PENUMBRA HEATING USING THERMAL SOFTWARE SYSTEM (TSS) V17

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ABSTRACT

Typical thermal analysis for spacecraft in Low Earth Orbits (LEO) uses a solar cylinder to represent the demarcation between sunlit orbit positions and full shaded umbra positions. The transition from full solar heating to full shadow is a simple linear interpolation. In LEO, the time crossing from full Sun exposure to complete darkness is on the order of 10 seconds. With such a quick transition, a linear interpolation is a good choice. The penumbra crossing time for GEO is on the order of a few minutes while an orbit at the Moon's distance takes about an hour. Hence, the farther out the orbit more time is spent during the transition that is not a linear change in heating. Thermal Software System v17 can be used to determine penumbra transition heating. The Orbit application calculates the spacecraft orbital position for penumbra entry and exit with a set of solar cones and allows the user to specify how many points to calculate heating within the penumbra. The Heatrate application can calculate the penumbra heating while in this partially shaded region. As High Earth Orbit (HEO) becomes more and more important the need for accurate penumbra calculations, given its longer duration, is required. Examples of non-linear transition heating at LEO, GEO, and Moon's orbit will be given for the penumbra.

ACRONYMS

GEO	Geosynchronous Equatorial Orbit
HEO	High Earth Orbit (Moon’s orbital distance)
hh:mm:ss	hours : minutes : seconds
LEO	Low Earth Orbit
mm:ss.ss	minutes : seconds . fraction of second
TSS	Thermal Software System
UT	Universal Time

INTRODUCTION

Typical thermal analysis for spacecraft uses a solar cylinder to represent the demarcation between sunlit orbit positions and full shaded umbra positions. A solar cylinder is generated by using the planet diameter and a center line along the vector between the Earth's and Sun's center points. The shadow side, or side opposite of the Sun illumination, is defined as the solar shadow cylinder, see Figure 1 (a). The entrance into the umbra shadow is the intersection between the orbital path, moving along the positive velocity vector, and the solar shadow cylinder. The solar shadow cylinder is a good approximation of the actual eclipse mechanism and has been used for many years to quickly approximate solar heating. During the transition, at the solar shadow cylinder from full solar heating to full shadow, a simple linear interpolation is used. In Low Earth Orbit (LEO), the time crossing from full Sun exposure to complete darkness is on the order of 10 seconds. With such a quick transition compared to 1 ½ hours for an entire orbit, a linear interpolation is a good choice. It should also be noted that the same process albeit in the reverse order is performed when the orbital path exits the solar shadow cylinder along a positive velocity vector.

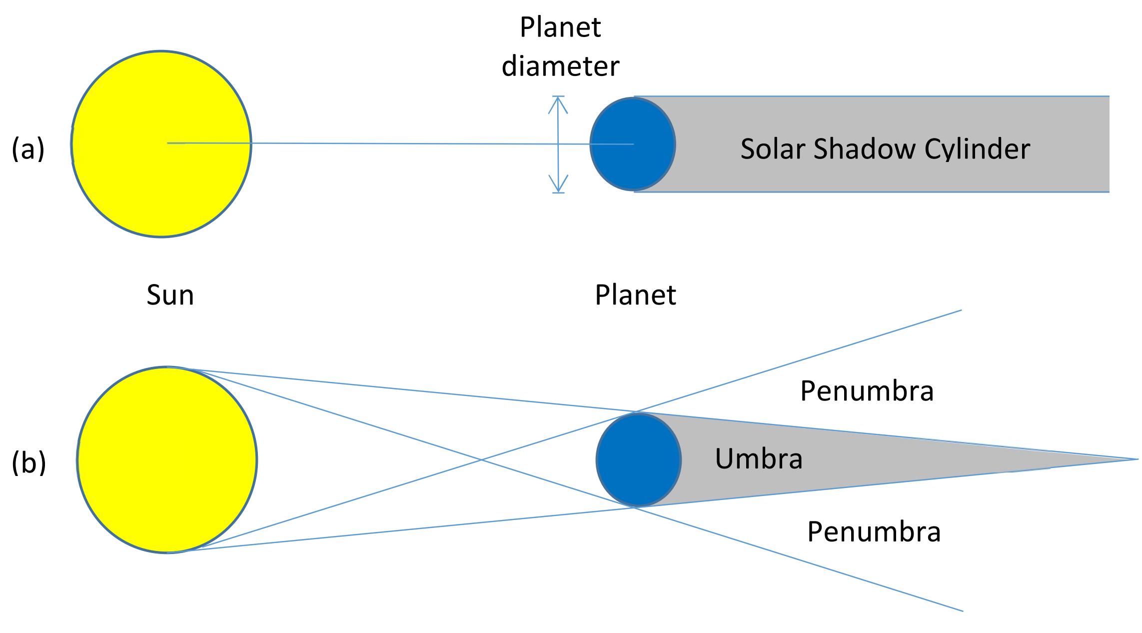

Figure 1. Method to determine Penumbra (partial shadow) and Umbra (full shadow).

