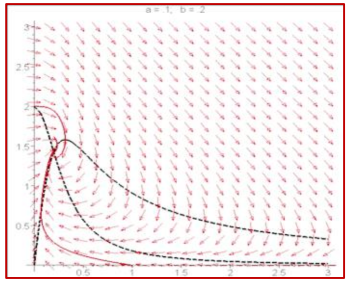
LHP operations exhibit many fundamental characteristics of nonlinear dynamical systems

Selkov Model of Glycolysis

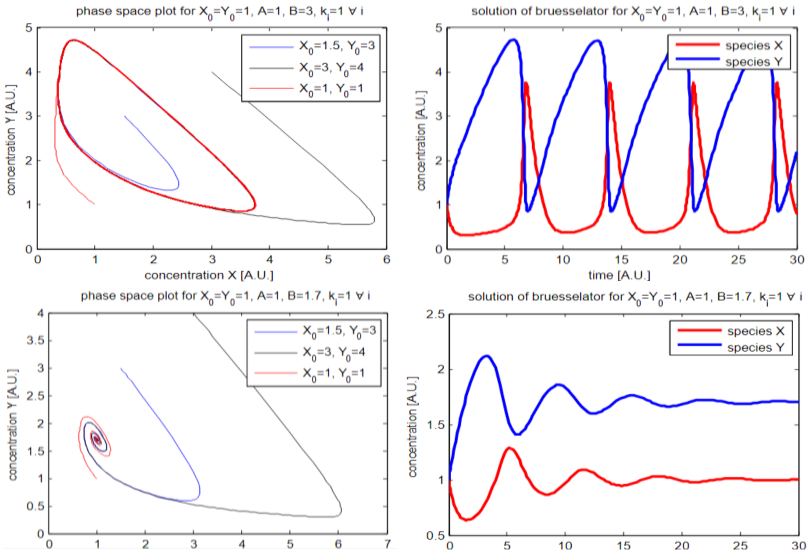
$$\frac{dx}{dt} = -x + ay + x^2 y$$

$$\frac{dy}{dt} = b - ay - x^2 y$$

Hopf bifurcations similar to those of LFHA oscillation In LHP operation



Brusselator Model of Autocatalytic Reaction



Phenomena in other dynamical systems may be drawn upon to study LHP oscillatory behaviors

Summary / Future Works

- Multi-Scale Singular Perturbation Method
  - inspired by *Prandtl's Boundary Layer Theory* $\Rightarrow$ isolate / separate computational domains into somewhat independent regimes for specific sets of characteristics
  - LHP fluid subsystem has at least two distinct time scales: one for vapor dynamics and one for liquid counterparts (*in addition to those of thermal environment*)
  - modeling of LHPs is perhaps NOT as intimidating as thought
- LHP Dynamical System Analysis
  - verified Hopf Bifurcations in LHP operation
  - characteristics almost identical to those of many other systems $\Rightarrow$ leveraged to serve as roadmap for future research
- Potential operational issues to be investigated
  - externally–forced periodic operating conditions $\rightarrow$ resonance?
  - multiple LHPs thermally–coupled in Thermal Control System
  - unstable "subcritical" Hopf bifurcation in oscillatory regimes