For a solar shadow cylinder entry location, the output for solar heating array has two values. The first value in full Sun and a subsequent second value of zero to represent the full shadow. Historically, the time between these two values was user defined and the default was 1 second. Hence, in previous versions of Thermal Software System (TSS), the full Sun time was 0.5 seconds before the solar shadow cylinder and the completely shadowed time was 0.5 seconds after the solar shadow cylinder. The user could adjust the transition time but it was somewhat ad hoc. There was no orbital analysis from TSS to guide the user to determine the proper transition time. The same process in the reverse order is performed when the orbital path exits the solar shadow cylinder, where again, the exit location has two values in the solar heating array.

For a simple 1-meter square, placed in LEO and Sun oriented so the surface normal points directly to the Sun, the solar shadow cylinder heating is shown in Figure 2. The solar heating is linearly interpolated, over one second, at approximate times 1053 seconds and 4939 seconds and the orbit duration is about 6000 seconds. In this simple example the craft begins and ends in the shadow.

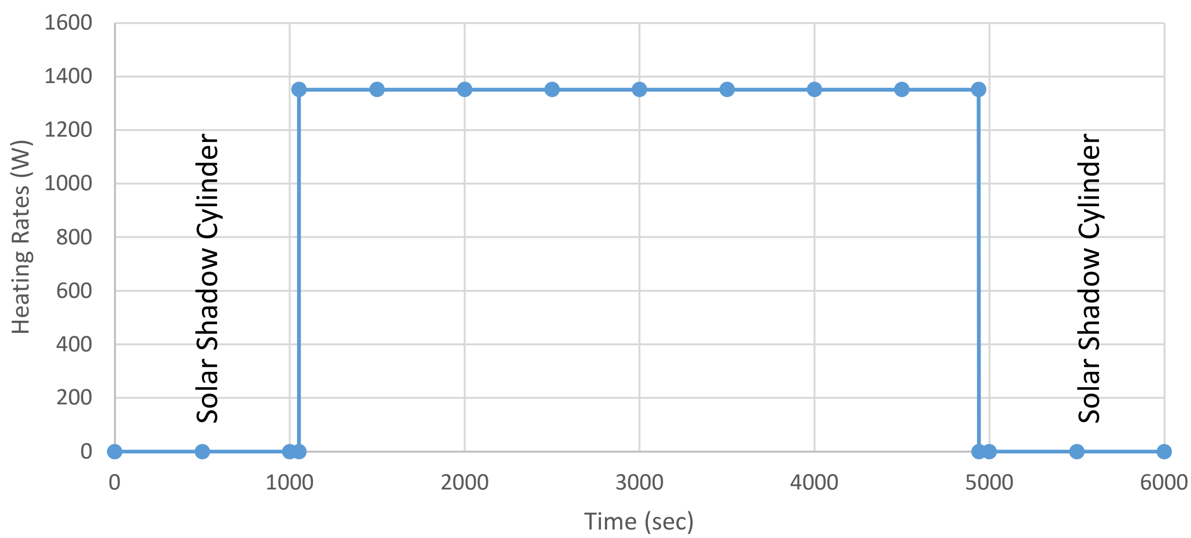

Figure 2. Solar Shadow Cylinder Heating

The raw array data is shown below for a solar shadow cylinder, the time-array #1 is in seconds and the heat-rate-array #2 is in Watts. The arrays show the default 1 second transitions from zero to full Sun near 1053 seconds and full Sun to zero near 4939 seconds.

```
C
C--- TIME-VARIABLE HEAT RATE TABLES
C
    1       =      $ TIME ARRAY
                 0.00000,   499.94619,   999.89239,  1052.76274,  1053.76274,
              1499.83858,  1999.78477,  2499.73097,  2999.67716,  3499.62335,
              3999.56955,  4499.51574,  4938.86930,  4939.86930,  4999.46193,
              5499.40813,  5999.35432
C
    2       =      $ HEAT RATE ARRAY FOR NODE: MAIN.1
                  0.0000,      0.0000,      0.0000,      0.0000,      1351.4
                  1351.4,      1351.4,      1351.4,      1351.4,      1351.4
                  1351.4,      1351.4,      1351.4,      0.0000,      0.0000
                  0.0000,      0.0000
```

Umbra is Latin for “shadow” and paene is Latin for “almost, nearly”; Penumbra is a contraction of paene and umbra meaning nearly shadow.[1] The actual eclipse method uses two cones that describe the umbra and penumbra shown in Figure 1 (b). Notice the stark visual difference between Figure 1 (a) and Figure 1 (b), this difference is over emphasized due the closeness of the two spheres. Figure 3 shows the Earth-Sun umbra cone (purple) and penumbra cone (grey). The two cones extend 30.7 Earth diameters that approximates the distance to the Moon. The Earth and geosynchronous equatorial orbit (GEO) are shown at the lower left in the image to provide scaling.

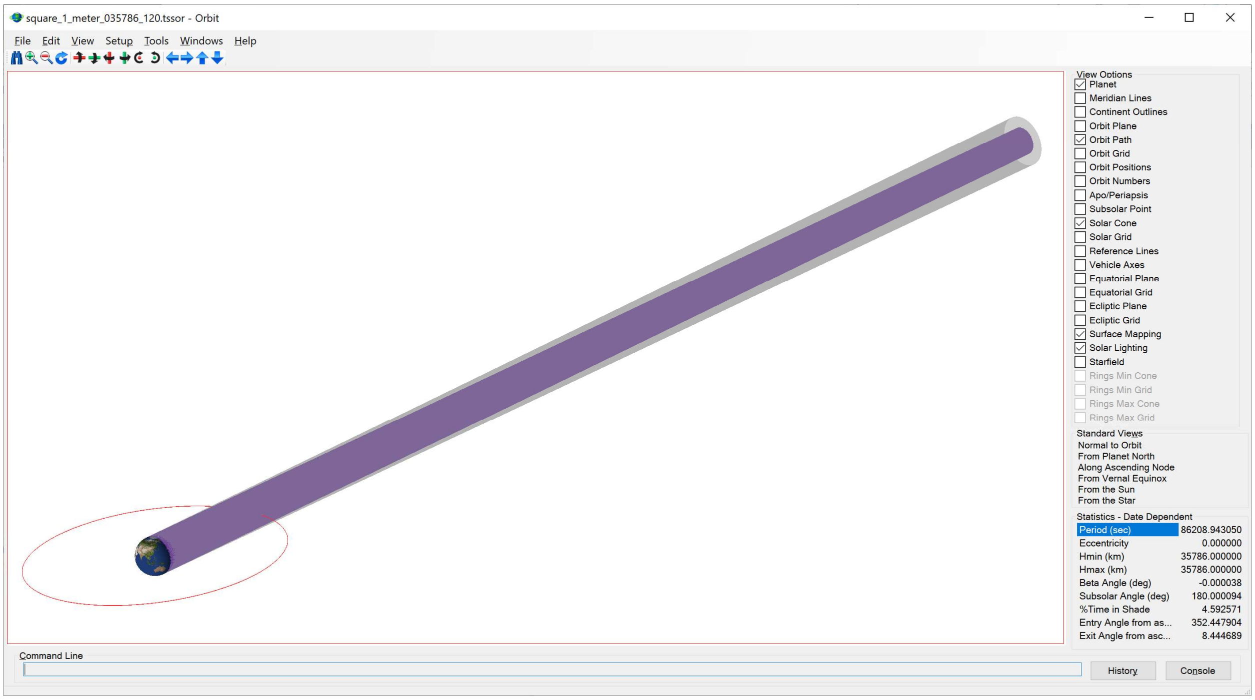

Figure 3. Earth Penumbra (outer cone) and Umbra (inner cone).

METHODS

TSS v17 can be used to determine penumbra transition heating by specifying the number of points during the transition and writing out the time varying heat arrays using the calculated transition times where the orbit intersects the penumbra and umbra (i.e. not user defined or 1 second transition default).

Figure 4 shows the ecliptic plane, equatorial plane, and start location of the test cases. Shown are the Sun (yellow circle) at the center of the image and Earth (blue circle) orbiting the Sun in the ecliptic plane in a counter clockwise direction. The North Pole points up and is perpendicular to the equatorial plane. The North Pole is inclined by 23.4°. The three test cases will begin in shadow at the Autumnal Equinox (September 23, 2023 at 6:49:36 UT) and orbit in Earth’s Equatorial plane. These test cases are circular and at progressively further distances, LEO, GEO, and Moon’s orbital distance (i.e. High Earth Orbit (HEO)), to capture unique results at known distances. The Moon's orbital distance is not the same as the Moon's Orbit. To get consistent results we are staying in the Earth's Equatorial plane, the Moon's Orbit is inclined by 5.145°. The initial starting location is on the vector that passes through the points defined by the Autumnal Equinox, Sun, and Vernal Equinox positions. The starting point for all orbits begins in the umbra shadow.

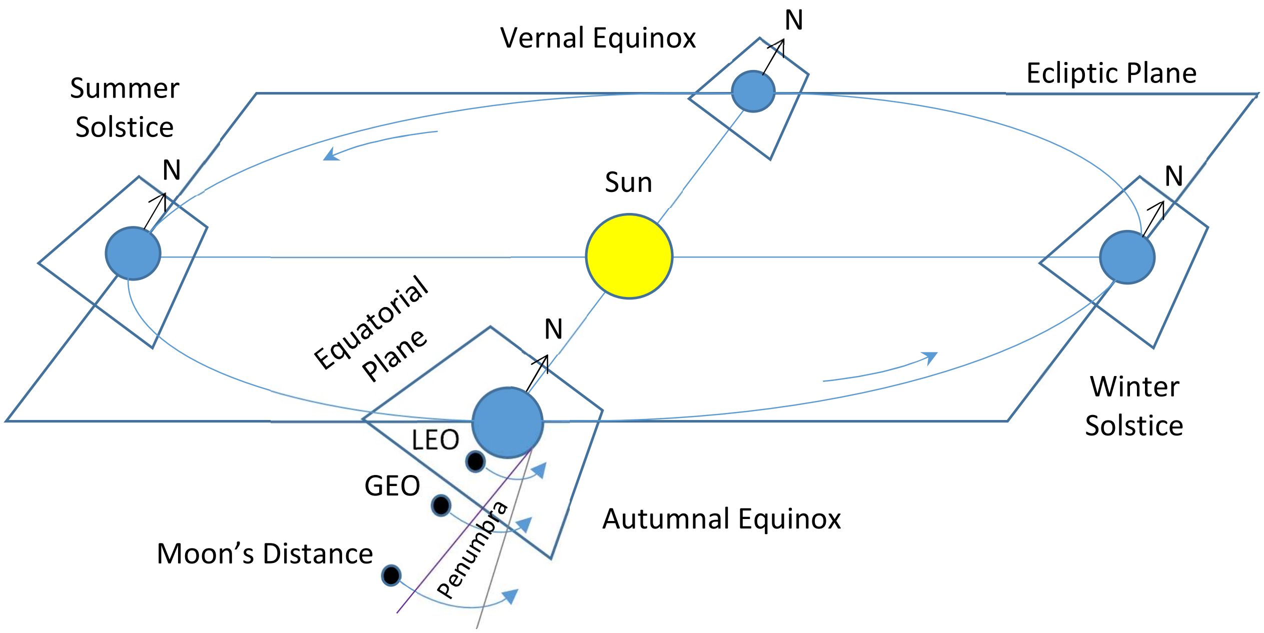

Figure 4. Ecliptic Plane and Equatorial Plane with Start Locations

Table 1 provides penumbra exit crossing times for LEO, GEO, and the Moon's orbital distance. The units for the crossing times are in minutes, seconds, and fraction of seconds. The interaction with the penumbra can be quite complex for other orbits. For example, a satellite can enter and exit the penumbra without entering the umbra.

Table 1. Orbital Penumbra Crossing Time for LEO, GEO, and Moon's Orbital Distance

Orbit type	Circular Orbit Altitude (km)	Penumbra Crossing Time (mm:ss.ss)
LEO	762	00:08.90
GEO	35786	02:08.40
Moon's orbital distance	384399	64:46.15

As discussed earlier, the LEO crossing time is about 10 seconds. The penumbra crossing time for GEO is on the order of a few minutes while an orbit at the Moon's distance takes about an hour. Hence, the farther out the orbit more time is spent during the transition that, as will be shown, is not a linear change in heating. TSS v17 can be used to determine penumbra transition heating. The Orbit application calculates the spacecraft orbital position for penumbra entry and exit with a set of solar cones and allows the user to specify how many points to calculate heating within the penumbra. The Heatrate application can calculate the penumbra heating while in this partially shaded region. As HEO becomes more and more important the need for accurate penumbra calculations, given its longer crossing duration, is required. Examples of non-linear transition heating at LEO, GEO, and Moon's orbit will be given for the penumbra exit crossing.

RESULTS

The Penumbra Crossing Time in minutes is shown for circular orbits in Figure 5, from LEO to the Moon's distance, calculated with TSS's Orbit application.

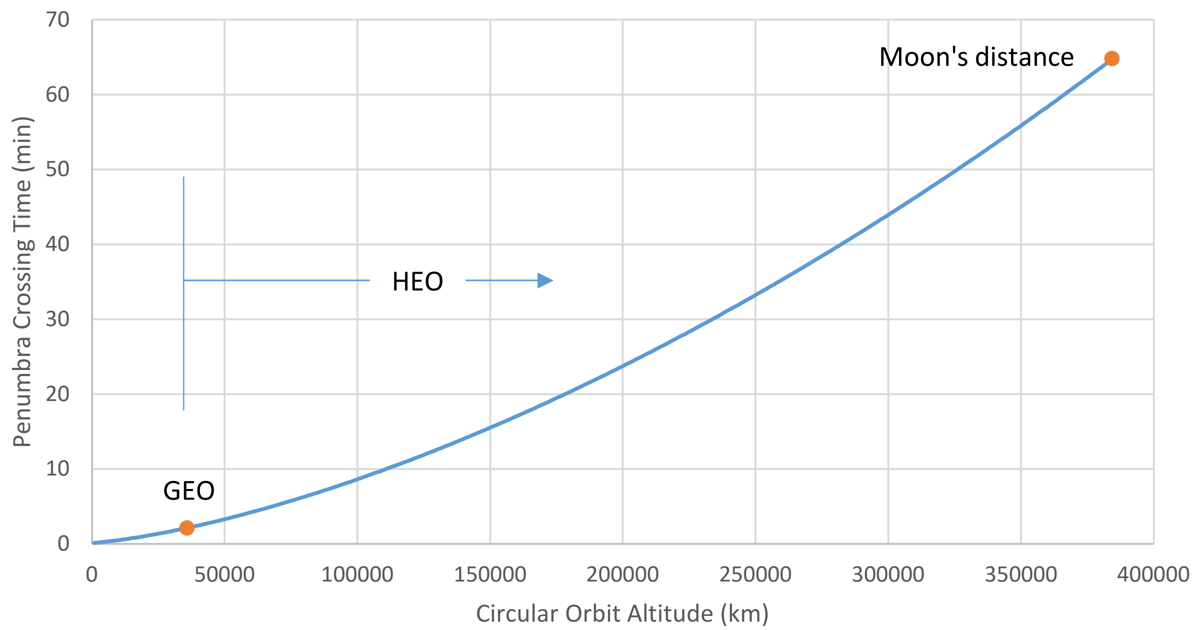

Figure 5. Penumbra Crossing Time vs Altitude for Circular Orbit

The LEO, GEO, and Moon's orbital distance heating for Solar has been calculated using TSS. The exit penumbra begins in full shadow, moves through the penumbra with increasing incident flux, and ends in full Sun. Figure 6 shows the LEO penumbra exit solar flux as the spacecraft moves through the LEO exit penumbra. Figure 7 shows the GEO penumbra solar exit flux as the spacecraft moves through the GEO exit penumbra. Figure 8 shows the Moon's orbit distance penumbra exit solar flux as the spacecraft moves through the exit penumbra. These figures show the Heatrate penumbra calculations (blue) and interpolated values (orange) for the exit penumbra, both in W/m^2, vs. Time, in seconds.

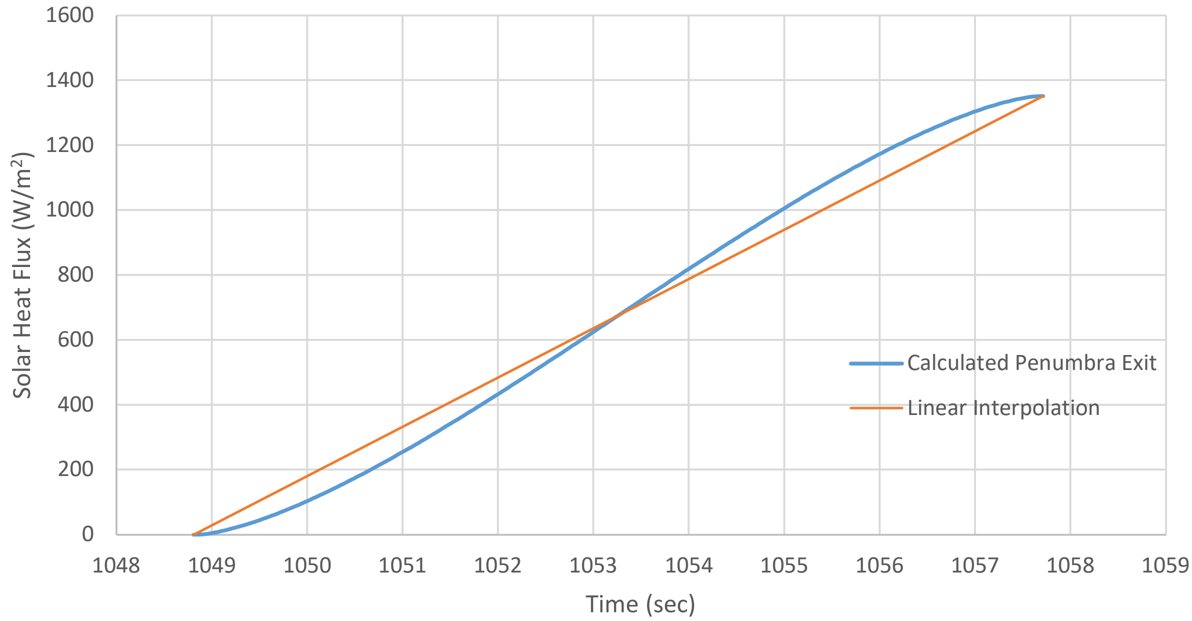

Figure 6. LEO Penumbra Exit Solar Incident Flux.

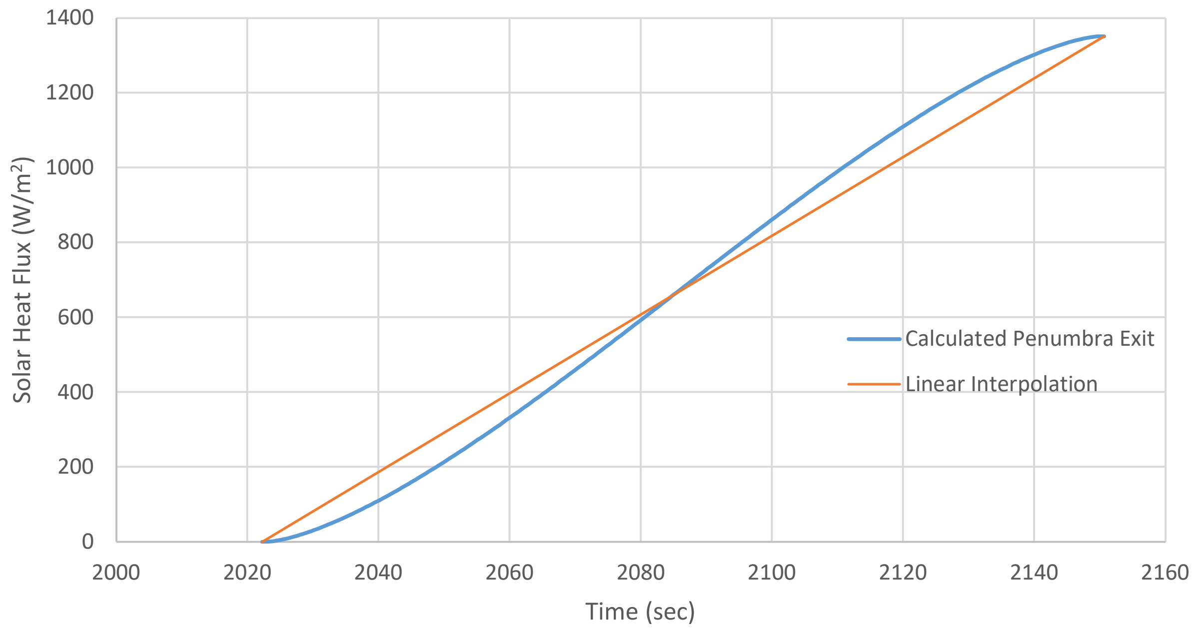

Figure 7. GEO Penumbra Exit Solar Incident Flux.

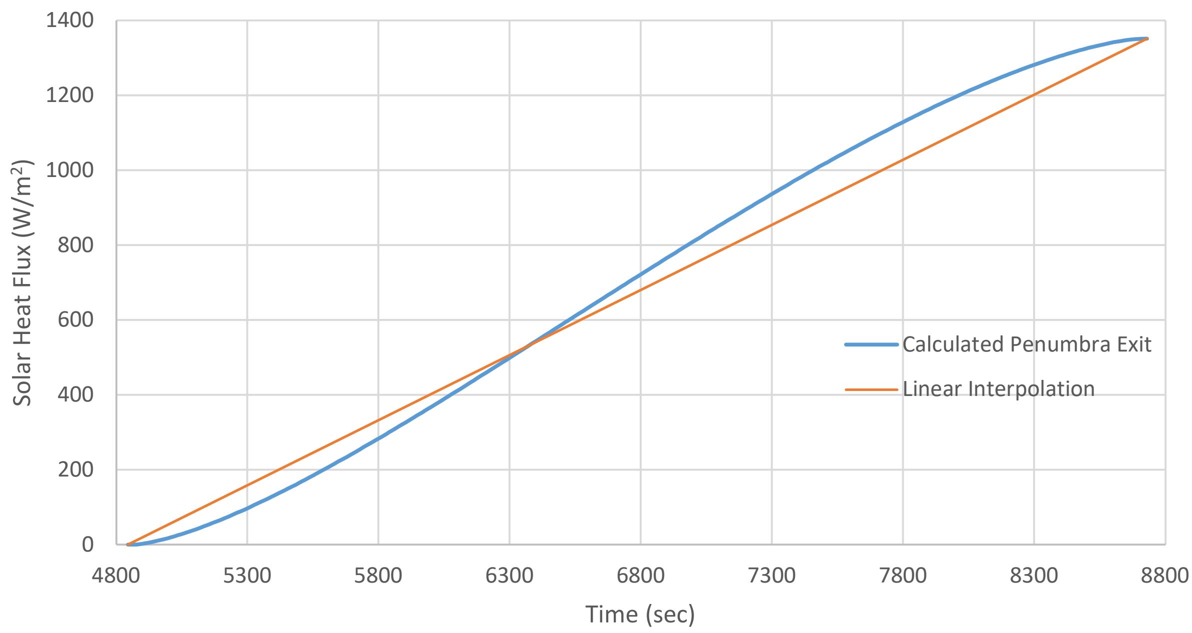

Figure 8. Circular Moon's Orbit Distance Penumbra Exit Solar Flux.

As can be seen by Figs. 6, 7, and 8, the penumbra transition is not linear for LEO, GEO, or Moon's orbit. Table 2 shows the energy integrated under the curve for LEO, GEO, and the Moon's orbit distance, for both penumbra calculations and linear interpolated values. Thereby, giving the total incident energies. The percent difference increases for the penumbra calculations over the linear interpolated case as the circular orbit altitude increases from 0.02% to 3.43%.

Table 2. Integrated Energy for LEO, GEO, and Moon's orbit distance Exiting Penumbra

Circular Orbit	Penumbra Calculations Total Energy (J)	Linear Interpolated Total Energy (J)	Difference (J)	Percent Difference
LEO	**6,017.65**	**6,016.43**	**1.22**	**0.02%**
GEO	**87,090.76**	**86,764.01**	**326.75**	**0.38%**
Moon's distance	**2,719,124.00**	**2,625,924.71**	**93,199.30**	**3.43%**

The increase in total energy is not just increasing with the size of the penumbra region that can be seen in Fig. 3, but is also influenced by the spacecraft speed. The spacecraft speed decreases as the circular orbit altitude increases resulting in increased time spent within the penumbra region. For these orbits, the longer the time spent in the penumbra the larger the total energy.

Linear interpolation in these examples appears to have less energy and the percent difference increases with higher altitude. It is suspected that the opposite result would be found at the penumbra entrance.

DISCUSSION

The results show the incident solar flux transition crossing the penumbra exit, from full shadow to full Sun, is not linear. For comparison, a linear transition is effectively modeled by two squares, shown in Figure 9 (a). Here a smaller square (yellow) is eclipsed by a larger square (blue).   The smaller square being exposed at a constant speed.

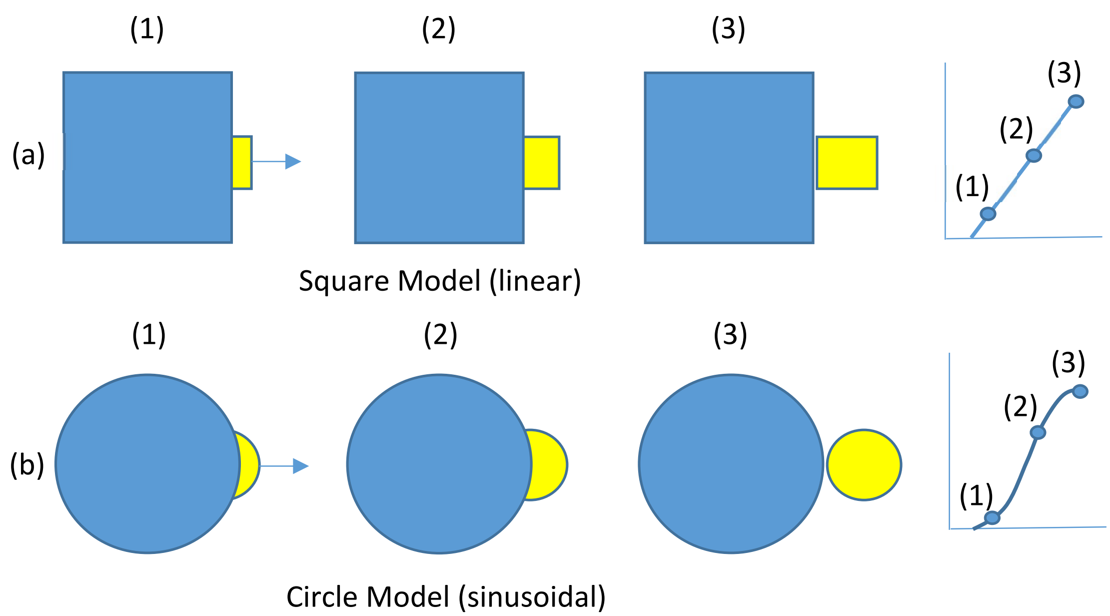

Figure 9. Linear vs. Sinusoidal Transition

Clearly the actual penumbra transition is not represented accurately by two squares but, as previously mentioned, it does closely approximate the total energy. And the view to the square increases linearly.

A “more” accurate model for the circular-orbit penumbra calculations (independent from the “much more” accurate 3D TSS ray tracing method) is shown in Figure 9 (b). Here a smaller circle (yellow) is eclipsed by a larger circle (blue). The smaller circle being exposed at a constant speed.

The mathematics to describe these two methods, square and circle, are now presented. The linear transition heat flux, $\dot{Q}_{linear}$, that is a function of time, t, is shown in Equation 1.

$$\dot{Q}_{linear}(t) = \dot{Q}_{max}\left(\frac{t-t_{start}}{t_{end}-t_{start}}\right) \quad (1)$$

Where $\dot{Q}_{max}$ is the solar flux after the transition (i.e. maximum solar flux), t_{start} is the start time, and t_{end} is the end time of the penumbra transition. The sinusoidal transition heat flux, $\dot{Q}_{sinusoidal}$, is shown by Equation 2 and Equation 3. Note the first term in Eq. 2 is the linear heat flux, Eq. 1.

$$\dot{Q}_{sinusoidal}(t) = \dot{Q}_{linear}(t) + A\sin(\theta(t) + \pi) \quad (2)$$

$$\theta(t) = 2\pi\left(\frac{t-t_{start}}{t_{end}-t_{start}}\right) \quad (3)$$

Where A is the sinusoidal flux amplitude and the sine function is phase shifted by π. θ(t) is the sine angle that ranges from 0 to 2π.

If one knows the sinusoidal flux (from TSS), penumbra entrance and exit times, and the solar flux, the only remaining unknown in Eq. 2 and 3 is the sinusoidal flux amplitude. The amplitude can be found as the maximum absolute value difference between the linear interpolated values and the sinusoidal curve over all times.

A simple test case to determine Amplitude, A, is shown in Figure 10. Here the sinusoidal curve is blue and the linear interpolated line is orange. The maximum amplitude A is equal to 3.0 by taking the absolute value of the difference between the sinusoidal value, 2, and the linear interpolated value, 5, at time 5.

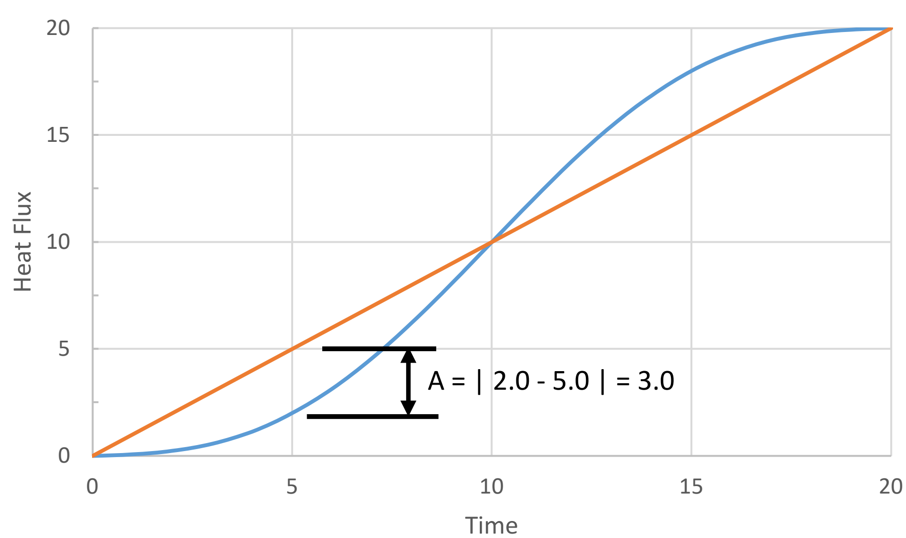

Figure 10 Simple Example of Sinusoidal Amplitude Calculation

For our three circular orbit cases at LEO, GEO, and the Moon's orbit distance, Table 3 shows the calculated values for amplitude A. Again, TSS v17 Orbit and Heatrate use a 3-dimensional calculation method with the Earth as an intervening surface to determine penumbra and umbra shadowing. The square and circle methods are just mathematical descriptions to represent the obtained results and provide an alternate direct calculation method.

Table 3 Sinusoidal Heat Flux for Circular Orbits

Circular Orbit	Sinusoidal Flux Amplitude, A (W/m^2)
LEO	**81.982**
GEO	**83.880**
Moon's orbital distance	**101.085**

It can be argued the larger amplitude for the Moon's orbital distance is the result of the sinusoidal response being skewed by the curvature of the Earth's eclipsing surface. The further out the orbit the more curve the eclipsing surface, in this case the Earth, has making the penumbra exit crossing have more time spent above the linear interpolation. Close in at LEO the curvature of the Earth's eclipsing surface is barely noticeable.

Two final thoughts, the antumbra is a long way out for the Sun-Earth system, approximately 1.4 million kilometers. TSS can calculate antumbra heating that acts much like penumbra except the partial solar is from an annular eclipse.

Lastly, the orbital mechanics in the region near the Moon (outer HEO) is not Keplerian, as discussed here. TSS already has the capability to determine multibody trajectories and future work to combine the umbra and penumbra eclipses with multibody dynamics is underway.

CONCLUSIONS

Thermal Software System v17 can determine penumbra heating that, as shown, is not a linear transition like the historic solar shadow cylinder. Future work will allow multibody calculations to determine penumbra heating for multiple bodies by finding and analyzing eclipses. A general methodology for calculating sinusoidal heat flux for circular orbits has been provided and compared to a linear interpolation for solar shadow cylinder transition.

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CONTACT

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REFERENCES

1. Wikipedia Foundation, Inc. (2023, May 16). *"Umbra, penumbra, and antumbra." Accessed 7 July 2023*. Retrieved July 7, 2023, from, Wikipedia: The Free Encyclopedia: en.wikipedia.org/wiki/Umbra,_penumbra_and_antumbra